

Problem 1 [Gruppen Prob. 2.2]

Assume that in an experiment at the LHC one expects to measure 10 Higgs particles in hundred days of running. Determine the probability of detecting:

- (a) 5 Higgs particles in 100 days.
- (b) 2 Higgs particles in 10 days.
- (a) no Higgs particles in 100 days.

Problem 2 [Knoll Prob. 3.10]

In a given application, a 10-min measurement resulted in a statistical uncertainty of 2.8%. How much additional time must be allocated to reduce the statistical uncertainty to 1.0%?

Problem 3 [Knoll Prob. 3.11]

A designer has the choice in a specific application of either doubling the signal from the source or reducing the background by a factor 2. From the standpoint of counting statistics, which should be chosen under the following conditions:

- (a) The signal is large compared with background.
- (b) The signal is small compared with background.

Problem 4 [Knoll Prob. 3.12]

A flow counter shows an average background rate of 2.87 counts/min. What is the probability that a given 2-min count will contain (a) exactly 5 counts and, (b) at least 1 count? What length of the counting time is required to ensure, with $>99\%$ probability that at least one count is recorded?

Problem 5 [Knoll Prob. 3.13]

The following data are obtained from sources A and B of the same isotope:

		Timing Period
Source A + background	251 counts	5 min
Source B + background	717 counts	2 min
Background	51 counts	10 min

What is the ratio of the activity of source A to source A, and what is the percent standard deviation in this ratio?

Problem 6 [Knoll Prob. 3.14]

The background count from a detector was measured to be 845 over a 30-min period. A source to be measured increases the total counting rate to about 80 counts /min. Estimate the time the source should be counted to determine the counting rate due to the source alone to within a fractional standard deviation of 3%?

Problem 7 [Knoll Prob. 3.15]

Thirty different students have measured the background counting rate with the same apparatus. Each used the same procedure, consisting of recording the number of counts in five 1-min intervals and taking their average. A set of numbers from a typical student is shown below:

25	=	count in first minute
35	=	count in second minute
30	=	count in third minute
23	=	count in fourth minute
27	=	count in fifth minute
total		$\overline{140}$

- Do these data seem reasonable assuming all the fluctuations are statistical? Substantiate your conclusion quantitatively.
- Based on the above data, what is the expected standard deviation of the mean?
- Estimate the sample variance of the 30 numbers representing a similar calculation of the mean background rate by each of the 30 students.
- Again, assuming only statistical variation, estimate the standard deviation of the final answer for the mean obtained by averaging all 30 independent values.

Problem 8 [Knoll Prob. 3.16]

The following set of 25 counts was recorded under identical detector conditions and counting times. Apply the chi-squared test to determine whether the observed fluctuations are consistent with expectations from Poisson statistics.

3626	3711	3677	3678	3465
3731	3617	3630	3624	3574
3572	3572	3615	3652	3601
3689	3578	3605	3595	3540
3625	3569	3591	3636	3629

Problem 9 [Knoll Prob. 3.17]

An average of five sequential 2-min counts of a constant source by Lab Group A gave a resulting value of 2162.4 counts/min. Lab Group B then used the same source and detector in identical conditions and arrived at a value of 2081.5 counts/min based on four sequential 5-min counts. Is the difference between these two results statistically significant?

Problem 10 [Knoll Prob. 3.18]

You are asked to calibrate the activity of a ^{137}Cs gamma-ray source by comparison with a standard ^{137}Cs reference source of approximately the same activity. The standard source has a quoted activity of $3.5 \pm 0.05 \mu\text{Ci}$ and either source alone gives rise to a counter rate of about 1000 per second in the available counter. Background rates are negligible. Assuming that each source is counted separately for equal counting times, how much total time will be required to determine the unknown activity to within 2% expected standard deviation?

Problem 11 [Knoll Prob. 3.19]

A particular counting system has a stable average background rate (measured over a long time) of 50 counts/min. A decaying radioisotope source was introduced and a 10-min count showed a total of 1683 counts. After a delay of 24 h, the 10-min count was repeated, this time giving a total of 914 counts.

- (a) What is the half-life of the source?
- (b) What is the expected standard deviation of the half-life value due to the counting statistics?

Problem 12 [Knoll Prob. 3.24]

A measurement of ^{137}Cs contamination on an air filter is made on a daily basis. The measurement consists of placing the filter in a counting system with an absolute gamma ray counting efficiency of 15% at 662 keV for a period of 30 min. A new uncontaminated filter is then substituted in the counting system and another 30-min background count recorded. The background count averages 100 counts per minute.

