3
The Nucleus and Nuclear Radiation

3.1
Nuclear Structure

The nucleus of an atom of atomic number Z and mass number A consists of Z protons and $N = A - Z$ neutrons. The atomic masses of all individual atoms are nearly integers, and A gives the total number of *nucleons* (i.e., protons and neutrons) in the nucleus. A species of atom, characterized by its nuclear constitution—its values of Z and A (or N)—is called a *nuclide*. It is conveniently designated by writing the appropriate chemical symbol with a subscript giving Z and superscript giving A. For example, ${}_{1}^{1}\mathrm{H}$, and ${}_{1}^{2}\mathrm{H}$, and ${}_{92}^{238}\mathrm{U}$ are nuclides. Nuclides of an element that have different A (or N) are called *isotopes*; nuclides having the same number of neutrons are called *isotones*; for example, ${}_{82}^{206}\mathrm{Pb}$ and ${}_{80}^{204}\mathrm{Hg}$ are isotones with $N = 124$. Hydrogen has three isotopes, ${}_{1}^{1}\mathrm{H}$, ${}_{1}^{2}\mathrm{H}$, and ${}_{1}^{3}\mathrm{H}$, all of which occur naturally. Deuterium, ${}_{1}^{2}\mathrm{H}$, is stable; tritium, ${}_{1}^{3}\mathrm{H}$, is radioactive. Fluorine has only a single naturally occurring isotope, ${}_{9}^{19}\mathrm{F}$; all of its other isotopes are man-made, radioactive, and short lived. The measured atomic weights of the elements reflect the relative abundances of the isotopes found in nature, as the next example illustrates.

Example

Chlorine is found to have two naturally occurring isotopes: ${}_{17}^{35}\mathrm{Cl}$, which is 76% abundant, and ${}_{17}^{37}\mathrm{Cl}$, which is 24% abundant. The atomic weights of the two isotopes are 34.97 and 36.97. Show that this isotopic composition accounts for the observed atomic weight of the element.

Solution

Taking the weighted average of the atomic weights of the two isotopes, we find for the atomic weight of Cl, $0.76 \times 34.97 + 0.24 \times 36.97 = 35.45$, as observed. (See periodic table in back of book.)

Since the electron configuration of the different isotopes of an element is the same, isotopes cannot be separated chemically. The existence of isotopes does cause a very slight perturbation in atomic energy levels, leading to an observed "isotope shift" in some spectral lines. In addition, the different nuclear spins of

Atoms, Radiation, and Radiation Protection. James E. Turner

ISBN: 978-3-527-40606-7

different isotopes of the same element are responsible for hyperfine structure in the spectra of elements. As we mentioned at the end of Section 2.8, the existence of isotopes has a big effect on the vibration–rotation spectra of molecules.

Nucleons are bound together in a nucleus by the action of the strong, or nuclear, force. The range of this force is only of the order of nuclear dimensions, $\sim 10^{-15}$ m, and it is powerful enough to overcome the Coulomb repulsion of the protons in the nucleus. Figure 3.1(a) schematically shows the potential energy of a proton as a function of the distance r separating its center and the center of a nucleus. The potential energy is zero at large separations. As the proton comes closer, its potential energy increases, due to the work done against the repulsive Coulomb force that

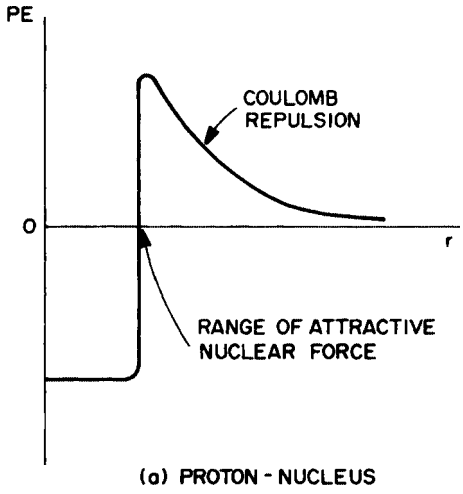

Fig. 3.1 (a) Potential energy (PE) of a proton as a function of its separation r from the center of a nucleus, (b) Potential energy of a neutron and a nucleus as a function of r. The uncharged neutron has no repulsive Coulomb barrier to overcome when approaching a nucleus.

acts between the two positive charges. Once the proton comes within range of the attractive nuclear force, though, its potential energy abruptly goes negative and it can react with the nucleus. If conditions are right, the proton's total energy can also become negative, and the proton will then occupy a bound state in the nucleus. As we learned in the Rutherford experiment in Section 2.2, a positively charged particle requires considerable energy in order to approach a nucleus closely. In contrast, the nucleus is accessible to a neutron of any energy. Because the neutron is uncharged, there is no Coulomb barrier for it to overcome. Figure 3.1(b) shows the potential-energy curve for a neutron and a nucleus.

Example

Estimate the minimum energy that a proton would have to have in order to react with the nucleus of a stationary Cl atom.

Solution

In terms of Fig. 3.1(a), the proton would have to have enough energy to overcome the repulsive Coulomb barrier in a head-on collision. This would allow it to just reach the target nucleus. We can use Eq. (2.2) to estimate how far apart the centers of the proton and nucleus would then be, when they "just touch." With $A = 1$ and $A = 35$ in Eq. (2.2), we obtain for the radii of the proton (r_p) and the chlorine nucleus (r_{Cl})

$$r_p = 1.3 \times 1^{1/3} \times 10^{-15} = 1.3 \times 10^{-15} \text{ m}, \tag{3.1}$$

$$r_{Cl} = 1.3 \times 35^{1/3} \times 10^{-15} = 4.3 \times 10^{-15} \text{ m}. \tag{3.2}$$

The proton has unit positive charge, $e = 1.60 \times 10^{-19}$ C, and the chlorine ($Z = 17$) nucleus has a charge $17e$. The potential energy of the two charges separated by the distance $r_p + r_{Cl} = 5.6 \times 10^{-15}$ m is therefore (Appendix C)

$$\text{PE} = \frac{8.99 \times 10^9 \times 17 \times (1.60 \times 10^{-19})^2}{5.6 \times 10^{-15}}$$

$$= 7.0 \times 10^{-13} \text{ J} = 4.4 \text{ MeV}. \tag{3.3}$$

(Problem 9 in Chapter 2 is worked like this example.)

Like an atom, a nucleus is itself a quantum-mechanical system of bound particles. However, the nuclear force, acting between nucleons, is considerably more complicated and more uncertain than the electromagnetic force that governs the structure and properties of atoms and molecules. In addition, wave equations describing nuclei cannot be solved with the same degree of numerical precision that atomic wave equations can. Nevertheless, many detailed properties of nuclei have been worked out and verified experimentally. Both the proton and the neutron are "spin-$\frac{1}{2}$" particles and hence obey the Pauli exclusion principle. Just as excited electron states exist in atoms, excited states can exist in nuclei. Whereas an atom has an infinite number of bound excited states, however, a nucleus has only a finite number, if any. This difference in atomic and nuclear structure is attributable to

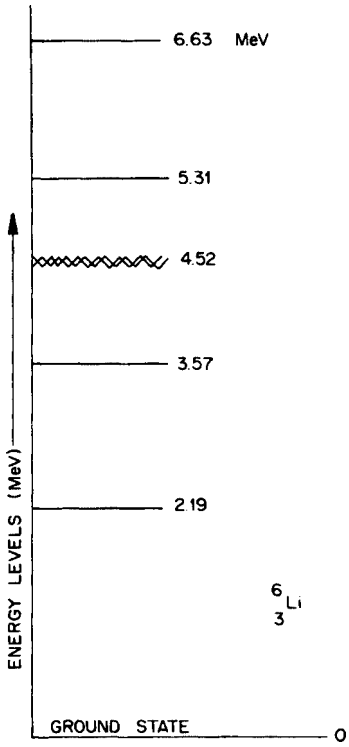

Fig. 3.2 Energy levels of the $^{6}_{3}$Li nucleus, relative to the ground state of zero energy.

the infinite range of the Coulomb force as opposed to the short range and limited, though large, strength of the nuclear force. The energy-level diagram of the $^{6}_{3}$Li nucleus in Fig. 3.2 shows that it has a number of bound excited states.[1)] The deuteron and alpha particle (nuclei of $^{2}_{1}$H and $^{4}_{2}$He) are examples of nuclei that have no bound excited states.

3.2 Nuclear Binding Energies

Changes can occur in atomic nuclei in a number of ways, as we shall see throughout this book. Nuclear reactions can be either exothermic (releasing energy) or endothermic (requiring energy in order to take place). The energies associated with nuclear changes are usually in the MeV range. They are thus $\sim 10^6$ times

1 The "level" at 4.52 MeV is very short lived and therefore does not have a sharp energy. All quantum-mechanical energy levels have a natural width, a manifestation of the uncertainty relation for energy and time, $\Delta E\, \Delta t \gtrsim \hbar$ [Eq. (2.31)]. The lifetimes of atomic states ($\sim 10^{-8}$ s) are long and permit precise knowledge of their energies ($\Delta E \sim 10^{-7}$ eV). For many excited nuclear states, the lifetime Δt is so short that the uncertainty in their energy, ΔE, is large, as is the case here.

greater than the energies associated with the valence electrons that are involved in chemical reactions. This factor characterizes the enormous difference in the energy released when an atom undergoes a nuclear transformation as compared with a chemical reaction.

The energy associated with exothermic nuclear reactions comes from the conversion of mass into energy. If the mass loss is ΔM, then the energy released, Q, is given by Einstein's relation, $Q = (\Delta M)c^2$, where c is the velocity of light. In this section we discuss the energetics of nuclear transformations.

We first establish the quantitative relationship between atomic mass units (AMU) and energy (MeV). By definition, the ^{12}C atom has a mass of exactly 12 AMU. Since its gram atomic weight is 12 g, it follows that

$$1\ \text{AMU} = 1/(6.02 \times 10^{23}) = 1.66 \times 10^{-24}\ \text{g} = 1.66 \times 10^{-27}\ \text{kg}. \tag{3.4}$$

Using the Einstein relation and $c = 3 \times 10^8\ \text{m s}^{-1}$, we obtain

$$\begin{aligned} 1\ \text{AMU} &= (1.66 \times 10^{-27})(3 \times 10^8)^2 \\ &= 1.49 \times 10^{-10}\ \text{J} \qquad (3.5) \\ &= \frac{1.49 \times 10^{-10}\ \text{J}}{1.6 \times 10^{-13}\ \text{J MeV}^{-1}} = 931\ \text{MeV}. \qquad (3.6) \end{aligned}$$

More precisely, 1 AMU = 931.49 MeV.

We now consider one of the simplest nuclear reactions, the absorption of a thermal neutron by a hydrogen atom, accompanied by emission of a gamma ray. This reaction, which is very important for understanding the thermal-neutron dose to the body, can be represented by writing

$$^1_0\text{n} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + {}^0_0\gamma, \tag{3.7}$$

the photon having zero charge and mass. The reaction can also be designated $^1_1\text{H}(\text{n},\gamma)^2_1\text{H}$. To find the energy released, we compare the total masses on both sides of the arrow. Appendix D contains data on nuclides which we shall frequently use. The atomic weight M of a nuclide of mass number A can be found from the mass difference, Δ, given in column 3. The quantity $\Delta = M - A$ gives the difference between the nuclide's atomic weight and its atomic mass number, expressed in MeV. (By definition, $\Delta = 0$ for the ^{12}C atom.) Since we are interested only in energy differences in the reaction (3.7), we obtain the energy released, Q, directly from the values of Δ, without having to calculate the actual masses of the neutron and individual atoms. Adding the Δ values for ^1_0n and ^1_1H and subtracting that for ^2_1H, we find

$$Q = 8.0714 + 7.2890 - 13.1359 = 2.2245\ \text{MeV}. \tag{3.8}$$

This energy appears as a gamma photon emitted when the capture takes place (the thermal neutron has negligible kinetic energy).

The process (3.7) is an example of energy release by the fusion of light nuclei. The binding energy of the deuteron is 2.2245 MeV, which is the energy required to separate the neutron and proton. As the next example shows, the binding energy of any nuclide can be calculated from a knowledge of its atomic weight (obtainable from Δ) together with the known individual masses of the proton, neutron, and electron.

Example

Find the binding energy of the nuclide $^{24}_{11}$Na.

Solution

One can work in terms of either AMU or MeV. The atom consists of 11 protons, 13 neutrons, and 11 electrons. The total mass in AMU of these separate constituents is, with the help of the data in Appendix A,

$$11(1.0073) + 13(1.0087) + 11(0.00055) = 24.199 \text{ AMU}. \tag{3.9}$$

From Appendix D, $\Delta = -8.418$ MeV gives the difference $M - A$. Thus, the mass of the $^{24}_{11}$Na nuclide is less than 24 by the amount 8.418 MeV/(931.49 MeV AMU^{-1}) = 0.0090371 AMU. Therefore, the nuclide mass is $M = 23.991$ AMU. Comparison with (3.9) gives for the binding energy

$$\text{BE} = 24.199 - 23.991 = 0.208 \text{ AMU} = 194 \text{ MeV}. \tag{3.10}$$

This figure represents the total binding energy of the atom—nucleons plus electrons. However, the electron binding energies are small compared with nuclear binding, which accounts for essentially all of the 194 MeV. Thus the binding energy per nucleon in $^{24}_{11}$Na is 194/24 = 8.08 MeV. [Had we worked in MeV, rather than AMU, the data from Appendix A give, in place of (3.9), 2.2541×10^4 MeV. Expressed in MeV, $A = 24 \times 931.49 = 2.2356 \times 10^4$ MeV. With $\Delta = -8.418$ MeV we have $M = A + \Delta = 2.2348 \times 10^4$ MeV. Thus the binding energy of the atom is $(2.2541 - 2.2348) \times 10^4 = 193$ MeV.]

The average binding energy per nucleon is plotted as a function of atomic mass number in Fig. 3.3. The curve has a broad maximum at about 8.5 MeV from $A = 40$ to 120.[2)] It then drops off as one goes either to lower or higher A. The implication from this curve is that the *fusion* of light elements releases energy, as does *the fission* of heavy elements. Both transformations are made exothermic through the increased average nucleon binding energy that results. The $^1_1\text{H}(\text{n},\gamma)^2_1\text{H}$ reaction considered earlier is an example of the release of energy through fusion. With a few exceptions, the average binding energies for all nuclides fall very nearly on the single curve shown. The nuclides $^4_2$He, $^{12}_6$C, and $^{16}_8$O show considerably tighter

2 The fact that the average nucleon binding energy is nearly constant over such a wide range of A is a manifestation of the saturation property of nuclear forces, mentioned at the end of Section 2.2.

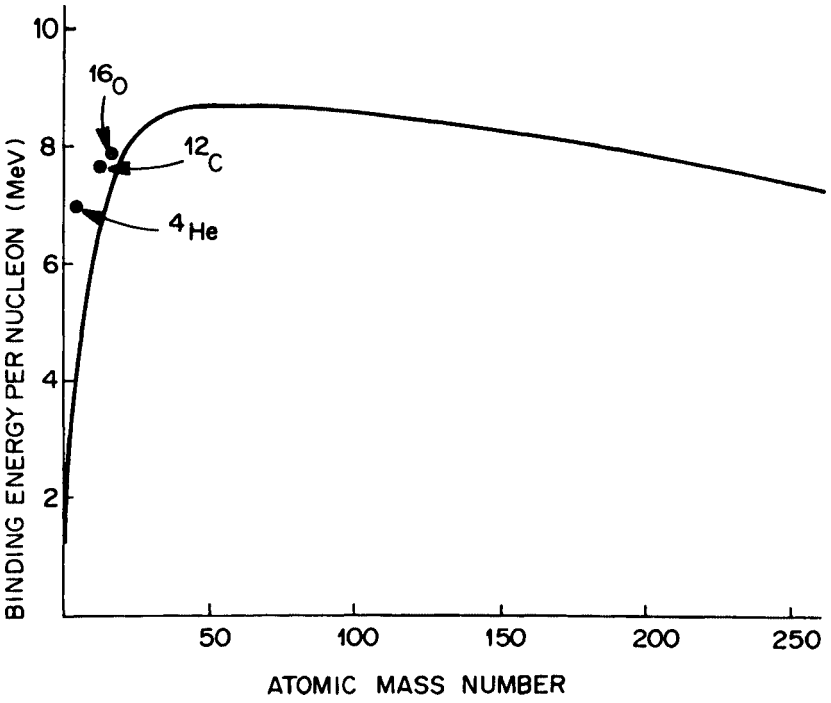

Fig. 3.3 Average energy per nucleon as a function of atomic mass number.

binding than their immediate neighbors. These nuclei are all "multiples" of the alpha particle, which appears to be a particularly stable nuclear subunit. (No nuclides with $A = 5$ exist for longer than $\sim 10^{-21}$ s.[3])

The loss of mass that accompanies the binding of particles is not a specifically nuclear phenomenon. The mass of the hydrogen atom is smaller than the sum of the proton and electron masses by 1.46×10^{-8} AMU. This is equivalent to an energy 1.46×10^{-8} AMU $\times$ 931 MeV/AMU^{-1} $= 1.36 \times 10^{-5}$ MeV $= 13.6$ eV, the binding energy of the H atom.