- (a) Where should the critical level L_c be set (in units of counts per 30-min measurement) to declare the positive presence of contamination?
- (b) Under these conditions, find the minimum detectable amount (MDA) of ^{137}Cs on the filter.

Problem 13 [Knoll Prob. 3-6]

From the following list, single out those measurements for which the square root of a typical measurement is a proper estimate of the standard deviation:

- (a) A 1-min count from a detector.
- (b) A 5-min count from a detector.
- (c) The net counts from a detector over a 1-min period after background subtraction.
- (d) The counting rate expressed as counts per second based on a 100-s measurement.
- (e) The average of five sequential 1-min count.
- (f) The sum of five sequential 1-min counts.

Problem 14 [Knoll Prob. 3.7]

A source is counted for a 1-min and gives 561 counts. The source is removed and a 1-min background count gives 410 counts. What is the net count due to the source alone and its associated standard deviation?

Problem 15 [Knoll Prob. 3.8]

A 10-min count of a source + background gives a total of 846 counts. Background alone counted for 10 min gives a total of 73 counts. What is the net counting rate due to the source alone and its associated standard deviation?

Problem 16 [Knoll Prob. 3.9]

The measurement described in the previous problem is to be repeated, but in this case the available 20 min is to be subdivided optimally between two separated counts. Find the optimum allocation of time that minimizes the expected standard deviation in the net source counting rate. By what factor has the expected statistical error been reduced from the situation of the previous problems.

Problem 17 [Knoll Prob. 3.1]

A series of 100 measurements of a physical quantity have been made that show a random fluctuation characterized by a sample variance of 2% of the mean value. If the series is lengthened to 1000 measurements made under the same conditions, estimate the sample variance of the larger set of data.

Problem 18 [Knoll Prob. 3.10]

In a given application, a 10-min measurement resulted in a statistical uncertainty of 2.8%. How much additional time must be allocated to reduce the statistical uncertainty to 1.0%?

Problem 19 [Knoll Prob. 3.11]

A designer has the choice in a specific application of either doubling the signal from the source or reducing the background by a factor 2. From the standpoint of counting statistics, which should be chosen under the following conditions:

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The following data are obtained from sources A and B of the same isotope:

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The background count from a detector was measured to be 845 over a 30-min period. A source to be measured increases the total counting rate to about 80 counts /min. Estimate the time the source should be counted to determine the counting rate due to the source alone to within a fractional standard deviation of 3%?

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25	=	count in first minute
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30	=	count in third minute
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27	=	count in fifth minute
total		$\overline{140}$

$$\text{mean} = \frac{140}{5} = 280 \frac{\text{counts}}{\text{min}}.$$

- Do these data seem reasonable assuming all the fluctuations are statistical? Substantiate your conclusion quantitatively.
- Based on the above data, what is the expected standard deviation of the mean?
- Estimate the sample variance of the 30 numbers representing a similar calculation of the mean background rate by each of the 30 students.
- Again, assuming only statistical variation, estimate the standard deviation of the final answer for the mean obtained by averaging all 30 independent values.

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The following set of 25 counts was recorded under identical detector conditions and counting times. Apply the chi-squared test to determine whether the observed fluctuations are consistent with expectations from Poisson statistics.

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An average of five sequential 2-min counts of a constant source by Lab Group A gave a resulting value of 2162.4 counts/min. Lab Group B then used the same source and detector in identical conditions and arrived at a value of 2081.5 counts/min based on four sequential 5-min counts. Is the difference between these two results statistically significant?

Problem 26 [Knoll Prob. 3.18]

You are asked to calibrate the activity of a ^{137}Cs gamma-ray source by comparison with a standard ^{137}Cs reference source of approximately the same activity. The standard source has a quoted activity of $3.5 \pm 0.05 \mu\text{Ci}$ and either source alone gives rise to a counter rate of about 1000 per second in the available counter. Background rates are negligible. Assuming that each source is counted separately for equal counting times, how much total time will be required to determine the unknown activity to within 2% expected standard deviation?

Problem 27 [Knoll Prob. 3.19]

A particular counting system has a stable average background rate (measured over a long time) of 50 counts/min. A decaying radioisotope source was introduced and a 10-min count showed a total of 1683 counts. After a delay of 24 h, the 10-min count was repeated, this time giving a total of 914 counts.

- (a) What is the half-life of the source?
- (b) What is the expected standard deviation of the half-life value due to the counting statistics?

Problem 28 [Knoll Prob. 3.2]

Find the probability that exactly 8 heads will occur in 12 tosses of a coin.