We turn now to the subject of radioactivity, the property that some atomic species, called radionuclides, have of undergoing spontaneous nuclear transformation. All of the heaviest elements are radioactive; $^{209}_{83}$Bi is the only stable nuclide with $Z > 82$. All elements have radioactive isotopes, the majority being man-made. The various kinds of radioactive decay and their associated nuclear energetics are described in the following sections.

3 Various forms of shell models have been studied for nuclei, analogous to an atomic shell model. The alpha particle consists of two spin-$\frac{1}{2}$ protons and two spin-$\frac{1}{2}$ neutrons in s states, forming the most tightly bound, "inner" nuclear shell. Generally, nuclei with even numbers of protons and neutrons ("even–even" nuclei) have the largest binding energies per nucleon.

3.3 Alpha Decay

Almost all naturally occurring alpha emitters are heavy elements with $Z \geq 83$. The principal features of alpha decay can be learned from the example of ^{226}Ra:

$$^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}. \tag{3.11}$$

The energy Q released in the decay arises from a net loss in the masses $M_{\text{Ra,N}}$, $M_{\text{Rn,N}}$, and $M_{\text{He,N}}$, of the radium, radon, and helium nuclei:

$$Q = M_{\text{Ra,N}} - M_{\text{Rn,N}} - M_{\text{He,N}}. \tag{3.12}$$

This nuclear mass difference is very nearly equal the atomic mass difference, which, in turn, is equal to the difference in Δ values.[4)] Letting Δ_{P}, Δ_{D}, and Δ_{He} denote the values of the parent, daughter, and helium atoms, we can write a general equation for obtaining the energy release in alpha decay:

$$Q_\alpha = \Delta_{\text{P}} - \Delta_{\text{D}} - \Delta_{\text{He}}. \tag{3.13}$$

Using the values in Appendix D for the decay of $^{226}_{88}$Ra to the ground state of $^{222}_{86}$Rn, we obtain

$$Q = 23.69 - 16.39 - 2.42 = 4.88 \text{ MeV}. \tag{3.14}$$

The Q value (3.14) is shared by the alpha particle and the recoil radon nucleus, and we can calculate the portion that each acquires. Since the radium nucleus was at rest, the momenta of the two decay products must be equal and opposite. Letting m and v represent the mass and initial velocity of the alpha particle and M and V those of the recoil nucleus, we write

$$mv = MV. \tag{3.15}$$

Since the initial kinetic energies of the products must be equal to the energy released in the decay, we have

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2 = Q. \tag{3.16}$$

Substituting $V = mv/M$ from Eq. (3.15) into (3.16) and solving for v^2, one finds

$$v^2 = \frac{2MQ}{m(m+M)}. \tag{3.17}$$

4 Specifically, the relatively slight difference in the binding energies of the 88 electrons on either side of the arrow in (3.11) is neglected when atomic mass loss is equated to nuclear mass loss. In principle, nuclear masses are needed; however, atomic masses are much better known. These small differences are negligible for most purposes.

One thus obtains for the alpha-particle energy

$$E_\alpha = \tfrac{1}{2}mv^2 = \frac{MQ}{m+M}. \tag{3.18}$$

With the roles of the two masses interchanged, it follows that the recoil energy of the nucleus is

$$E_N = \tfrac{1}{2}MV^2 = \frac{mQ}{m+M}. \tag{3.19}$$

As a check, we see that $E_\alpha + E_N = Q$. Because of its much smaller mass, the alpha particle, having the same momentum as the nucleus, has much more energy. For ^{226}Ra, it follows from (3.14) and (3.18) that

$$E_\alpha = \frac{222 \times 4.88}{4+222} = 4.79 \text{ MeV}. \tag{3.20}$$

The radon nucleus recoils with an energy of only 0.09 MeV.

The conservation of momentum and energy, Eqs. (3.15) and (3.16), fixes the energy of an alpha particle uniquely for given values of Q and M. Alpha particles therefore occur with discrete values of energy.

Appendix D gives the principal radiations emitted by various nuclides. We consider each of those listed for ^{226}Ra. Two alpha-particle energies are shown: 4.785 MeV, occurring with a frequency of 94.4% of all decays, and 4.602 MeV, occurring 5.5% of the time. The Q value for the less frequent alpha particle can be found from Eq. (3.18):

$$Q = \frac{(m+M)E_\alpha}{M} = \frac{226 \times 4.60}{222} = 4.68 \text{ MeV}. \tag{3.21}$$

The decay in this case goes to an excited state of the ^{222}Rn nucleus. Like excited atomic states, excited nuclear states can decay by photon emission. Photons from the nucleus are called gamma rays, and their energies are generally in the range from tens of keV to several MeV. Under the gamma rays listed in Appendix D for ^{226}Ra we find a 0.186-MeV photon emitted in 3.3% of the decays, in addition to another that occurs very infrequently (following alpha decay to still another excited level of higher energy in the daughter nucleus). We conclude that emission of the higher energy alpha particle ($E_\alpha = 4.79$ MeV) leaves the daughter ^{222}Rn nucleus in its ground state. Emission of the 4.60 MeV alpha particle leaves the nucleus in an excited state with energy $4.79 - 4.60 = 0.19$ MeV above the ground state. A photon of this energy can then be emitted from the nucleus, and, indeed, one of energy 0.186 MeV is listed for 3.3% of the decays. As an alternative to photon emission, under certain circumstances an excited nuclear state can decay by ejecting an atomic electron, usually from the K or L shell. This process, which produces the electrons listed (e^-), is called internal conversion, and will be discussed in Section 3.6.[5)] For

5 In atoms, an Auger electron can be ejected from a shell in place of a photon, accompanying an electronic transition (Sect. 2.11).

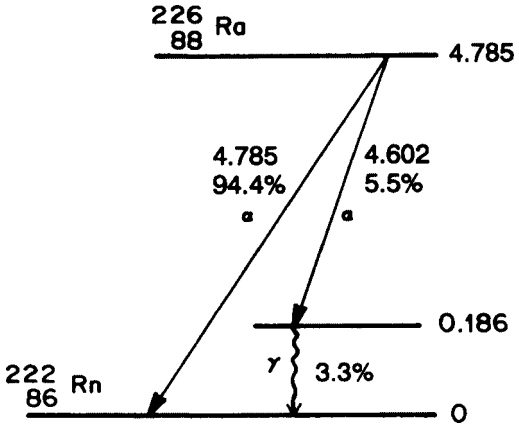

Fig. 3.4 Nuclear decay scheme of $^{226}_{88}$Ra.

^{226}Ra, since the excited state occurs in 5.5% of the total disintegrations and the 0.186 MeV photon is emitted only 3.3% of the time, it follows that internal conversion occurs in about 2.2% of the total decays. As we show in more detail in Section 3.6, the energy of the conversion electron is equal to the excited-state energy (in this case 0.186 MeV) minus the atomic-shell binding energy. The listing in Appendix D shows one of the e^- energies to be 0.170 MeV. In addition, since internal conversion leaves a K- or L-shell vacancy in the daughter atom, one also finds among the photons emitted the characteristic X rays of Rn. Finally, as noted in the radiations listed in Appendix D for ^{226}Ra, various kinds of radiation are emitted from the radioactive daughters, in this case ^{222}Rn, ^{218}Po, ^{214}Pb, ^{214}Bi, and ^{214}Po.

Decay-scheme diagrams, such as that shown in Fig. 3.4 for ^{226}Ra, conveniently summarize the nuclear transformations. The two arrows slanting downward to the left[6] show the two modes of alpha decay along with the alpha-particle energies and frequencies. Either changes the nucleus from that of ^{226}Ra to that of ^{222}Rn. When the lower energy particle is emitted, the radon nucleus is left in an excited state with energy 0.186 MeV above the ground state. (The vertical distances in Fig. 3.4 are not to scale.) The subsequent gamma ray of this energy, which is emitted almost immediately, is shown by the vertical wavy line. The frequency 3.3% associated with this photon emission implies that an internal-conversion electron is emitted in the other 2.2% of the total number of disintegrations. Radiations not emitted directly from the nucleus (i.e., the Rn X rays and the internal-conversion electron) are not shown on such a diagram, which represents the nuclear changes. Relatively infrequent modes of decay could also be shown, but are not included in Fig. 3.4 (see Fig. 3.7). (A small round-off error occurs in the energies.)

The most energetic alpha particles are found to come from radionuclides having relatively short half-lives. An early empirical finding, known as the Geiger–Nuttall law, implies that there is a linear relationship between the logarithm of the range R

6 By convention, going left represents a decrease in Z and right, an increase in Z. Photon emission is represented by a vertical wavy line.

of an alpha particle in air and the logarithm of the emitter's half-life T. The relation can be expressed in the form

$$-\ln T = a + b \ln R, \tag{3.22}$$

where a and b are empirical constants.

To conclude this section, we briefly consider the possible radiation-protection problems that alpha emitters can present. As we shall see in Chapter 5, alpha particles have very short ranges and cannot even penetrate the outer, dead layer of skin. Therefore, they generally pose no direct external hazard to the body. Inhaled, ingested, or entering through a wound, however, an alpha source can present a hazard as an internal emitter. Depending upon the element, internal emitters tend to seek various organs and irradiate them. Radium, for example, seeks bone, where it can become lodged and irradiate an individual over his or her lifetime. In addition to the internal hazard, one can generally expect gamma rays to occur with an alpha source, as is the case with radium. Also, many alpha emitters have radioactive daughters that present radiation-protection problems.

3.4 Beta Decay (β^-)

In beta decay, a nucleus simultaneously emits an electron, or negative beta particle, ${}^{0}_{-1}\beta$, and an antineutrino, ${}^{0}_{0}\bar{\nu}$. Both of these particles are created at the moment of nuclear decay. The antineutrino, like its antiparticle[7] the neutrino, ${}^{0}_{0}\nu$, has no charge and little or no mass;[8] they have been detected only in rather elaborate experiments.

As an example of beta decay, we consider ^{60}Co:

$${}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}. \tag{3.23}$$

In this case, the value of Q is equal to the difference between the mass of the ^{60}Co nucleus, $M_{\mathrm{Co,N}}$, and that of the ^{60}Ni nucleus, $M_{\mathrm{Ni,N}}$, plus one electron (m):

$$Q = M_{\mathrm{Co,N}} - (M_{\mathrm{Ni,N}} + m). \tag{3.24}$$

The nickel atom has one more electron than the cobalt atom. Therefore, if we neglect differences in atomic-electron binding energies, Eq. (3.24) implies that Q is

7 The Dirac equation predicts the existence of an antiparticle for every spin-$\frac{1}{2}$ particle and describes its relationship to the particle. Other examples include the positron, ${}^{0}_{+1}\beta$, antiparticle to the electron; the antiproton; and the antineutron. Creation of a spin-$\frac{1}{2}$ particle is always accompanied by creation of a related particle, which can be the antiparticle, such as happens in the creation of an electron–positron pair. Particle–antiparticle pairs can annihilate, as electrons and positrons do. A bar over a symbol is used to denote an antiparticle: for example, $\nu, \bar{\nu}$. Several kinds of neutrinos have been found—electron, muon, and tau.

8 Experimentally, the neutrino and antineutrino masses cannot be larger than about 30 eV.

simply equal to the difference in the masses of the ^{60}Co and ^{60}Ni *atoms*.[9] Therefore, it follows that one can compute the energy released in beta decay from the difference in the values Δ_P and Δ_D, of the parent and daughter atoms:

$$Q_{\beta^-} = \Delta_P - \Delta_D. \tag{3.25}$$

Using the data from Appendix D, we find for the energy released in a β^- transformation of ^{60}Co to the ground state of ^{60}Ni

$$Q = -61.651 - (-64.471) = 2.820 \text{ MeV}. \tag{3.26}$$

In accordance with (3.23), this energy is shared by the beta particle, antineutrino, and recoil ^{60}Ni nucleus. The latter, because of its relatively large mass, receives negligible energy, and so

$$E_{\beta^-} + E_{\bar{\nu}} = Q, \tag{3.27}$$

where E_{β^-} and $E_{\bar{\nu}}$ are the initial kinetic energies of the electron and antineutrino. Depending on the relative directions of the momenta of the three decay products (β^-, $\bar{\nu}$, and recoil nucleus), E_{β^-} and $E_{\bar{\nu}}$ can each have any value between zero and Q, subject to the condition (3.27) on their sum. Thus the spectrum of beta-particle energies E_{β^-} is continuous, with $0 \leq E_{\beta^-} \leq Q$, in contrast to the discrete spectra of alpha particles, as required by Eq. (3.18). Alpha particles are emitted in a decay into two bodies, which must share energy and momentum in a unique way, giving rise to discrete alpha spectra. Beta particles are emitted in a decay into three bodies, which can share energy and momentum in a continuum of ways, resulting in continuous beta spectra. The shape of a typical spectrum is shown in Fig. 3.5.[10] The maximum beta-particle energy is always equal to the Q value for the nuclear transition. As a rule of thumb, the average beta energy is about one-third of Q: $\overline{E}_{\beta^-} \sim Q/3$.

To construct the decay scheme for ^{60}Co we consult Appendix D. We see that 99 + % of the decays occur with $Q = 0.318$ MeV and that both of the gamma photons occur with almost every disintegration. Therefore, almost every decay must go through an excited state of the daughter ^{60}Ni nucleus with an energy at least $1.173 + 1.332 = 2.505$ MeV above the ground state. Adding the maximum beta energy to this gives $2.505 + 0.318 = 2.823$ MeV, the value [Eq. (3.26), except for round-off] calculated for a transition all the way to the ground state of the ^{60}Ni nucleus. Therefore, we conclude that the ^{60}Co nucleus first emits a beta particle, with $Q = 0.318$ MeV, which is followed successively by the two gamma rays. It remains

9 We can think of adding and subtracting 27 electron masses in Eq. (3.24), giving $Q = (M_{Co,N} + 27m) - (M_{Ni,N} + 28m)$. Neglecting the difference in electron binding energies, then, we have $Q = M_{Co,A} - M_{Ni,A}$, where the subscript A denotes the atomic masses. It follows that Q is equal to the difference in Δ values for the two atoms.

10 Beta-ray spectra exhibit a variety of shapes. The spectra of some 100 nuclides of importance for radiation protection and biomedical applications are presented by W. G. Gross, H. Ing, and N. Freedman, "A Short Atlas of Beta-Ray Spectra," Phys. Med. Biol. **28**, 1251–1260 (1983).

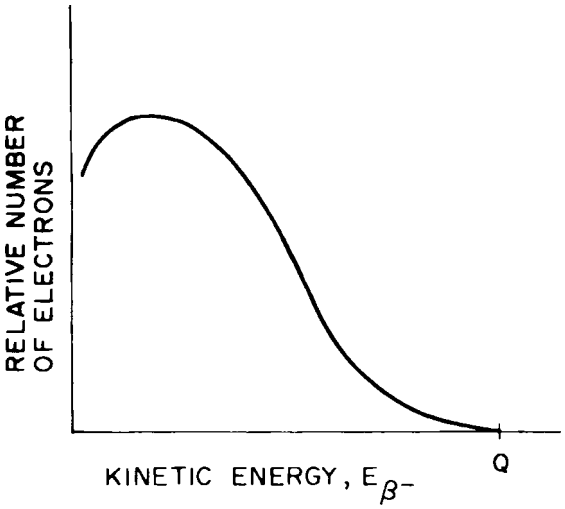

Fig. 3.5 Shape of typical beta-particle energy spectrum.

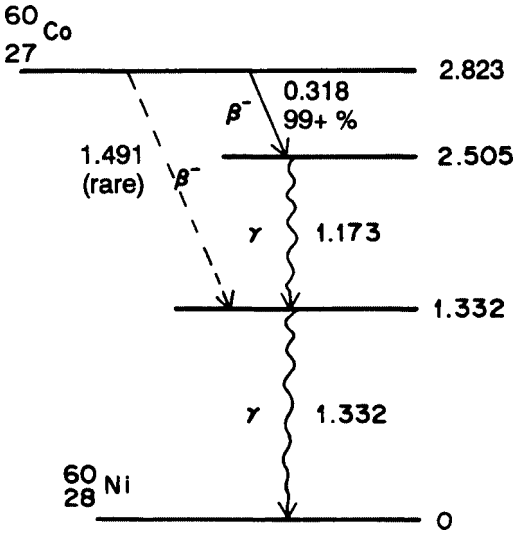

Fig. 3.6 Decay scheme of $^{60}_{27}$Co.

to determine the energy of the nuclear excited state from which the second photon is emitted: 1.173 MeV or 1.332 MeV? Appendix D lists a rare beta particle with $Q = 1.491$ MeV. This decay must go to a level in the daughter nucleus having an energy $2.823 - 1.491 = 1.332$ MeV above the ground state. Thus we can conclude that the 1.332 MeV photon is emitted last in the transition to the ground state. The decay scheme is shown in Fig. 3.6. The arrows drawn slanting toward the right indicate the increase in atomic number that results from β^- decay. The rare mode is shown with a dashed line. No significant internal conversion occurs with this radionuclide.

A number of beta emitters have no accompanying gamma rays. Examples of such pure beta emitters are ^{3}H, ^{14}C, ^{32}P, ^{90}Sr, and ^{90}Y. Mixed beta–gamma emitters include ^{60}Co, ^{137}Cs, and many others. A number of radionuclides emit beta particles in decaying to several levels of the daughter nucleus, thus giving rise to complex beta spectra. A few radioisotopes can decay by emission of either an alpha or a beta particle. For example, $^{212}_{83}Bi$ decays by alpha emission 36% of the time and by beta emission 64% of the time.

Beta rays can have sufficient energy to penetrate the skin and thus be an external radiation hazard. Internal beta emitters are also a hazard. As is the case with ^{60}Co, many beta radionuclides also emit gamma rays. High-energy beta particles (i.e., in the MeV range) can emit bremsstrahlung, particularly in heavy-metal shielding. The bremsstrahlung from a beta source may be the only radiation that escapes the containment.

3.5
Gamma-Ray Emission

As we have seen, one or more gamma photons can be emitted from the excited states of daughter nuclei following radioactive decay. Transitions that result in gamma emission leave Z and A unchanged and are called *isomeric*; nuclides in the initial and final states are called *isomers*.

As the examples in the last two sections illustrate, the gamma-ray spectrum from a radionuclide is discrete. Furthermore, just as optical spectra are characteristic of the chemical elements, a gamma-ray spectrum is characteristic of the particular radionuclides that are present. By techniques of gamma-ray spectroscopy (Chapter 10), the intensities of photons at various energies can be measured to determine the distribution of radionuclides in a sample. When ^{60}Co is present, for example, photons of energy 1.173 MeV and 1.332 MeV are observed with equal frequency. (Although these are called "^{60}Co gamma rays," we note from Fig. 3.6 that they are actually emitted by the daughter ^{60}Ni nucleus.) Radium can also be easily detected by its gamma-ray spectrum, which is more complex than indicated by Fig. 3.4. Since individual photons are registered in a spectrometer, gamma rays from infrequent modes of radioactive decay can often be readily measured. Figure 3.7 shows a more detailed decay scheme for ^{226}Ra, which involves three excited states of the daughter ^{222}Rn nucleus and the emission of photons of four different energies. Transitions from the highest excited level (0.601 MeV) to the next (0.448 MeV) and from there to ground are "forbidden" by selection rules.

Example

Like $^{60}_{27}Co$, another important gamma-ray source is the radioisotope $^{137}_{55}Cs$. Consult Appendix D and work out its decay scheme.

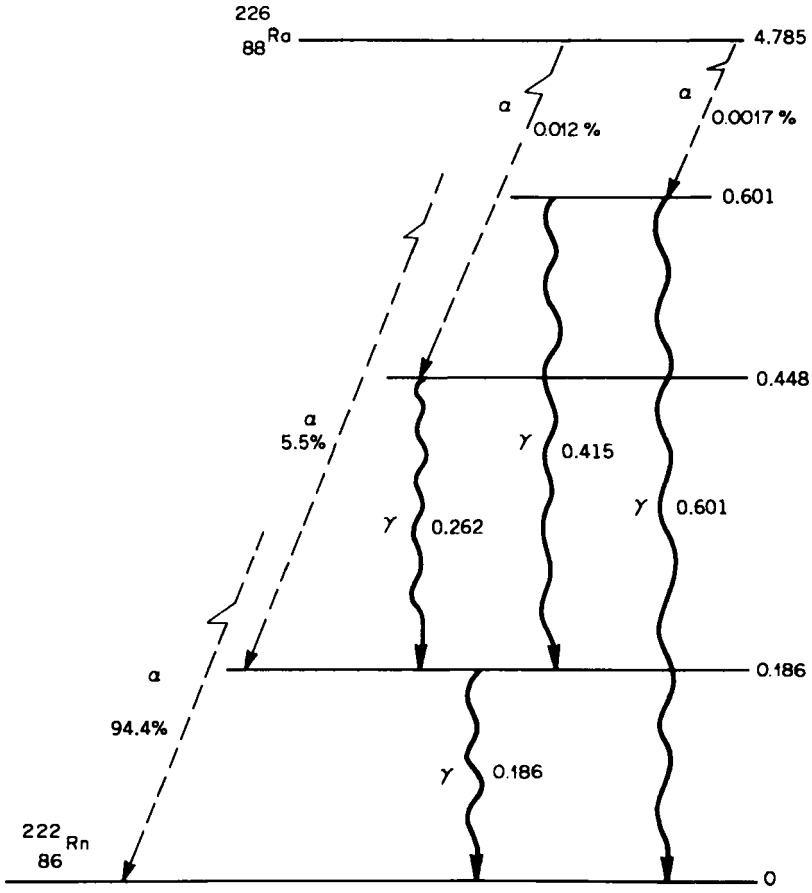

Fig. 3.7 Detailed decay scheme for $^{226}_{88}$Ra, showing origin of photons found in its gamma spectrum (position of initial $^{226}_{88}$Ra energy level not to scale).

Solution

Also like $^{60}_{27}$Co, $^{137}_{55}$Cs is a β^- emitter that leaves its daughter, stable $^{137}_{56}$Ba, in an excited state that results in gamma emission. The decay is represented by

$$^{137}_{55}\text{Cs} \rightarrow {}^{137}_{56}\text{Ba} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}. \tag{3.28}$$

From the Δ values in Appendix D, we obtain for decay to the daughter ground state $Q = -86.9 + 88.0 = 1.1$ MeV. Comparison with the radiations listed in the Appendix indicates that decay by this mode takes place 5% of the time, releasing 1.174 MeV. Otherwise, the decay in 95% of the cases leaves the daughter nucleus in an excited state with energy $1.174 - 0.512 = 0.662$ MeV. A photon of this energy is shown with 85% frequency. Therefore, internal conversion occurs in $95 - 85 = 10\%$ of the disintegrations, giving rise to the conversion electrons, e^-, with the energies shown. Characteristic Ba X rays are emitted following the inner-shell vacancies created in

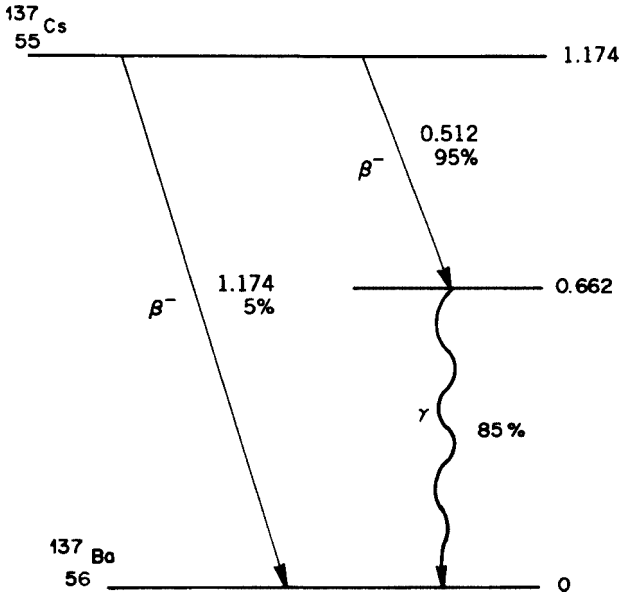

Fig. 3.8 Decay scheme of $^{137}_{55}$Cs.

the atom by internal conversion. The decay scheme of $^{137}_{55}$Cs is drawn in Fig. 3.8; the spectrum of electrons emitted by the source is shown schematically in Fig. 3.9.

The lifetimes of nuclear excited states vary, but $\sim 10^{-10}$ s can be regarded as typical. Thus, gamma rays are usually emitted quickly after radioactive decay to an excited daughter state. In some cases, however, selection rules prevent photon emission for an extended period of time. The excited state of $^{137}_{56}$Ba following the decay of $^{137}_{55}$Cs has a half-life of 2.55 min. Such a long-lived nuclear state is termed *metastable* and is designated by the symbol m: $^{137m}_{56}$Ba.

Another example of a metastable nuclide is $^{99m}_{43}$Tc, which results from the beta decay of the molybdenum isotope $^{99}_{42}$Mo. $^{99m}_{43}$Tc has a half-life of 6.02 h in making an isomeric transition (IT) to the ground state:

$$^{99m}_{43}\text{Tc} \rightarrow {}^{99}_{43}\text{Tc} + {}^{0}_{0}\gamma. \tag{3.29}$$

The energy released in an isomeric transition is simply equal to the difference in Δ values of the parent and daughter atoms:

$$Q_{\text{IT}} = \Delta_{\text{P}} - \Delta_{\text{D}}. \tag{3.30}$$

Example

Work out the decay scheme of $^{99m}_{43}$Tc with the help of the data given in Appendix D.

Solution

Using Eq. (3.30) and the given values of Δ, we obtain for the energy released in going to the ground state in the transition (3.29), $Q = 87.33 - 87.18 = 0.15$ MeV. This

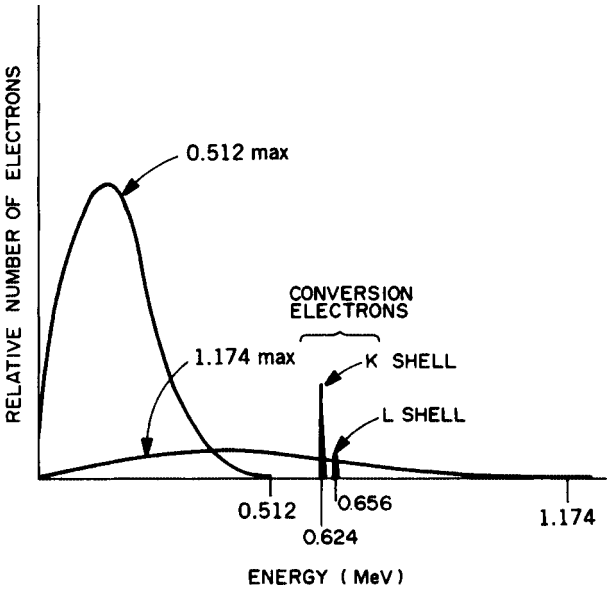

Fig. 3.9 Sources of electrons from $^{137}_{55}$Cs and their energy spectra. There are two modes of β^- decay, with maximum energies of 0.512 MeV (95%) and 1.174 MeV (5%). Internal-conversion electrons also occur, with discrete energies of 0.624 MeV (from the K shell) and 0.656 MeV (L shell) with a total frequency of 10%. See decay scheme in Fig. 3.8. The total spectrum of emitted electrons is the sum of the curves shown here.

transition is responsible for the gamma photon listed in Appendix D, 0.140 (89%). By implication, internal conversion must occur the other 11% of the time, and one finds two electron energies (e^-), one a little less than the photon energy (by an amount that equals the L-shell electron binding energy in the Tc atom). Because internal conversion leaves inner-shell vacancies, a $^{99m}_{43}$Tc source also emits characteristic Tc X rays, as listed. Since $^{99}_{43}$Tc decays by β^- emission into stable $^{99}_{44}$Ru, this daughter radiation also occurs.

The way in which gamma rays penetrate matter is fundamentally different from that of alpha and beta particles. Because of their charge, the latter lose energy almost continually as a result of electromagnetic forces that the electrons in matter exert on them. A shield of sufficient thickness can be used to absorb a beam of charged particles completely. Photons, on the other hand, are electrically neutral. They can therefore travel some depth in matter without being affected. As discussed in Section 8.5, monoenergetic photons, entering a uniform medium, have an exponential distribution of flight distances before they experience their first interaction. Although the intensity of a beam of gamma rays is steadily attenuated by passage through matter, some photons can traverse even thick shields with no interaction. Protection from gamma and X radiation is the subject of Chapter 15.

3.6 Internal Conversion

Internal conversion is the process in which the energy of an excited nuclear state is transferred to an atomic electron, most likely a K- or L-shell electron, ejecting it from the atom. It is an alternative to emission of a gamma photon from the nucleus.[11] We had examples of internal conversion in discussing the decay of ^{137}Cs and ^{99m}Tc. In the case of the latter, however, the situation is somewhat more involved than described in the last example. The dominant mode of the isomeric transition to the ground state takes place in two steps. The first, internal conversion with release of nuclear energy by ejection of a 2-keV orbital electron, consumes virtually all of the 6.02-h half-life of ^{99m}Tc. The second step, nuclear emission of the 140-keV gamma photon, then follows almost immediately, in $<10^{-9}$ s. The relatively soft gamma ray and the 6.02-h half-life, together with the ease of production (neutron bombardment of ^{98}Mo in a reactor) as well as the special chemical properties of the element, endow ^{99m}Tc with extremely useful properties for medical imaging. An example is shown in Fig. 3.10.

The internal conversion coefficient α for a nuclear transition is defined as the ratio of the number of conversion electrons N_e and the number of competing gamma photons N_γ for that transition:

$$\alpha = \frac{N_e}{N_\gamma}. \tag{3.31}$$

The kinetic energy E_e of the ejected atomic electron is very nearly equal to the excitation energy E^* of the nucleus minus the binding energy E_B of the electron in its atomic shell:

$$E_e = E^* - E_B. \tag{3.32}$$

The conversion coefficients α increase as Z^3, the cube of the atomic number, and decrease with E^*. Internal conversion is thus prevalent in heavy nuclei, especially in the decay of low-lying excited states (small E^*). Gamma decay predominates in light nuclei.

3.7 Orbital Electron Capture

Some nuclei undergo a radioactive transformation by capturing an atomic electron, usually from the K shell, and emitting a neutrino. An isotope of palladium under-

11 Internal conversion does not occur as a two-step process in which a photon is emitted by the nucleus and then absorbed by the atomic electron. The mechanism is entirely different. A similar observation was made in regard to Auger electrons in Section 2.11.

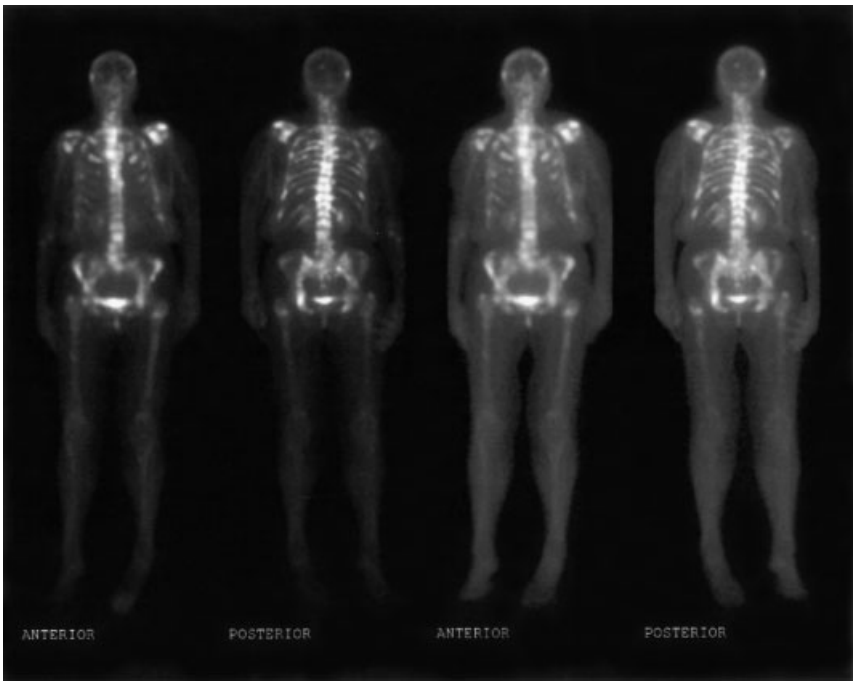

Fig. 3.10 Images of a woman with a history of breast carcinoma, now with extensive metastatic disease throughout the spine, ribs, and pelvis. Additional lesions can be seen in the skull, shoulders, and proximal femurs. Patient was administered 25 mCi ^{99m}Tc-hydroxymethylene diphosphonate (^{99m}Tc-HDP) and imaged 2–3 hours post injection. Anterior and posterior whole-body images were acquired simultaneously in approximately 15 min with a large field of view, dual head gamma camera. Images on left are displayed with a linear gray scale; those on right use a logarithmic gray scale, enhancing the soft-tissue activity. (Courtesy Glenn J. Hathaway, School of Nuclear Medicine, University of Tennessee Medical Center, Knoxville, TN.)

goes this process of electron capture (EC), going to a metastable state of the nucleus of the daughter rhodium:

$$^{103}_{46}\mathrm{Pd} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{103\mathrm{m}}_{45}\mathrm{Rh} + {}^{0}_{0}\nu. \tag{3.33}$$

The neutrino acquires the entire energy Q released by the reaction.