Problem 29 [Knoll Prob. 3.24]

A measurement of ^{137}Cs contamination on an air filter is made on a daily basis. The measurement consists of placing the filter in a counting system with an absolute gamma ray counting efficiency of 15% at 662 keV for a period of 30 min. A new uncontaminated filter is then substituted in the counting system and another 30-min background count recorded. The background count averages 100 counts per minute.

- (a) Where should the critical level L_c be set (in units of counts per 30-min measurement) to declare the positive presence of contamination?
- (b) Under these conditions, find the minimum detectable amount (MDA) of ^{137}Cs on the filter.

Problem 30 [Knoll Prob. 3.3]

A given large population consists of 75% males. Random samples of 15 people are taken from this population, and the number of males tallied for each sample. Find the predicted mean, variance, and standard deviation of the expected results.

Problem 31 [Knoll Prob. 3.4]

Find the probability that no sixes turn up in 10 rolls of a die.

Problem 32 [Knoll Prob. 3.5]

A computer programmer averages one error per 60 program statements.

- (a) Find the mean and standard deviation of the number of errors expected in a 250 statements.
- (b) Find the probability that a 100 statement program is free of errors.

Problem 33 [Knoll Prob. 3-6]

From the following list, single out those measurements for which the square root of a typical measurement is a proper estimate of the standard deviation:

- (a) A 1-min count from a detector.
- (b) A 5-min count from a detector.
- (c) The net counts from a detector over a 1-min period after background subtraction.
- (d) The counting rate expressed as counts per second based on a 100-s measurement.
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Problem 34 [Knoll Prob. 3.7]

A source is counted for a 1-min and gives 561 counts. The source is removed and a 1-min background count gives 410 counts. What is the net count due to the source alone and its associated standard deviation?

Problem 35 [Knoll Prob. 3.8]

A 10-min count of a source + background gives a total of 846 counts. Background alone counted for 10 min gives a total of 73 counts. What is the net counting rate due to the source alone and its associated standard deviation?

Problem 36 [Knoll Prob. 3.9]

The measurement described in the previous problem is to be repeated, but in this case the available 20 min is to be subdivided optimally between two separated counts. Find the optimum allocation of time that minimizes the expected standard deviation in the net source counting rate. By what factor has the expected statistical error been reduced from the situation of the previous problems.

Problem 37 [Poston Prob. 3.1]

The following counts were obtained in successive one-minute runs with a certain detector: 68, 75, 62, 59, 77, 71, 80, 64, 67, 72, 67, 70, 65, 79, 84, 66, 55, 59, 68, 90, 78, 75, 59, 45, 75, 58, 92, 71, 67, 89. Calculate the mean, variance and standard deviation for that sample. Calculate the average count and standard deviation for the first 12 data points. Are they greatly different?

Problem 38 [Poston Prob. 3.2]

A one-minute count of a sample gives a count of 100, whereas a one-minute count of background alone gives a count of 36. What is the standard error of each of the counts?

What is the best estimate, and its associated error for the counts due to the source?

Problem 39 [Poston Prob. 3.3]

How many counts must be recovered to have a probable error (\pm one standard deviation) of 5% of the counts?

Problem 40 [Poston Prob. 3.4]

Repeated measurements have been made on a sample yielding an average count of 880. Within what range should 90% of the measurements fall?

Problem 41 [Poston Prob. 3.5]

In the data series in Problem 3.1:

- (a) Was the counter operating properly? Why do you think so?
- (b) What is the maximum variation in this sequence that would be acceptable; at what point could a result be rejected?
- (c) What should be reported as the sample count at the 95% confidence level?

Problem 42 [Poston Prob. 3.6]

A given sample has count rates of 35 cpm and a background of 5 cpm. How long does the sample need to be counted for a 2% accuracy in the sample activity?

If only one hour is available for a count-rate determination, including background, what is the best attainable accuracy for this sample?

Problem 43 [Poston Prob. 3.7]

^{14}C , half-time 5730 yr, is used to date archaeological samples. In a particular experiment the total count rate was about 10 counts/min and the background 6.4 counts/min. How long would this sample have to be counted to determine its activity to an accuracy of 5%? If the above sample was 10,000 years old, what is the oldest sample whose age could be determined in that detector to any significant accuracy, assuming a maximum counting time of one day per sample?

Problem 44 [Poston Prob. 3.8]

A sample was counted for 10 min and the average count rate was 28 cpm. The background was counted for 20 min and the average count rate was 17 cpm. What is the probable error in the sample activity measured?