To find Q, we note that the captured electron releases its total mass, $m - E_B$, to the nucleus when it is absorbed there, E_B being the mass equivalent of the binding energy of the electron in the atomic shell. Therefore, in terms of the masses $M_{\mathrm{Pd,N}}$ and $M_{\mathrm{m_{Rh},N}}$ of the parent and daughter nuclei, the energy released by the reaction (3.33) is given by

$$Q = M_{\mathrm{Pd,N}} + m - E_B - M_{\mathrm{m_{Rh},N}}. \tag{3.34}$$

Since the palladium atom has one more electron than the rhodium atom, it follows that (neglecting the small difference in the electron binding energies) Q is equal to

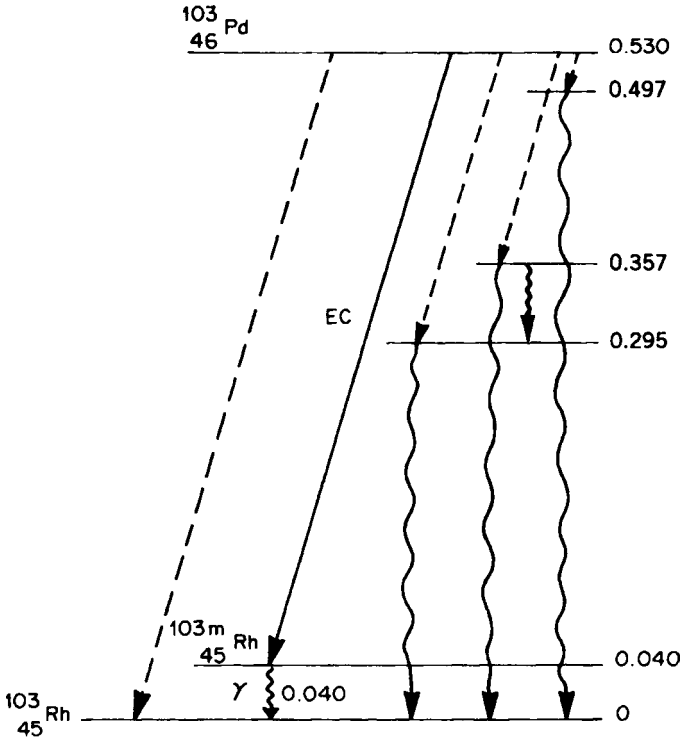

Fig. 3.11 Decay scheme of $^{103}_{46}$Pd.

the difference in the two *atomic* masses, less the energy E_B.[12] Since the difference in the atomic masses is equal to the difference in the parent and daughter Δ values, we can write the general expression for the energy release by electron capture:

$$Q_{EC} = \Delta_P - \Delta_D - E_B. \tag{3.35}$$

Orbital electron capture thus cannot take place unless $\Delta_P - \Delta_D > E_B$. For the K shell of palladium, $E_B = 0.024$ MeV. Using the Δ values from Appendix D in Eq. (3.35), we find that, for the decay to ^{103m}Rh,

$$Q = -87.46 - (-87.974) - 0.024 = 0.490 \text{ MeV}. \tag{3.36}$$

In subsequently decaying to the ground state, ^{103m}Rh releases an energy $-87.974 + 88.014 = 0.040$ MeV, as found from the values in Appendix D.

A decay scheme for ^{103}Pd is given in Fig. 3.11. Since electron capture decreases the atomic number of the nucleus, it is symbolized by an arrow pointing downward toward the left. The solid arrow represents the transition to ^{103m}Rh that we just analyzed. The presence of the gamma rays listed for ^{103}Pd in Appendix D implies that

12 A similar argument was given in the footnote after Eq. (3.24), except that E_B was not involved there.

EC sometimes leaves the nucleus in other excited levels, as shown. These transitions, as well as one directly to the ground state, are indicated by the dashed arrows. It is not possible from the information given in Appendix D to specify the frequency of these transitions relative to that represented by the solid arrow. Since electron capture necessarily leaves an inner-atomic-shell vacancy, characteristic X rays of the daughter are always emitted. (Electron capture is detected through the observation of characteristic X rays and Auger electrons as well as the recoil of the daughter nucleus.)

The radiations listed for ^{103m}Rh in Appendix D can also be explained. The photon with energy 0.040 MeV is shown in Fig. 3.11. Its 0.07% frequency implies that 99.03% of the time the metastable nucleus decays to the ground state by internal conversion, resulting in the ejection of atomic electrons (e^-) with the energies shown. (The present instance affords an example of internal conversion being favored over gamma emission in the decay of low-lying excited states in heavy nuclei, mentioned at the end of the last section.) Internal conversion also leaves a vacancy in an atomic shell, and hence the characteristic X rays of Rh are also found with a ^{103m}Rh source.

The neutrino emitted in electron capture has a negligible interaction with matter and offers no radiation hazard, as far as is known. Characteristic X rays of the daughter will always be present. In addition, if the capture does not leave the daughter in its ground state, gamma rays will occur.

3.8 Positron Decay (β^+)

Some nuclei, such as $^{22}_{11}$Na, disintegrate by emitting a positively charged electron (positron, β^+) and a neutrino:

$$^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + {}^{0}_{1}\beta + {}^{0}_{0}\nu. \tag{3.37}$$

Positron decay has the same net effect as electron capture, reducing Z by one unit and leaving A unchanged. The energy released is given in terms of the masses $M_{\mathrm{Na,N}}$ and $M_{\mathrm{Ne,N}}$ of the sodium and neon nuclei by

$$Q = M_{\mathrm{Na,N}} - M_{\mathrm{Ne,N}} - m. \tag{3.38}$$

Thus the mass of the parent nucleus must be greater than that of the daughter nucleus by at least the mass m of the positron it creates. As before, we need to express Q in terms of atomic masses, $M_{\mathrm{Na},A}$ and $M_{\mathrm{Ne},A}$. Since Na has 11 electrons and Ne 10, we write

$$Q = M_{\mathrm{Na,N}} + 11m - (M_{\mathrm{Ne,N}} + 10m) - 2m \tag{3.39}$$

$$= M_{\mathrm{Na},A} - M_{\mathrm{Ne},A} - 2m, \tag{3.40}$$

where the difference in the atomic binding of the electrons has been neglected in writing the last equality. In terms of the values Δ_P and Δ_D of the parent and daughter, the energy released in positron decay is given by

$$Q_{\beta^+} = \Delta_P - \Delta_D - 2mc^2. \tag{3.41}$$

Therefore, for positron emission to be possible, the mass of the parent atom must be greater than that of the daughter by at least $2mc^2 = 1.022$ MeV. Using the information from Appendix D, we find for the energy released via positron emission in the decay (3.37) to the ground state of $^{22}_{10}$Ne

$$Q_{\beta^+} = -5.182 - (-8.025) - 1.022 = 1.821 \text{ MeV}. \tag{3.42}$$

Electron capture, which results in the same net change as positron decay, can compete with (3.37):

$$^{0}_{-1}\text{e} + ^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} + ^{0}_{0}\nu. \tag{3.43}$$

Neglecting the electron binding energy in the $^{22}_{11}$Na atom, we obtain from Eq. (3.35) for the energy released by electron capture

$$Q_{EC} = -5.182 + 8.025 = 2.843 \text{ MeV}. \tag{3.44}$$

[Comparison of Eqs. (3.35) and (3.41) shows that the Q value for EC is greater than that for β^+ decay by 1.022 MeV when E_B is neglected.]

We next develop the decay scheme for $^{22}_{11}$Na. Appendix D indicates that β^+ emission occurs 89.8% of the time and EC 10.2%. A gamma ray with energy 1.275 MeV occurs with 100% frequency, indicating that either β^+ emission or EC leaves the daughter nucleus in an excited state with this energy. The positron decay scheme is shown in Fig. 3.12(a) and that for electron capture in (b). The two are combined in (c) to show the complete decay scheme for $^{22}_{11}$Na. The energy levels are drawn relative to the ground state of $^{22}_{10}$Ne as having zero energy. The starting EC level is $2mc^2$ higher than the starting level for β^+ decay.

Additional radiations are given in Appendix D for $^{22}_{11}$Na. Gamma rays of energy 0.511 MeV are shown with 180% frequency. These are annihilation photons that are present with all positron emitters. A positron slows down in matter and then annihilates with an atomic electron, giving rise to two photons, each having energy $mc^2 = 0.511$ MeV and traveling in opposite directions. Since a positron is emitted in about 90% of the decay processes, the frequency of an annihilation photon is 1.8 per disintegration of a $^{22}_{11}$Na atom. The remaining radiation shown, Ne X rays, comes as the result of the atomic-shell vacancy following electron capture.

As this example shows, electron capture and positron decay are competitive processes. However, whereas positron emission cannot take place when the parent–daughter atomic mass difference is less than $2mc^2$, electron capture can, the only restriction being $\Delta_P - \Delta_D > E_B$, as implied by Eq. (3.35). The nuclide $^{126}_{53}$I can decay by three routes: EC (60.2%), β^- (36.5%), or β^+ (3.3%).[13)]

13 In general, the various possible decay modes for a nuclide are those for which $Q > 0$ for a transition to the daughter ground state.

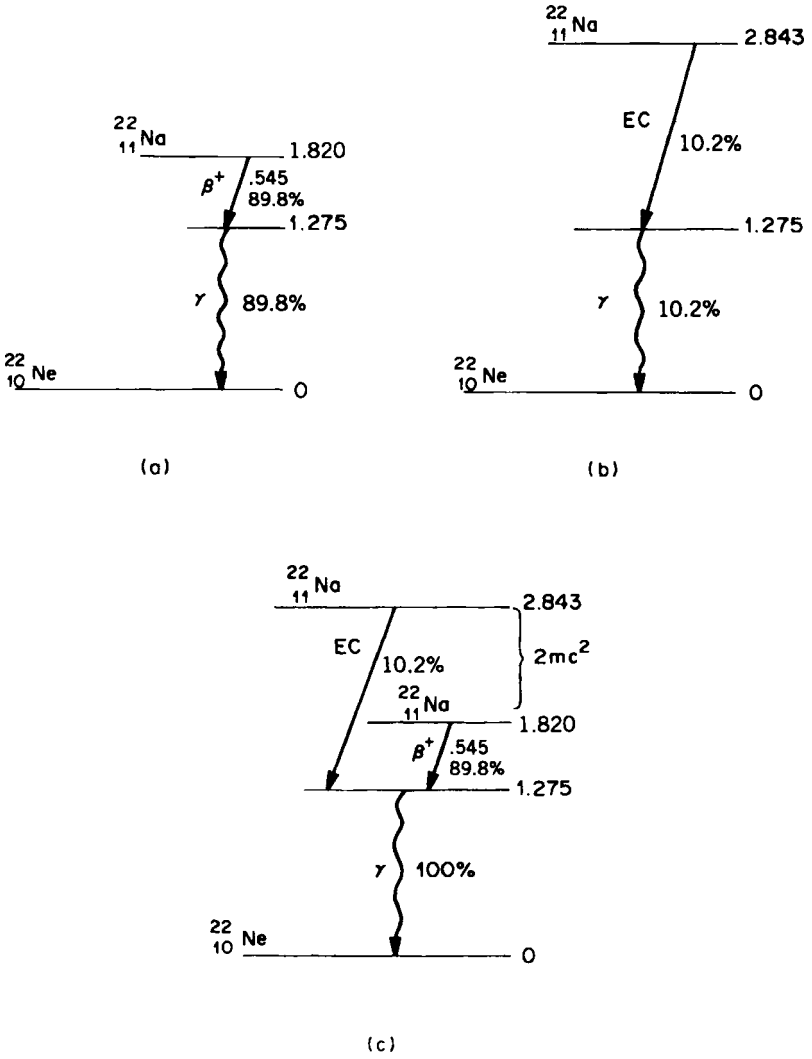

Fig. 3.12 Decay scheme of $^{22}_{11}$Na.

The radiation-protection problems associated with positron emitters include all those of β^- emitters (direct radiation and possible bremsstrahlung) and then some. As already mentioned, the 0.511-MeV annihilation photons are always present. In addition, because of the competing process of electron capture, characteristic X rays can be expected.

Example

Refer to Appendix D and deduce the decay scheme of $^{26}_{13}$Al.

Solution

This nuclide decays by β^+ emission (81.8%) and EC (18.2%) into $^{26}_{12}$Mg. The energy release for EC with a transition to the daughter ground state is, from the Δ values,

$$Q_{EC} = -12.211 + 16.214 = 4.003 \text{ MeV}. \tag{3.45}$$

Here we have neglected the small binding energy of the atomic electron. The corresponding value for β^+ decay to the ground state is $Q_{\beta^+} = 4.003 - 1.022 = 2.981$ MeV. A 1.809-MeV gamma photon is emitted with 100% frequency, and so we can assume that both EC and β^+ decay modes proceed via an excited daughter state of this energy. Adding this to the maximum β^+ energy, we have $1.809 + 1.174 = 2.983$ MeV $= Q_{\beta^+}$. Therefore, the positron decay occurs as shown in Fig. 3.13(a). Its 81.8% frequency accounts for the annihilation photons listed with 164% frequency in Appendix D. The other 18.2% of the decays via EC also go through the level at 1.809 MeV. An additional photon of energy 1.130 MeV and frequency 2.5% is listed in Appendix D. This can arise if a fraction of the EC transformations go to a level with energy $1.809 + 1.130 = 2.939$ MeV above ground. The complete decay scheme is shown in Fig. 3.13(b). (Some small inconsistencies result from round-off.)

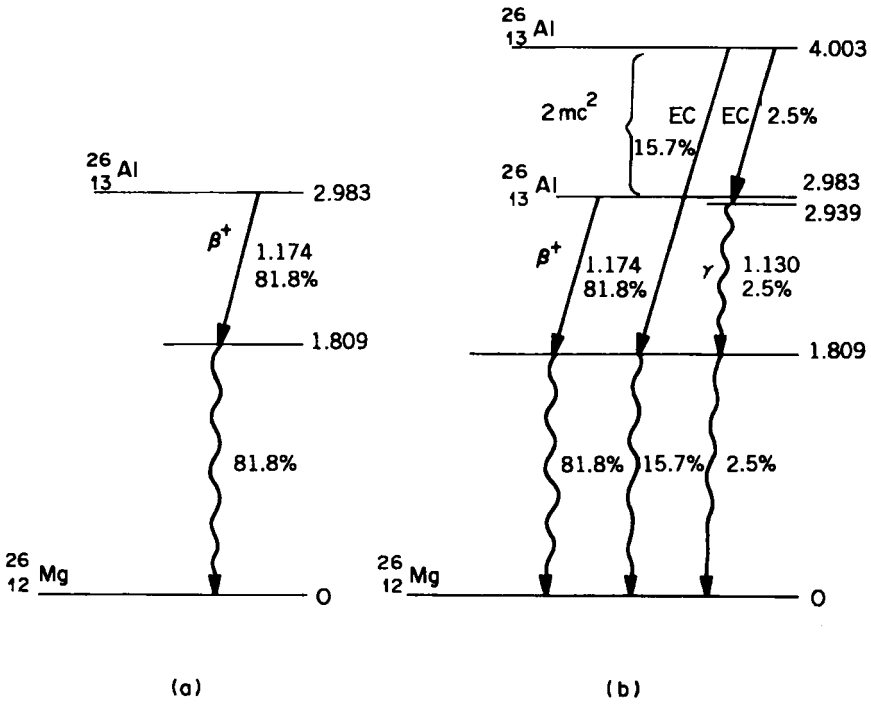

Fig. 3.13 Decay scheme of $^{26}_{13}$Al (see example in text).

Table 3.1 Formulas for Energy Release, Q, in Terms of Mass Differences, Δ_P and Δ_D, of Parent and Daughter Atoms

Type of decay	Formula	Reference
α	$Q_\alpha = \Delta_P - \Delta_D - \Delta_{He}$	Eq. (3.13)
β^-	$Q_{\beta^-} = \Delta_P - \Delta_D$	Eq. (3.25)
γ	$Q_{IT} = \Delta_P - \Delta_D$	Eq. (3.30)
EC	$Q_{EC} = \Delta_P - \Delta_D - E_B$	Eq. (3.35)
β^+	$Q_{\beta^+} = \Delta_P - \Delta_D - 2mc^2$	Eq. (3.41)

This completes the description of the various types of radioactive decay. The formulas for finding the energy release Q from the mass differences Δ of the parent and daughter atoms are summarized in Table 3.1.

3.9 Suggested Reading

1 Attix, F. H., *Introduction to Radiological Physics and Radiation Dosimetry*, Wiley, New York, NY (1986). [Types of radioactive disintegrations and decay schemes are discussed in Chapter 5.]

2 Cember, H., *Introduction to Health Physics*, 3rd ed., McGraw-Hill, New York, NY (1996). [See especially Chapter 4 on radioactivity.]

3 Evans, R. D., *The Atomic Nucleus*, McGraw-Hill, New York, NY (1955). [A comprehensive and classic text.]

4 Faw, R. E., and Shultis, J. K., *Radiological Assessment*, Prentice-Hall, Englewood Cliffs, NJ (1993). [Appendix B gives detailed decay-scheme data for some 200 radionuclides.]

5 Glasstone, S., *Sourcebook on Atomic Energy*, 3rd ed., D. Van Nostrand, Princeton, NJ (1967). [Describes much of the development of nuclear physics.]

6 Martin, J. E., *Physics for Radiation Protection*, John Wiley, New York, NY (2006). [Treats basic physics to deal with practical problems in radiation protection. Much useful data on radioactivity and radionuclides for the practitioner.]

7 Walker, Philip M. and Carroll, James J., "Ups and Downs of Nuclear Isomers," *Physics Today* **58**(6), 39–44 (2005).

A wealth of information and detailed data on radioactivity, decay schemes, isotopes, and related subjects is available through key-word searches on the World Wide Web. One extensive source, for example, with links to a number of related sites, is Lawrence Berkeley National Laboratory's Isotope Project Home Page, http://ie.lbl.gov.

3.10 Problems

1. Gallium occurs with two natural isotopes, ^{69}Ga (60.2% abundant) and ^{71}Ga (39.8%), having atomic weights 68.93 and 70.92. What is the atomic weight of the element?
2. The atomic weight of lithium is 6.941. It has two natural isotopes, ^{6}Li and ^{7}Li, with atomic weights of 6.015 and 7.016. What are the relative abundances of the two isotopes?
3. What minimum energy would an alpha particle need in order to react with a ^{238}U nucleus?
4. Calculate the energy released when a thermal neutron is absorbed by deuterium.
5. Calculate the total binding energy of the alpha particle.
6. How much energy is released when a ^{6}Li atom absorbs a thermal neutron in the reaction $^{6}_{3}\mathrm{Li}(\mathrm{n}, \alpha)^{3}_{1}\mathrm{H}$?
7. What is the mass of a ^{6}Li atom in grams?
8. Calculate the average binding energy per nucleon for the nuclide $^{40}_{19}$K.
9. The atomic weight of ^{32}P is 31.973910. What is the value of Δ in MeV?
10. Show that 1 AMU $= 1.49 \times 10^{-10}$ J.
11. Calculate the gamma-ray threshold for the reaction $^{12}C(\gamma, \mathrm{n})^{11}\mathrm{C}$.
12. **(a)** Calculate the energy released by the alpha decay of $^{222}_{86}$Rn.
    (b) Calculate the energy of the alpha particle.
    (c) What is the energy of the recoil polonium atom?
13. The $^{238}_{92}$U nucleus emits a 4.20-MeV alpha particle. What is the total energy released in this decay?
14. The $^{226}_{88}$Ra nucleus emits a 4.60-MeV alpha particle 5.5% of the time when it decays to $^{222}_{86}$Rn.
    (a) Calculate the Q value for this decay.
    (b) What is the recoil energy of the ^{222}Rn atom?
15. The Q value for alpha decay of $^{239}_{94}$Pu is 5.25 MeV. Given the masses of the ^{239}Pu and ^{4}He atoms, 239.052175 AMU and 4.002603 AMU, calculate the mass of the $^{235}_{92}$U atom in AMU.
16. Calculate the Q value for the beta decay of the free neutron into a proton, $^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$.
17. **(a)** Calculate the energy released in the beta decay of $^{32}_{15}$P.
    (b) If a beta particle has 650 keV, how much energy does the antineutrino have?
18. Calculate the Q value for tritium beta decay.
19. Draw the decay scheme for $^{42}_{19}$K.

20. A $^{108}_{49}$In source emits a 633-keV gamma photon and a 606-keV internal-conversion electron from the K shell. What is the binding energy of the electron in the K shell?

21. **(a)** Draw the decay scheme for $^{198}_{79}$Au.
(b) Estimate the K-shell electron binding energy from the data given in Appendix D.

22. Draw the decay scheme for $^{59}_{26}$Fe, labeling energies and frequencies (percentages) for each transition.

23. Draw the decay scheme for $^{203}_{80}$Hg.

24. Nuclide A decays into nuclide B by β^+ emission (24%) or by electron capture (76%). The major radiations, energies (MeV), and frequencies per disintegration are, in the notation of Appendix D:

β^+: 1.62 max (16%), 0.98 max (8%)
γ: 1.51 (47%), 0.64 (55%), 0.511 (48%, $\gamma^{\pm}$)
Daughter X rays
e^-: 0.614

(a) Draw the nuclear decay scheme, labeling type of decay, percentages, and energies.
(b) What leads to the emission of the daughter X rays?

25. Draw the decay scheme for $^{84}_{37}$Rb.

26. **(a)** Calculate the Q values for the decay of $^{57}_{28}$Ni by positron emission and by electron capture.
(b) Draw the decay scheme.

27. A parent nuclide decays by beta-particle emission into a stable daughter. The major radiations, energies (MeV), and frequencies are, in the notation of Appendix D:

β^-: 3.92 max (7%), 3.10 max (5%), 1.60 max (88%)
γ: 2.32 (34%), 1.50 (54%), 0.820 (49%)
e^-: 0.818, 0.805

(a) Draw the decay scheme.
(b) What is the maximum energy that the antineutrino can receive in this decay?
(c) What is the value of the internal-conversion coefficient?
(d) Estimate the L-shell electron binding energy of the daughter nuclide.
(e) Would daughter X rays be expected also? Why or why not?

28. Calculate the recoil energy of the technetium atom as a result of photon emission in the isomeric transition $^{99m}_{43}\text{Tc} \rightarrow {}^{99}_{43}\text{Tc} + \gamma$.

29. Refer to the decay scheme of $^{137}_{55}$Cs in Fig. 3.8. The binding energies of the K- and L-shell electrons of the daughter $^{137}_{56}$Ba atom are 38 keV and 6 keV.
(a) What are the energies of the internal-conversion electrons ejected from these shells?

(b) What is the wavelength of the barium K_α X ray emitted when an L-shell electron makes a transition to the K shell?
(c) What is the value of the internal-conversion coefficient?

30. **(a)** Calculate the Q value for K orbital-electron capture by the $^{37}_{18}$Ar nucleus, neglecting the electron binding energy.
(b) Repeat (a), including the binding energy, 3.20 keV, of the K-shell electron in argon.
(c) What becomes of the energy released as a result of this reaction?

31. What is the maximum possible positron energy in the decay of $^{35}_{18}$Ar?

32. Explain the origins of the radiations listed in Appendix D for $^{85}_{39}$Y. Draw the decay scheme.

33. The nuclide $^{65}_{30}$Zn decays by electron capture (98.5%) and by positron emission (1.5%).
(a) Calculate the Q value for both modes of decay.
(b) Draw the decay scheme for ^{65}Zn.
(c) What are the physical processes responsible for each of the major radiations listed in Appendix D?
(d) Estimate the binding energy of a K-shell electron in copper.

34. Does $^{26m}_{13}$Al decay to the ground state of its daughter $^{26}_{12}$Mg?

35. Show that $^{55}_{26}$Fe, which decays by electron capture, cannot decay by positron emission.

36. The isotope $^{126}_{53}$I can decay by EC, β^-, and β^+ transitions.
(a) Calculate the Q values for the three modes of decay to the ground states of the daughter nuclei.
(b) Draw the decay scheme.
(c) What kinds of radiation can one expect from a ^{126}I source?

3.11 Answers

2. 7.49% ^{6}Li, 92.51% ^{7}Li
4. 6.26 MeV
9. –24.303 MeV
15. 235.0439 AMU
16. 0.782 MeV
20. 27 keV
28. 0.106 eV
29. **(a)** 0.624 MeV and 0.656 MeV
(b) 0.388 Å
(c) 0.118
31. 4.94 MeV

4
Radioactive Decay

4.1
Activity

The rate of decay, or transformation, of a radionuclide is described by its activity, that is, by the number of atoms that decay per unit time. The unit of activity is the becquerel (Bq), defined as one disintegration per second: 1 Bq = 1 s^{-1}. The traditional unit of activity is the curie (Ci), which was originally the activity ascribed to 1 g of ^{226}Ra. The curie is now defined as 1 Ci = 3.7×10^{10} Bq, exactly.

4.2
Exponential Decay

The activity of a pure radionuclide decreases exponentially with time, as we now show. If N represents the number of atoms of a radionuclide in a sample at any given time, then the change dN in the number during a short time dt is proportional to N and to dt. Letting λ be the constant of proportionality, we write

$$dN = -\lambda N\, dt. \tag{4.1}$$

The negative sign is needed because N decreases as the time t increases. The quantity λ is called the decay, or transformation, constant; it has the dimensions of inverse time (e.g., s^{-1}). The decay rate, or activity, A, is given by

$$A = -\frac{dN}{dt} = \lambda N. \tag{4.2}$$

We separate the variables in Eq. (4.1) by writing

$$\frac{dN}{N} = -\lambda\, dt. \tag{4.3}$$

Integration of both sides gives

$$\ln N = -\lambda t + c, \tag{4.4}$$

Atoms, Radiation, and Radiation Protection. James E. Turner

ISBN: 978-3-527-40606-7

where c is an arbitrary constant of integration, fixed by the initial conditions. If we specify that N_0 atoms of the radionuclide are present at time $t = 0$, then Eq. (4.4) implies that $c = \ln N_0$. In place of (4.4) we write

$$\ln N = -\lambda t + \ln N_0, \tag{4.5}$$

$$\ln \frac{N}{N_0} = -\lambda t \tag{4.6}$$

or

$$\frac{N}{N_0} = e^{-\lambda t}. \tag{4.7}$$

Equation (4.7) describes the exponential radioactive decay law. Since the activity of a sample and the number of atoms present are proportional, activity follows the same rate of decrease,

$$\frac{A}{A_0} = e^{-\lambda t}, \tag{4.8}$$

where A_0 is the activity at time $t = 0$. The dose rate at a given location in the neighborhood of a fixed radionuclide source also falls off at the same exponential rate.

The function (4.8) is plotted in Fig. 4.1. During successive times T, called the half-life of the radionuclide, the activity drops by factors of one-half, as shown. To

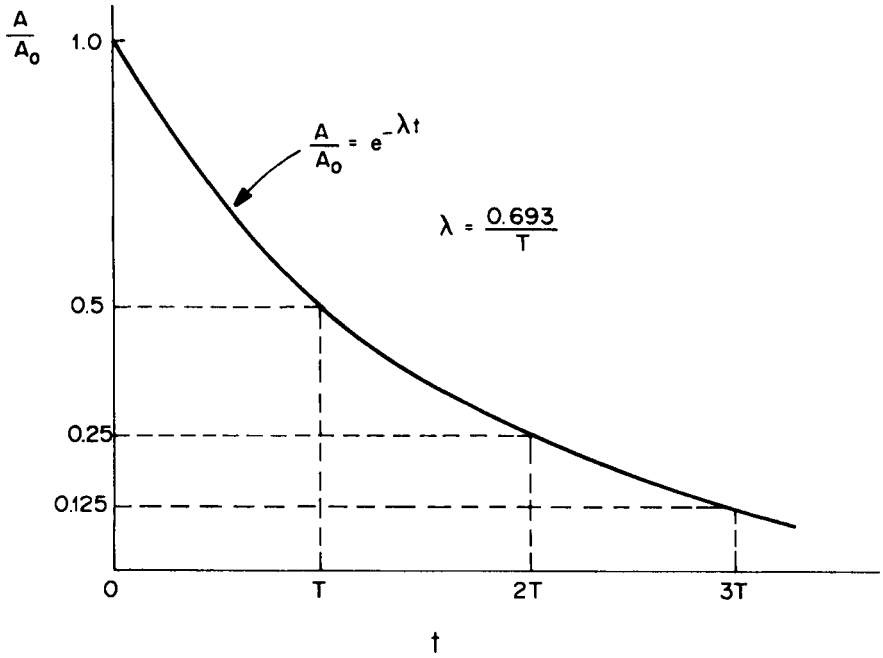

Fig. 4.1 Exponential radioactivity decay law, showing relative activity, A/A_0, as a function of time t; λ is the decay constant and T the half-life.

find T in terms of λ, we write from Eq. (4.8) at time $t = T$,

$$\tfrac{1}{2} = \mathrm{e}^{-\lambda T}. \tag{4.9}$$

Taking the natural logarithm of both sides gives

$$-\lambda T = \ln\left(\tfrac{1}{2}\right) = -\ln 2, \tag{4.10}$$

and therefore

$$T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. \tag{4.11}$$

Written in terms of the half-life, the exponential decay laws (4.7) and (4.8) become

$$\frac{N}{N_0} = \frac{A}{A_0} = \mathrm{e}^{-0.693t/T}. \tag{4.12}$$

The decay law (4.12) can be derived simply on the basis of the half-life. If, for example, the activity decreases to a fraction A/A_0 of its original value after passage of time t/T half-lives, then we can write

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{t/T}. \tag{4.13}$$

Taking the logarithm of both sides of Eq. (4.13) gives

$$\ln\frac{A}{A_0} = -\frac{t}{T}\ln 2 = -\frac{0.693t}{T}, \tag{4.14}$$

from which Eq. (4.12) follows.

Example

Calculate the activity of a 30-MBq source of $^{24}_{11}$Na after 2.5 d. What is the decay constant of this radionuclide?

Solution

The problem can be worked in several ways. We first find λ from Eq. (4.11) and then the activity from Eq. (4.8). The half-life $T = 15.0$ h of the nuclide is given in Appendix D. From (4.11),

$$\lambda = \frac{0.693}{T} = \frac{0.693}{15.0\ \mathrm{h}} = 0.0462\ \mathrm{h}^{-1}. \tag{4.15}$$

With $A_0 = 30$ MBq and $t = 2.5\ \mathrm{d} \times 24\ \mathrm{h\,d}^{-1} = 60.0$ h,

$$A = 30\ \mathrm{e}^{-(0.0462\ h^{-1} \times 60\ h)} = 1.88\ \mathrm{MBq}. \tag{4.16}$$

Note that the time units employed for λ and t must be the same in order that the exponential be dimensionless.

Example

A solution contains 0.10 μCi of ^{198}Au and 0.04 μCi of ^{131}I at time $t = 0$. What is the total beta activity in the solution at $t = 21$ d? At what time will the total activity decay to one-half its original value?

Solution

Both isotopes decay to stable daughters, and so the total beta activity is due to these isotopes alone. (A small fraction of ^{131}I decays into ^{131m}Xe, which does not contribute to the beta activity.) From Appendix D, the half-lives of ^{198}Au and ^{131}I are, respectively, 2.70 days and 8.05 days. At the end of 21 days, the activities A_{Au} and A_I of the nuclides are, from Eq. (4.12),

$$A_{Au} = 0.10e^{-0.693\times 21/2.70} = 4.56 \times 10^{-4}\ \mu\text{Ci} \tag{4.17}$$

and

$$A_I = 0.04e^{-0.693\times 21/8.05} = 6.56 \times 10^{-3}\ \mu\text{Ci}. \tag{4.18}$$

The total activity at $t = 21$ days is the sum of these two activities, 7.02×10^{-3} μCi. To find the time t in days at which the activity has decayed to one-half its original value of $0.10 + 0.04 = 0.14$ Ci, we write

$$0.07 = 0.1e^{-0.693t/2.70} + 0.04e^{-0.693t/8.05}. \tag{4.19}$$

This is a transcendental equation, which cannot be solved in closed form for t. The solution can be found either graphically or by trial and error, focusing in between two values of t that make the right-hand side of (4.19) >0.07 and <0.07. We present a combination of both methods. The decay constants of the two nuclides are, for Au, $0.693/2.70 = 0.257$ d^{-1} and, for I, $0.693/8.05 = 0.0861$ d^{-1}. The activities in μCi, as functions of time t, are

$$A_{Au}(t) = 0.10e^{-0.257t} \tag{4.20}$$

and

$$A_I(t) = 0.04e^{-0.0861t}. \tag{4.21}$$

Figure 4.2 shows a plot of these two activities and the total activity, $A(t) = A_{Au} + A_I$, calculated as functions of t from these two equations. Plotted to scale, the total activity $A(t)$ is found to reach the value 0.07 μCi near $t = 3.50$ d. We can improve on this approximate graphical solution. Direct calculation from Eqs. (4.20) and (4.21) shows that $A(3.50) = 0.0703$ and $A(3.60) = 0.0689$. Linear interpolation suggests the solution $t = 3.52$ d; indeed, one can verify that $A(3.52) = 0.0700$ μCi.