Problem 45 [June 2010]

In a laboratory, a measurement of background noise, which will be assumed to be constant for the following, taken for half an hour gave 480 counts. The same day, the measurement of a sample for 15 minutes, gave 350 counts. The measurement of the same sample, the next day, but this time for 10 minutes, gave 204 events.

- (a) What is the net count of this sample for each of the 2 days, normalized to the days, normalized to a duration of 15 minutes

- (b) From this result, can you determine if the sample is greater or less than 1 day?

Problem 46 [June 2011]

The same radiation generates on two detectors A and B

- $N_A = 100$ ion-e pairs
- $N_B = 10000$ ion-e pairs

If the Fano factors are $F_A = 1$ and $F_B = 0.1$, what is the ratio between the energy resolutions of the two detectors?

Problem 47 [June 2012]

In a radioactive installation, the measurement of possible ^{137}Cs contamination in an air filter is performed. The measurement consists of placing the air filter in a counting system with an absolute counting efficiency of 15% for a period of 30 min. The background is estimated by placing an uncontaminated filter immediately afterwards and for a further period of 30 minutes. The measurement of this background noise gives an average value of 100 counts/min.

- Where should the critical level, L_c , be, in units of counts/30 min, to assert that contamination exists?
- Under these conditions, what is the "Minimum Detectable Amount (MDA)" of ^{137}Cs in the system described above?

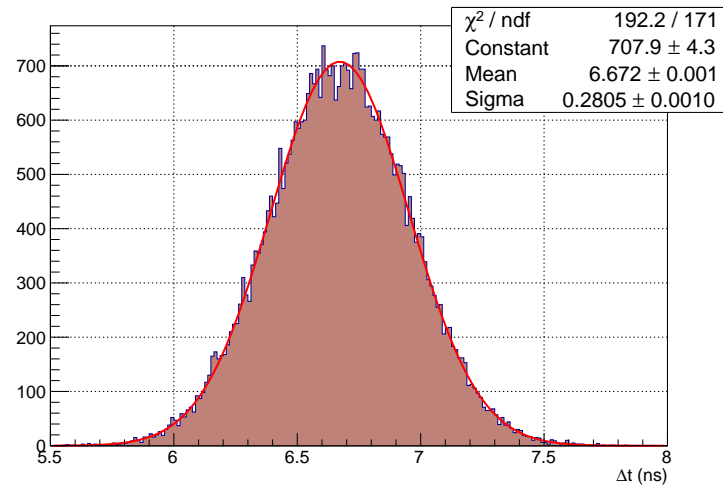
Problem 48 [[January 2013]

With a GM detector, we measure the activity of a source obtaining $N_g=40000$ counts during 10 minutes. To measure the background, the source is removed and $N_b=3600$ counts is obtained in 20 minutes. Calculate:

- The errors of the direct measurements of N_g and N_b
- The rates of the total number and the background noise (in strokes per minute) with their errors
- The net count rate (in counts per minute) and its error

Problem 49 Temporal resolution

Two detectors, placed at a distance of 2m, intercept a charged beam line. The time difference obtained with the coincidence of the two detectors can be fitted with a gaussian as seen in the figure:



If the time resolution of detector 1 is 198 ps, What's the time resolution of the detector 2?

Problem 50 Minimum Detectable Activity [January 2018]

Measurements of a sample and background for the same length of time yields:

$$N_T = 333 \text{ counts}$$

$$N_B = 296 \text{ counts}$$

The calibration constant converting counts into Bq ($A = \gamma N_S$) is $\gamma = 4.08 \text{ Bq/count}$.

- What's the measured activity?
- What's the critical count number (L_C)?
- Can we conclude that the sample has an activity?
- What's the minimum significant measured activity?
- What's the minimum detectable true activity?

Hint: For b) c) and d) consider that there is no real activity. For e), you should consider that there is an activity.

Problem 51 Radiation Statistics [January 2022]

Measurement of a sample and background for the same length of time yields $n_s = 333$ counts and $n_b = 296$ counts respectively. With this data, can we conclude that the sample has any detectable activity? Justify your answer.

Problem 52 Activity measurement [September 2022]

During a radiation measurement, the environmental background counted by the detector is $N_b = 180$ and the total number of counts (including the radiation source) is $N_t = 570$. All both measurements have been performed in 3 minutes.

- (a) What are the errors associated to N_b and N_t ?
- (b) What is the number of counts (and its error) coming from the radiation source?
- (c) What is the activity and its error? Provide the activity in counts per second.