The average, or mean, life τ of a radionuclide is defined as the average of all of the individual lifetimes that the atoms in a sample of the radionuclide experience. It is equal to the mean value of t under the exponential curve in Fig. 4.3. Therefore,

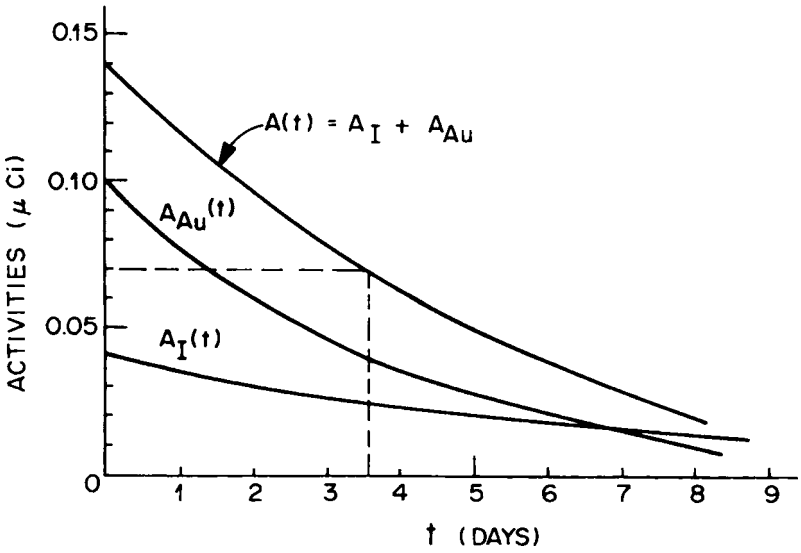

Fig. 4.2 Graphical solution to example in text.

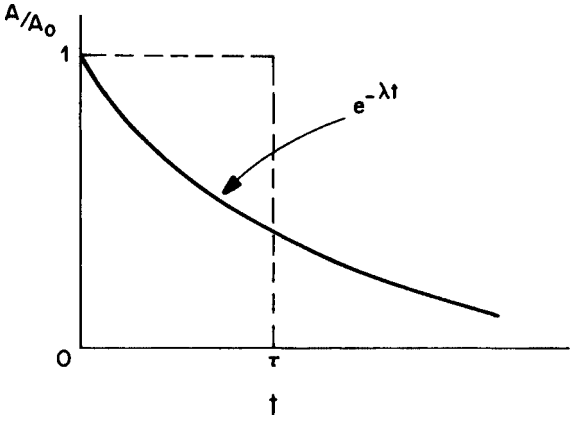

Fig. 4.3 The average life τ of a radionuclide is given by $\tau = 1/\lambda$.

τ defines a rectangle, as shown, with area equal to the area under the exponential curve:

$$1 \times \tau = \int_0^\infty e^{-\lambda t}\, dt = -\frac{1}{\lambda} e^{-\lambda t}\Big|_0^\infty = \frac{1}{\lambda}. \tag{4.22}$$

Thus the mean life is the reciprocal of the decay constant. In terms of the half-life, we have

$$\tau = \frac{1}{\lambda} = \frac{T}{0.693}, \tag{4.23}$$

showing that $\tau > T$.

4.3 Specific Activity

The specific activity of a sample is defined as its activity per unit mass, for example, $\mathrm{Bq\,g^{-1}}$ or $\mathrm{Ci\,g^{-1}}$. If the sample is a pure radionuclide, then its specific activity SA is determined by its decay constant λ, or half-life T, and by its atomic weight M as follows. Since the number of atoms per gram of the nuclide is $N = 6.02 \times 10^{23}/M$, Eq. (4.2) gives for the specific activity

$$\mathrm{SA} = \frac{6.02 \times 10^{23}\,\lambda}{M} = \frac{4.17 \times 10^{23}}{MT}. \tag{4.24}$$

If T is in seconds, then this formula gives the specific activity in $\mathrm{Bq\,g^{-1}}$. In practice, using the atomic mass number A in place of M usually gives sufficient accuracy.

Example

Calculate the specific activity of ^{226}Ra in $\mathrm{Bq\,g^{-1}}$.

Solution

From Appendix D, $T = 1600$ y and $M = A = 226$. Converting T to seconds, we have

$$\mathrm{SA} = \frac{4.17 \times 10^{23}}{226 \times 1600 \times 365 \times 24 \times 3600} \tag{4.25}$$

$$= 3.66 \times 10^{10}\ \mathrm{s^{-1}\,g^{-1}} = 3.7 \times 10^{10}\ \mathrm{Bq\,g^{-1}}. \tag{4.26}$$

This, by definition, is an activity of 1 Ci.

The fact that ^{226}Ra has unit specific activity in terms of $\mathrm{Ci\,g^{-1}}$ can be used in place of Eq. (4.24) to find SA for other radionuclides. Compared with ^{226}Ra, a nuclide of shorter half-life and smaller atomic mass number A will have, in direct proportion, a higher specific activity than ^{226}Ra. The specific activity of a nuclide of half-life T and atomic mass number A is therefore given by

$$\mathrm{SA} = \frac{1600}{T} \times \frac{226}{A}\,\mathrm{Ci\,g^{-1}}, \tag{4.27}$$

where T is expressed in years. (The equation gives $\mathrm{SA} = 1\ \mathrm{Ci\,g^{-1}}$ for ^{226}Ra.)

Example

What is the specific activity of ^{14}C?

Solution

With $T = 5730$ y and $A = 14$, Eq. (4.27) gives

$$\mathrm{SA} = \frac{1600}{5730} \times \frac{226}{14} = 4.51\ \mathrm{Ci\,g^{-1}}. \tag{4.28}$$

Alternatively, we can use Eq. (4.24) with $T = 5730 \times 365 \times 24 \times 3600 = 1.81 \times 10^{11}$ s, obtaining

$$\mathrm{SA} = \frac{4.17 \times 10^{23}}{14 \times 1.81 \times 10^{11}} = 1.65 \times 10^{11}\ \mathrm{Bq\,g^{-1}} \tag{4.29}$$

$$= \frac{1.65 \times 10^{11}\ \mathrm{Bq\,g^{-1}}}{3.7 \times 10^{10}\ \mathrm{Bq\,Ci^{-1}}} = 4.46\ \mathrm{Ci\,g^{-1}}, \tag{4.30}$$

in agreement with (4.28).

Specific activity need not apply to a pure radionuclide. For example, ^{14}C produced by the ^{14}N(n,p)^{14}C reaction can be extracted chemically as a "carrier-free" radionuclide, that is, without the presence of nonradioactive carbon isotopes. Its specific activity would be that calculated in the previous example. A different example is afforded by ^{60}Co, which is produced by neutron absorption in a sample of ^{59}Co (100% abundant), the reaction being ^{59}Co(n,γ)^{60}Co. The specific activity of the sample depends on its radiation history, which determines the fraction of cobalt atoms that are made radioactive. Specific activity is also used to express the concentration of activity in solution; for example, μCi mL^{-1} or Bq L^{-1}.

4.4 Serial Radioactive Decay

In this section we describe the activity of a sample in which one radionuclide produces one or more radioactive offspring in a chain. Several important cases will be discussed.

Secular Equilibrium ($T_1 \gg T_2$)

First, we calculate the total activity present at any time when a long-lived parent (1) decays into a relatively short-lived daughter (2), which, in turn, decays into a stable nuclide. The half-lives of the two radionuclides are such that $T_1 \gg T_2$; and we consider intervals of time that are short compared with T_1, so that the activity A_1 of the parent can be treated as constant. The total activity at any time is A_1 plus the activity A_2 of the daughter, on which we now focus. The rate of change, $\mathrm{d}N_2/\mathrm{d}t$, in the number of daughter atoms N_2 per unit time is equal to the rate at which they are produced, A_1, minus their rate of decay, $\lambda_2 N_2$:

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = A_1 - \lambda_2 N_2. \tag{4.31}$$

To solve for N_2, we first separate variables by writing

$$\frac{\mathrm{d}N_2}{A_1 - \lambda_2 N_2} = \mathrm{d}t, \tag{4.32}$$

where A_1 can be regarded as constant. Introducing the variable $u = A_1 - \lambda_2 N_2$, we have $\mathrm{d}u = -\lambda_2 \mathrm{d}N_2$ and, in place of Eq. (4.32),

$$\frac{\mathrm{d}u}{u} = -\lambda_2 \,\mathrm{d}t. \tag{4.33}$$

Integration gives

$$\ln(A_1 - \lambda_2 N_2) = -\lambda_2 t + c, \tag{4.34}$$

where c is an arbitrary constant. If N_{20} represents the number of atoms of nuclide (2) present at $t = 0$, then we have $c = \ln(A_1 - \lambda_2 N_{20})$. Equation (4.34) becomes

$$\ln \frac{A_1 - \lambda_2 N_2}{A_1 - \lambda_2 N_{20}} = -\lambda_2 t, \tag{4.35}$$

or

$$A_1 - \lambda_2 N_2 = (A_1 - \lambda_2 N_{20}) \mathrm{e}^{-\lambda_2 t}. \tag{4.36}$$

Since $\lambda_2 N_2 = A_2$, the activity of nuclide (2), and $\lambda_2 N_{20} = A_{20}$ is its initial activity, Eq. (4.36) implies that

$$A_2 = A_1(1 - \mathrm{e}^{-\lambda_2 t}) + A_{20}\mathrm{e}^{-\lambda_2 t}. \tag{4.37}$$

In many practical instances one starts with a pure sample of nuclide (1) at $t = 0$, so that $A_{20} = 0$, which we now assume. The activity A_2 then builds up as shown in Fig. 4.4. After about seven daughter half-lives ($t \gtrsim 7T_2$), $\mathrm{e}^{-\lambda_2 t} \ll 1$ and Eq. (4.37) reduces to the condition $A_1 = A_2$, at which time the daughter activity is equal to

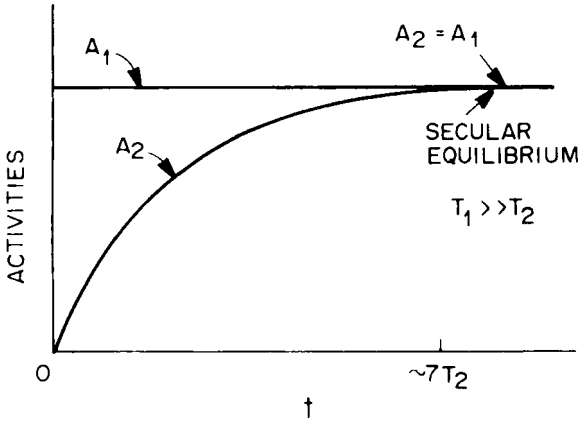

Fig. 4.4 Activity A_2 of relatively short-lived radionuclide daughter ($T_2 \ll T_1$) as a function of time t with initial condition $A_{20} = 0$. Activity of daughter builds up to that of the parent in about seven half-lives ($\sim 7T_2$). Thereafter, daughter decays at the same rate it is produced ($A_2 = A_1$), and secular equilibrium is said to exist.

that of the parent. This condition is called secular equilibrium. The total activity is $2A_1$. In terms of the numbers of atoms, N_1 and N_2, of the parent and daughter, secular equilibrium can be also expressed by writing

$$\lambda_1 N_1 = \lambda_2 N_2. \tag{4.38}$$

A chain of n short-lived radionuclides can all be in secular equilibrium with a long-lived parent. Then the activity of each member of the chain is equal to that of the parent and the total activity is $n+1$ times the activity of the original parent.

General Case

When there is no restriction on the relative magnitudes of T_1 and T_2, we write in place of Eq. (4.31)

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2. \tag{4.39}$$

With the initial condition $N_{20} = 0$, the solution to this equation is

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}), \tag{4.40}$$

as can be verified by direct substitution into (4.39). This general formula yields Eq. (4.38) when $\lambda_2 \gg \lambda_1$ and $A_{20} = 0$, and hence also describes secular equilibrium.

Transient Equilibrium ($T_1 \gtrsim T_2$)

Another practical situation arises when $N_{20} = 0$ and the half-life of the parent is greater than that of the daughter, but not greatly so. According to Eq. (4.40), N_2 and hence the activity $A_2 = \lambda_2 N_2$ of the daughter initially build up steadily. With the continued passage of time, $e^{-\lambda_2 t}$ eventually becomes negligible with respect to $e^{-\lambda_1 t}$, since $\lambda_2 > \lambda_1$. Then Eq. (4.40) implies, after multiplication of both sides by λ_2, that

$$\lambda_2 N_2 = \frac{\lambda_2 \lambda_1 N_{10} e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}. \tag{4.41}$$

Since $A_1 = \lambda_1 N_1 = \lambda_1 N_{10} e^{-\lambda_1 t}$ is the activity of the parent as a function of time, this relation says that

$$A_2 = \frac{\lambda_2 A_1}{\lambda_2 - \lambda_1}. \tag{4.42}$$

Thus, after initially increasing, the daughter activity A_2 goes through a maximum and then decreases at the same rate as the parent activity. Under this condition, illustrated in Fig. 4.5, transient equilibrium is said to exist. The total activity also

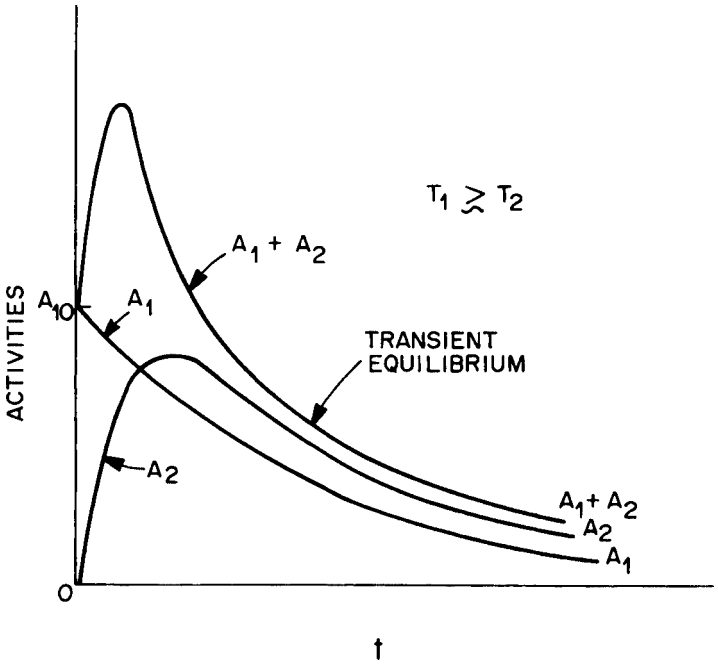

Fig. 4.5 Activities as functions of time when T_1 is somewhat larger than T_2 ($T_1 \gtrsim T_2$) and $N_{20} = 0$. Transient equilibrium is eventually reached, in which all activities decay with the half-life T_1 of the parent.

reaches a maximum, as shown in the figure, at a time earlier than that of the maximum daughter activity. Equation (4.42) can be differentiated to find the time at which the daughter activity is largest. The result is (Problem 25)

$$t = \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2}{\lambda_1}, \quad \text{for maximum } A_2. \tag{4.43}$$

The total activity is largest at the earlier time (Problem 26)

$$t = \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\lambda_2^2}{2\lambda_1\lambda_2 - \lambda_1^2}, \quad \text{for maximum } A_1 + A_2. \tag{4.44}$$

The time at which transient equilibrium is established depends on the individual magnitudes of T_1 and T_2. Secular equilibrium can be viewed as a special case of transient equilibrium in which $\lambda_2 \gg \lambda_1$ and the time of observation is so short that the decay of the activity A_1 is negligible. Under these conditions, the curve for A_1 in Fig. 4.5 would be flat, A_2 would approach A_1, and the figure would resemble Fig. 4.4.

No Equilibrium ($T_1 < T_2$)

When the daughter, initially absent ($N_{20} = 0$), has a longer half-life than the parent, its activity builds up to a maximum and then declines. Because of its shorter half-life, the parent eventually decays away and only the daughter is left. No equilibrium occurs. The activities in this case exhibit the patterns shown in Fig. 4.6.

Example

Starting with a 10.0-GBq ($= 10^{10}$ Bq) sample of pure ^{90}Sr at time $t = 0$, how long will it take for the total activity (^{90}Sr + ^{90}Y) to build up to 17.5 GBq?

Solution

Appendix D shows that $^{90}_{38}$Sr β^- decays with a half-life of 29.12 y into $^{90}_{39}$Y, which β^- decays into stable $^{90}_{40}$Zr with a half-life of 64.0 h. These two isotopes illustrate a long-lived parent ($T_1 = 29.12$ y) decaying into a short-lived daughter ($T_2 = 64.0$ h). Secular equilibrium is reached in about seven daughter half-lives, that is, in $7 \times 64 = 448$ h. At the end of this time, the ^{90}Sr activity A_1 has not diminished appreciably, the ^{90}Y activity A_2 has increased to the level $A_2 = A_1 = 10.0$ GBq, and the total activity is 20.0 GBq. In the present problem we are asked, in effect, to find the time at which the ^{90}Y activity reaches 7.5 GBq. The answer will be less than 448 h. Equation (4.37) with $A_{20} = 0$ applies here.[1)] The decay constant for ^{90}Y is $\lambda_2 = 0.693/T_2 = 0.693/64.0 = 0.0108\ \text{h}^{-1}$. With $A_1 = 10.0$ GBq, $A_2 = 7.5$ GBq, and $A_{20} = 0$, Eq. (4.37) gives

$$7.5 = 10.0(1 - e^{-0.0108t}), \tag{4.45}$$

where t is in hours. Rearranging, we have

$$e^{-0.0108t} = \tfrac{1}{4}, \tag{4.46}$$

giving $t = 128$ h. (In this example note that the ^{90}Y activity increases in an inverse fashion to the way a pure sample of ^{90}Y would decay. It takes two half-lives, $2T_2 = 128$ h, for the activity to build up to three-fourths its final value at secular equilibrium.)

Example

How many grams of ^{90}Y are in secular equilibrium with 1 mg of ^{90}Sr?

Solution

The amount of ^{90}Y will be that having the same activity as 1 mg of ^{90}Sr. The specific activity, SA, of ^{90}Sr ($T_1 = 29.12$ y) is [from Eq. (4.27)]

$$\text{SA}_1 = \frac{1600}{29.12} \times \frac{226}{90} = 138\ \text{Ci g}^{-1}. \tag{4.47}$$

1 Equation (4.40), describing the general case without restriction on the relative magnitudes of T_1 and T_2, can always be applied. To the degree of accuracy with which we are working, one will obtain the same numerical answer from the simplified Eq. (4.37), which already contains the appropriate approximations.

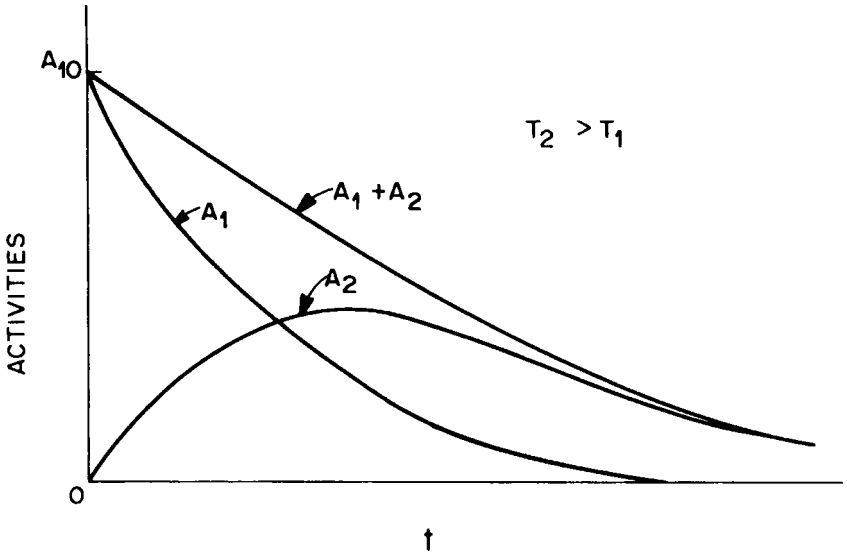

Fig. 4.6 Activities as functions of time when $T_2 > T_1$ and $N_{20} = 0$. No equilibrium conditions occur. Eventually, only the daughter activity remains.

Therefore, the activity of the 1 mg sample of ^{90}Sr is

$$A_1 = 10^{-3}\ \text{g} \times 138\ \text{Ci g}^{-1} = 0.138\ \text{Ci}, \tag{4.48}$$

which is also equal to the activity A_2 of the ^{90}Y. The latter has a specific activity

$$\text{SA}_2 = \frac{1600\ \text{y}}{64.0\ \text{h} \times \dfrac{1}{24}\dfrac{\text{d}}{\text{h}} \times \dfrac{1}{365}\dfrac{\text{y}}{\text{d}}} \times \frac{226}{90} \tag{4.49}$$

$$= 5.50 \times 10^5\ \text{Ci g}^{-1}. \tag{4.50}$$

Therefore, the mass of ^{90}Y in secular equilibrium with 1 mg of ^{90}Sr is

$$\frac{0.138\ \text{Ci}}{5.50 \times 10^5\ \text{Ci g}^{-1}} = 2.51 \times 10^{-7}\ \text{g} = 0.251\ \mu\text{g}. \tag{4.51}$$

Example

A sample contains 1 mCi of ^{191}Os at time $t = 0$. The isotope decays by β^- emission into metastable $^{191\text{m}}$Ir, which then decays by γ emission into ^{191}Ir. The decay and half-lives can be represented by writing

$$^{191}_{76}\text{Os} \xrightarrow[15.4\ \text{d}]{\beta^-} {}^{191\text{m}}_{77}\text{Ir} \xrightarrow[4.94\ \text{s}]{\gamma} {}^{191}_{77}\text{Ir}. \tag{4.52}$$

(a) How many grams of ^{191}Os are present at $t = 0$?
(b) How many millicuries of $^{191\text{m}}$Ir are present at $t = 25$ d?

(c) How many atoms of ^{191m}Ir decay between $t = 100$ s and $t = 102$ s?
(d) How many atoms of ^{191m}Ir decay between $t = 30$ d and $t = 40$ d?

Solution

As in the last two examples, the parent half-life is large compared with that of the daughter. Secular equilibrium is reached in about $7 \times 4.9 = 34$ s. Thereafter, the activities A_1 and A_2 of the ^{191}Os and ^{191m}Ir remain equal, as they are in secular equilibrium. During the periods of time considered in (b) and in (d), however, the osmium will have decayed appreciably; and so one deals with an example of transient equilibrium. The problem can be solved as follows.

(a) The specific activity of ^{191}Os is, from Eq. (4.27),

$$\mathrm{SA}_1 = \frac{1600 \times 365}{15.4} \times \frac{226}{191} = 4.49 \times 10^4 \ \mathrm{Ci\,g^{-1}}. \tag{4.53}$$

The mass of the sample, therefore, is

$$\frac{10^{-3} \ \mathrm{Ci}}{4.49 \times 10^4 \ \mathrm{Ci\,g^{-1}}} = 2.23 \times 10^{-8} \ \mathrm{g}. \tag{4.54}$$

(b) At $t = 25$ d,

$$A_2 = A_1 = 1 \times \mathrm{e}^{-0.693 \times 25/15.4} = 0.325 \ \mathrm{mCi}. \tag{4.55}$$

(c) Between $t = 100$ s and 102 s secular equilibrium exists with the osmium source essentially still at its original activity. Thus the ^{191m}Ir decay rate at $t = 100$ s is $A_2 = 1$ mCi $= 3.7 \times 10^7$ s^{-1}. During the next 2 s the number of ^{191m}Ir atoms that decay is $2 \times 3.7 \times 10^7 = 7.4 \times 10^7$.

(d) This part is like (c), except that the activities A_1 and A_2 do not stay constant during the time between 30 and 40 d. Since transient equilibrium exists, the numbers of atoms of ^{191m}Ir and ^{191}Os that decay are equal. The number of ^{191m}Ir atoms that decay, therefore, is equal to the integral of the ^{191}Os activity during the specified time (t in days):

$$3.7 \times 10^7 \int_{30}^{40} \mathrm{e}^{-0.693t/15.4} \, \mathrm{d}t = \frac{3.7 \times 10^7}{-0.0450} \mathrm{e}^{-0.0450t} \Big|_{30}^{40} \tag{4.56}$$

$$= -8.22 \times 10^8 (0.165 - 0.259)$$

$$= 7.73 \times 10^7. \tag{4.57}$$

4.5 Natural Radioactivity

All of the heavy elements ($Z > 83$) found in nature are radioactive and decay by alpha or beta emission. The nuclide $^{209}_{83}$Bi is the only one with atomic number greater than that of lead (82) that is stable. The heaviest elements decay into successive radioactive daughters, forming series of radionuclides that end when a stable species is produced. It is found that all of the naturally occurring heavy radionuclides belong to one of three series. Since the atomic mass number can change by only four units (viz., when alpha emission occurs), a given nuclide can be easily identified as belonging to one series or another by noting the remainder obtained when its mass number is divided by four. The uranium series, for example, begins with $^{238}_{91}$U and ends with stable $^{206}_{82}$Pb. When divided by four the mass number of every member of the uranium series has remainder two. The thorium series, starting with $^{232}_{90}$Th and ending with $^{208}_{82}$Pb, has remainder zero. The third group, the actinium series, which begins with $^{235}_{92}$U and ends with $^{207}_{82}$Pb, has remainder three. A fourth series, with remainder one, is the neptunium series. However, its longest-lived member, $^{237}_{93}$Np, has a half-life of 2.2×10^6 years, which is short on a geological time scale. Neptunium is not found in nature, but has been produced artificially, starting with $^{241}_{94}$Pu and ending with $^{209}_{82}$Pb. All four series contain one gaseous member (an isotope of Rn) and end in a stable isotope of Pb.

Primordial ^{238}U would be found in secular equilibrium with its much shorter-lived daughters, if undisturbed by physical or chemical processes in nature. It is more likely, however, that secular equilibrium will be found only among certain subsets of nuclides in the series. In this regard, a significant change occurs when ^{226}Ra decays into ^{222}Rn. The daughter, radon, is a noble gas, not bound chemically in the material where its parents resided. The half-life of ^{222}Rn is long enough for much of the gas to work its way out into the atmosphere. As seen from Table 1.1, radon (more precisely, its short-lived daughters) contributes an average of about one-half the effective dose to persons from natural background radiation. This important source of human exposure is discussed in the next section.

Several lighter elements have naturally occurring, primordial radioactive isotopes. One of the most important from the standpoint of human exposure is ^{40}K, which has an isotopic abundance of 0.0118% and a half-life of 1.28×10^9 years. The nuclide decays by β^- emission (89%) or EC (11%). The maximum β^- energy is 1.312 MeV. This isotope is an important source of human internal and external radiation exposure, because potassium is a natural constituent of plants and animals. In addition to the beta particle, ^{40}K emits a penetrating gamma ray (1.461 MeV) following electron capture (11%).

Other naturally occurring radionuclides are of cosmogenic origin. Only those produced as a result of cosmic-ray interactions with constituents of the atmosphere result in any mentionable exposure to man: ^{3}H, ^{7}Be, ^{14}C, and ^{22}Na. The reaction ^{14}N$(n, p)^{14}$C with atmospheric nitrogen produces radioactive ^{14}C, which has a half-life of 5730 y. The radioisotope, existing as CO_2 in the atmosphere, is utilized by

plants and becomes fixed in their structure through photosynthesis. The time at which ^{14}C was assimilated in a previously living specimen, used to make furniture or paper, for example, can be inferred from the relative amount of the isotope remaining in it today. Thus the age of such objects can be determined by radiocarbon dating. In modern times, the equilibrium of natural ^{14}C and ^{3}H in the atmosphere has been upset by the widespread burning of fossil fuels and by the testing of nuclear weapons in the atmosphere.

Example

How many alpha and beta particles are emitted by a nucleus of an atom of the uranium series, which starts as $^{238}_{92}U$ and ends as stable $^{206}_{82}Pb$?

Solution

Nuclides of the four heavy-element radioactive series decay either by alpha or beta emission. A single disintegration, therefore, either (1) reduces the atomic number by 2 and the mass number by 4 or (2) increases the atomic number by 1 and leaves the mass number unchanged. Since the atomic mass numbers of $^{238}_{92}U$ and $^{206}_{82}Pb$ differ by 32, it follows that 8 alpha particles are emitted in the series. Since this alone would reduce the atomic number by 16, as compared with the actual reduction of 10, a total of 6 beta particles must also be emitted.

4.6 Radon and Radon Daughters

As mentioned in the last section, the noble gas ^{222}Rn produced in the uranium series can become airborne before decaying. Soil and rocks under houses are ordinarily the principal contributors to indoor radon, which is typically four or five times more concentrated than radon outdoors, where greater air dilution occurs. Additional contributions to indoor radon come from outside air, building materials, and the use of water and natural gas. Circumstances vary widely in time and place, and so exceptions to generalizations are frequent.

Airborne radon itself poses little health hazard. As an inert gas, inhaled radon is not retained in significant amounts by the body. The potential health hazard arises when radon in the air decays, producing nongaseous radioactive daughters. When inhaled, the airborne daughters can be trapped in the respiratory system, where they are likely to decay before being removed by normal lung-clearing mechanisms of the body. Some of the daughter atoms in air are adsorbed onto micron- or submicron-sized aerosols or dust particles (the "attached fraction"). Others (the "unattached fraction") remain in the air as essentially free ions or in small molecular agglomerates (e.g., with several water molecules). Still other decay products plate out on various surfaces. In assessing hazard, it is common to characterize the radon daughters in an atmosphere by specifying the unattached fraction of each. When inhaled, this fraction is trapped efficiently, especially in the upper respiratory tract.

As shown in Fig. 4.7, ^{222}Rn decays into a series of short-lived daughters, two of which, ^{218}Po and ^{214}Po, are alpha emitters.[2)] When an alpha particle is emitted in the lung, it deposits all of its energy locally within a small thickness of adjacent tissue. An alpha particle from ^{214}Po, for example, deposits its 7.69 MeV of energy within about 70 μm. A 1-MeV beta particle from ^{214}Bi, on the other hand, deposits its energy over a much larger distance of about 4000 μm. The dose to the cells of the lung from the beta (and gamma) radiation from radon daughters is very small compared with that from the alpha particles. The "radon problem," technically, is that of alpha-particle irradiation of sensitive lung tissue by the short-lived daughters of radon and the associated risk of lung cancer.

The health hazard from radon is thus closely related to the air concentration of the potential alpha-particle energy of the short-lived daughters. Depending on local conditions, the daughters will be in various degrees of secular equilibrium with one another and with the parent radon in an atmosphere. Rather than using individual concentrations of the various progeny, one can characterize an atmosphere radiologically by means of a collective quantity: the potential alpha-energy concentration (PAEC). The PAEC is defined as the amount of alpha energy per unit volume of

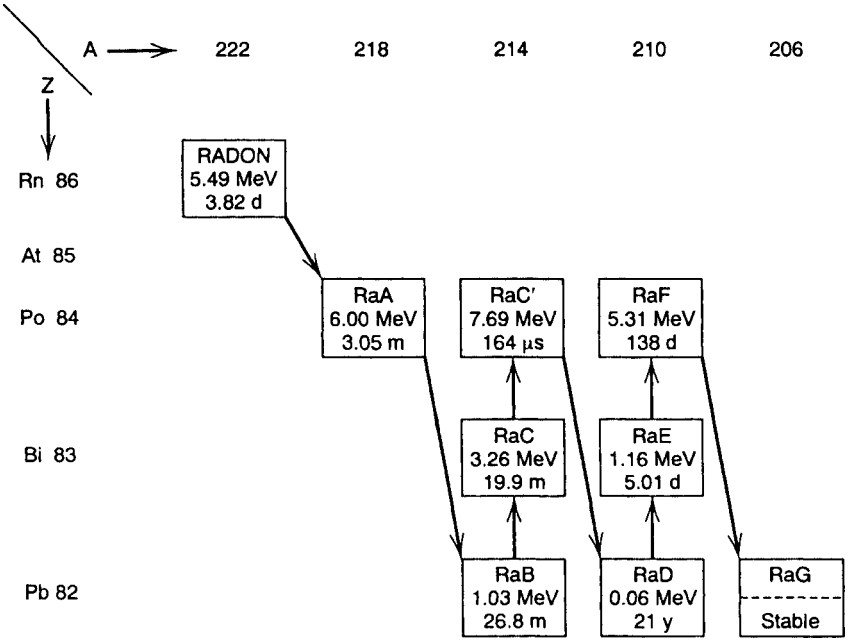

Fig. 4.7 Radon and radon daughters. Alpha emission is represented by an arrow slanting downward toward the right; beta emission, by a vertical arrow. Alpha-particle and average beta-particle energies and half-lives are shown in the boxes.

2 In earlier terminology, the successive short-lived daughters, ^{218}Po through ^{210}Pb, were called RaA, RaB, RaC, RaC′, and RaD, respectively.

undisturbed air that would ultimately be released from the particular mixture of short-lived daughters in their decay to ^{210}Pb. The PAEC can be expressed in J m^{-3} or MeV m^{-3}. For a given PAEC, the equilibrium-equivalent decay-product concentration (EEDC) is defined as the concentration of each decay product that would be present if secular equilibrium existed. The ratio of the EEDC and the concentration of radon is called the *equilibrium factor*. By definition, this factor is equal to unity if the radon and all of its short-lived daughters are in secular equilibrium. Equilibrium factors for most indoor atmospheres are in the range of 0.2 to 0.6, a factor of 0.5 often being assumed as a rule of thumb. A limitation of the quantities described in this paragraph is that they do not distinguish between the attached and unattached fractions.

Until now, we have discussed only ^{222}Rn, which is a member of the uranium series. Radon is also generated in the other two series of naturally occurring radionuclides. However, these isotopes of radon are of lesser radiological importance. The thorium series generates ^{220}Rn, which is also called thoron. The parent nuclide, ^{232}Th, is somewhat more abundant than ^{238}U, but has a longer half-life. As a result, the average rate of production of ^{220}Rn in the ground is about the same as that of ^{222}Rn. However, the shorter half-life of ^{220}Rn, 56 s, as compared with 3.82 d for ^{222}Rn, gives it a much greater chance to decay before becoming airborne. The contributions of the daughters of ^{220}Rn to lung dose are usually negligible compared with ^{222}Rn. The third (actinium) series produces ^{219}Rn, also called actinon, after several transformations from the relatively rare original nuclide ^{235}U. Its half-life is only 4 s, and its contribution to airborne radon is insignificant.

Example

Measurements of room air show the nuclide activity concentrations given in Table 4.1. Calculate the PAEC for this case.

Solution

The PAEC (and EEDC) pertain to the short-lived decay products and do not involve the radon itself, which is not retained by the lungs. To obtain the PAEC, we need to calculate the number of daughter atoms of each type per unit volume of air; multiply these numbers by the potential alpha-particle energy associated with each type of

Table 4.1

Nuclide	Activity Concentration (Bq m^{-3})
^{222}Rn	120
^{218}Po	93
^{214}Pb	90
^{214}Bi	76
^{214}Po	76

Table 4.2

Nuclide	A ($\mathrm{Bq\,m^{-3}}$)	λ ($\mathrm{s^{-1}}$)	N ($\mathrm{m^{-3}}$)	E (MeV)	NE ($\mathrm{MeV\,m^{-3}}$)
^{218}Po	93	3.79×10^{-3}	2.45×10^{4}	13.69	3.35×10^{5}
^{214}Pb	90	4.31×10^{-4}	2.09×10^{5}	7.69	1.61×10^{6}
^{214}Bi	76	5.83×10^{-4}	1.30×10^{5}	7.69	1.00×10^{6}
^{214}Po	76	4.23×10^{3}	1.80×10^{-2}	7.69	1.38×10^{-1}

atom; and then sum. The number of atoms N of a radionuclide associated with an activity A is given by Eq. (4.2), $N = A/\lambda$, where λ is the decay constant. For the first daughter, ^{218}Po, for example, we find from the half-life $T = 183$ s given in Fig. 4.7 (or Appendix D) that $\lambda = 0.693/T = 3.79 \times 10^{-3}\ \mathrm{s^{-1}}$. From the activity density $A = 93\ \mathrm{Bq\,m^{-3}}$ given in Table 4.1, it follows that the number density of ^{218}Po atoms is

$$N = \frac{A}{\lambda} = \frac{93\ \mathrm{Bq\,m^{-3}}}{3.79 \times 10^{-3}\ \mathrm{s^{-1}}} = 2.45 \times 10^{4}\ \mathrm{m^{-3}}, \tag{4.58}$$

where the units Bq and $\mathrm{s^{-1}}$ cancel. Each atom of ^{218}Po will emit a 6.00-MeV alpha particle. Each will also lead to the emission later of a 7.69-MeV alpha particle with the decay of its daughter ^{214}Po into ^{210}Pb. Thus, the presence of one ^{218}Po atom represents a potential alpha-particle energy $E = 6.00 + 7.69 = 13.69$ MeV from the short-lived radon daughters. Using N from Eq. (4.58), we find for the potential alpha-particle energy per unit volume contributed by the atoms of ^{218}Po, $NE = 3.35 \times 10^{5}\ \mathrm{MeV\,m^{-3}}$. Similar calculations can be made for the contributions of the other three daughters in Table 4.1 to the PAEC. The only modification for the others is that the potential alpha energy associated with each atom is 7.69 MeV (Fig. 4.7).

The complete calculation is summarized in Table 4.2. The individual nuclide contributions in the last column can be added to give the final answer, PAEC = $2.95 \times 10^{6}\ \mathrm{MeV\,m^{-3}}$. Note that the half-life of ^{214}Po, which is in secular equilibrium with ^{214}Bi (equal activity densities), is so short that very few atoms are present. Its contribution to the PAEC is negligible.

Example

Calculate the EEDC in the last example. What is the equilibrium factor?

Solution

By definition, the EEDC is the activity concentration of the short-lived radon daughters that would give a specified value of the PAEC under the condition of secular equilibrium. For the last example, the EEDC is the (equal) concentration that would appear in the second column of Table 4.2 for each nuclide that would result in the given value, PAEC = $2.95 \times 10^{6}\ \mathrm{MeV\,m^{-3}}$. The solution can be set up in more than one way. We compute the PAEC for secular equilibrium at unit activity density

Table 4.3

Nuclide	NE/A (MeV Bq^{-1})
^{218}Po	3.60×10^3
^{214}Pb	1.79×10^4
^{214}Bi	1.32×10^4
^{214}Po	1.82×10^{-3}

(1 Bq m^{-3}), from which the answer follows immediately. The contribution per unit activity from ^{218}Po, for example, is obtained from Table 4.2:

$$\frac{NE}{A} = \frac{3.35 \times 10^5 \text{ MeV m}^{-3}}{93 \text{ Bq m}^{-3}} = 3.60 \times 10^3 \text{ MeV Bq}^{-1}. \tag{4.59}$$

Values for the four nuclides are shown in Table 4.3. Adding the numbers in the second column gives a total of 3.47×10^4 MeV Bq^{-1}. This is the PAEC (MeV m^{-3}) per unit activity concentration (Bq m^{-3}) of each daughter in secular equilibrium. Therefore, for the last example we have

$$\begin{aligned} \text{EEDC} &= \frac{\text{PAEC}}{3.47 \times 10^4 \text{ MeV Bq}^{-1}} \\ &= \frac{2.95 \times 10^6 \text{ MeV m}^{-3}}{3.47 \times 10^4 \text{ MeV Bq}^{-1}} = 85.0 \text{ Bq m}^{-3}. \end{aligned} \tag{4.60}$$

The equilibrium factor is the ratio of this activity concentration and that of the radon (Table 4.1): $85.0/120 = 0.708$.

This example shows the useful relationship between an equilibrium concentration of 1 Bq m^{-3} of the short-lived radon daughters and the associated potential alpha-energy concentration:

$$\frac{3.47 \times 10^4 \text{ MeV m}^{-3}}{1 \text{ Bq m}^{-3}} = 3.47 \times 10^4 \text{ MeV Bq}^{-1}. \tag{4.61}$$

Note that the potential alpha energy per unit activity of the daughters in secular equilibrium is independent of the actual concentration of the daughters in air.

An older unit for the PAEC is the working level (WL), defined as a potential alpha-particle energy concentration of 1.3×10^5 MeV L^{-1} of air for the short-lived radon daughters. This value corresponds to the presence of 100 pCi L^{-1} = 3.7 Bq L^{-1} of the daughters in secular equilibrium, that is, to an EEDC of 3.7 Bq L^{-1}. The exposure of persons to radon daughters is often expressed in working-level months (WLM), with a working month defined as 170 h. The WLM represents the integrated exposure of an individual over a specified time period. Concentrations of radon itself are sometimes reported, rather than PAECs or WLs, which pertain to the daughters. As a rule of thumb, 1 WL of radon daughters is often associated with a radon concentration of 200 pCi L^{-1}, corresponding to an equilibrium factor of 0.5.

Example

A person spends an average of 14 hours per day at home, where the average concentration of radon is 1.5 pCi L^{-1} (a representative value for many residences). What is his exposure in WLM over a six-month period?

Solution

Using the rule of thumb just given, we estimate the daughter concentration to be (1.5/200) (1 WL) = 0.0075 WL. The exposure time in hours for the six months (= 183 d) is 183 × 14 = 2560 h. Since there are 170 h in a working month, the exposure time is 2560/170 = 15.1 working months. The person's exposure to radon daughters over the six-month period is therefore 0.0075 × 15.1 = 0.11 WLM.

4.7 Suggested Reading

1 Attix, F. H., *Introduction to Radiological Physics and Radiation Dosimetry*, Wiley, New York, NY (1986). [Chapter 6 discusses radioactive and serial decay.]

2 Bodansky, D., Robkin, M. A., and Stadler, D. R., eds., *Indoor Radon and Its Hazards*, University of Washington Press, Seattle, WA (1987).

3 Cember, H., *Introduction to Health Physics*, 3rd ed., McGraw-Hill, New York, NY (1996). [Chapter 4 treats radioactive transformations, serial decay, and naturally occurring radioactivity.]

4 Evans, R. D., *The Atomic Nucleus*, McGraw-Hill, New York, NY (1955). [Chapter 15 describes serial decay, decay schemes, equilibrium, and other topics.]

5 Faw, R. E., and Shultis, J. K., *Radiological Assessment*, Prentice-Hall, Englewood Cliffs, NJ (1993). [Chapter 4 gives and excellent coverage of exposure to natural sources of radiation, including radon.]

6 Magill, Joseph, and Galy, Jean, *Radioactivity-Radionuclides-Radiation*, Springer Verlag, New York, NY (2005). [This book discusses a number of environmental and human radiological health issues. It contains extensive nuclear data. A CD-ROM is included.]

7 National Research Council, *Health Risks of Radon and Other Internally Deposited Alpha-Emitters: BEIR IV*, National Academy Press, Washington, DC (1988). [A comprehensive report (602 pp.) from the Committee on the Biological Effects of Ionizing Radiation.]

8 Nazaroff, W. W., and Nero, A. V., Jr., eds., *Radon and its Decay Products in Indoor Air*, Wiley, New York, NY (1988).

9 NCRP Report No. 97, *Measurement of Radon and Radon Daughters in Air*, National Council on Radiation Protection and Measurements, Bethesda, MD (1988). [Contains basic data on the three natural decay series and characteristics of radon daughters discussed here. Main emphasis of the Report is on measurements.]

10 NCRP Report No. 103, *Control of Radon in Houses*, National Council on Radiation Protection and Measurements, Bethesda, MD (1989).

4.8 Problems

1. What is the value of the decay constant of ^{40}K?
2. What is the decay constant of tritium?
3. The activity of a radioisotope is found to decrease by 30% in 1 wk. What are the values of its
   (a) decay constant
   (b) half-life
   (c) mean life?
4. What percentage of the original activity of a radionuclide remains after
   (a) 5 half-lives
   (b) 10 half-lives?
5. The isotope ^{132}I decays by β^- emission into stable ^{132}Xe with a half-life of 2.3 h.
   (a) How long will it take for $\frac{7}{8}$ of the original ^{132}I atoms to decay?
   (b) How long it will take for a sample of ^{132}I to lose 95% of its activity?
6. A very old specimen of wood contained 10^{12} atoms of ^{14}C in 1986.
   (a) How many ^{14}C atoms did it contain in the year 9474 B.C.?
   (b) How many ^{14}C atoms did it contain in 1986 B.C.?
7. A radioactive sample consists of a mixture of ^{35}S and ^{32}P. Initially, 5% of the activity is due to the ^{35}S and 95% to the ^{32}P. At what subsequent time will the activities of the two nuclides in the sample be equal?
8. The gamma exposure rate at the surface of a shielded ^{198}Au source is 10 R h^{-1} (roentgen/hour, Sec. 12.2). What will be the exposure rate in this position after 2 wk?
9. Compute the specific activity of
   (a) ^{238}U
   (b) ^{90}Sr
   (c) ^{3}H.
10. How many grams of ^{32}P are there in a 5 mCi source?
11. How many atoms are there in a 1.16-MBq source of
    (a) ^{24}Na?
    (b) ^{238}U?
12. An encapsulated ^{210}Po radioisotope was used as a heat source, in which an implanted thermocouple junction converts heat into electricity with an efficiency of 15% to power a small transmitter for an early space probe.

(a) How many curies of ^{210}Po are needed at launch time if the transmitter is to be supplied with 100 W of electricity 1 y after launch?
(b) Calculate the number of grams of ^{210}Po needed.
(c) If the transmitter shuts off when the electrical power to it falls below 1 W, how long can it be expected to operate after launch?
(d) What health physics precautions would you recommend during fabrication, encapsulation, and handling of the device?

13. The Cassini spacecraft went into orbit about the planet, Saturn, in July 2004, after a nearly seven-year journey from Earth. On-board electrical systems were powered by heat from three radioisotope thermoelectric generators, which together utilized a total of 32.7 kg of ^{238}Pu, encapsulated as PuO_2. The isotope has a half-life of 86.4 y and emits an alpha particle with an average energy of 5.49 MeV. The daughter ^{234}U has a half-life of 2.47×10^5 y.
(a) Calculate the specific thermal-power generation rate of ^{238}Pu in W g^{-1}.
(b) How much total thermal power is generated in the spacecraft?

14. A 0.2-g sample of $^{85}_{36}$Kr gas, which decays into stable $^{85}_{37}$Rb, is accidentally broken and escapes inside a sealed warehouse measuring $40 \times 30 \times 20$ m. What is the specific activity of the air inside?

15. A 6.2-mg sample of ^{90}Sr is in secular equilibrium with its daughter ^{90}Y.
(a) How many Bq of ^{90}Sr are present?
(b) How many Bq of ^{90}Y are present?
(c) What is the mass of ^{90}Y present?
(d) What will the activity of the ^{90}Y be after 100 y?

16. A sample contains 1.0 GBq of ^{90}Sr and 0.62 GBq of ^{90}Y.
(a) What will be the total activity of the sample 10 days later?
(b) What will be the total activity of the sample 29.12 years later?

17. Consider the following β^- nuclide decay chain with the half-lives indicated:

$$^{210}_{82}\text{Pb} \xrightarrow[22\text{ y}]{\beta^-} {}^{210}_{83}\text{Bi} \xrightarrow[5.0\text{ d}]{\beta^-} {}^{210}_{84}\text{Po}.$$

A sample contains 30 MBq of ^{210}Pb and 15 MBq of ^{210}Bi at time $t = 0$.
(a) Calculate the activity of ^{210}Bi at time $t = 10$ d.

(b) If the sample was originally pure ^{210}Pb, then how old is it at time $t = 0$?

18. What is the mean life of a ^{226}Ra atom?

19. ^{59}Fe has a half-life of 45.53 d.
(a) What is the mean life of a ^{59}Fe atom?
(b) Calculate the specific activity of ^{59}Fe.
(c) How many atoms are there in a 10-mCi source of ^{59}Fe?

20. At time $t = 0$ a sample consists of 2 Ci of ^{90}Sr and 8 Ci of ^{90}Y.
(a) What will the activity of ^{90}Y be in the sample after 100 h?
(b) At what time will the ^{90}Y activity be equal to 3 Ci?

21. ^{136}Cs (half-life = 13.7 d) decays (β^-) into ^{136m}Ba (half-life = 0.4 s), which decays (γ) into stable ^{136}Ba:

$$^{136}\text{Cs} \xrightarrow[13.7\ \text{d}]{\beta^-} {}^{136\text{m}}\text{Ba} \xrightarrow[0.4\ \text{s}]{\gamma} {}^{136}\text{Ba}.$$

(a) Calculate the decay constant of ^{136}Cs.
(b) Calculate the specific activity of ^{136}Cs.
(c) Starting with a pure 10^{10}-Bq sample of ^{136}Cs at time $t = 0$, how many atoms of ^{136m}Ba decay between time $t_1 = 13.7$ d (exactly) and time $t_2 = 13.7\ \text{d} + 5\ \text{s}$ (exactly)?

22. Show that Eq. (4.40) leads to secular equilibrium, $A_1 = A_2$, under the appropriate conditions.

23. Show by direct substitution that the solution given by Eq. (4.40) satisfies Eq. (4.39).

24. A 40-mg sample of pure ^{226}Ra is encapsulated.
(a) How long will it take for the activity of ^{222}Rn to build up to 10 mCi?
(b) What will be the activity of ^{222}Rn after 2 years?
(c) What will be the activity of ^{222}Rn after 1000 years?
(d) What is the ratio of the specific activity of ^{222}Rn to that of ^{226}Ra?

25. Verify Eq. (4.43).

26. **(a)** Verify Eq. (4.44).
(b) Show that the time of maximum total activity occurs earlier than the time of maximum daughter activity in Fig. 4.5.
(c) Does Eq. (4.43) apply to A_2 when there is no equilibrium (Fig. 4.6)?

27. To which of the natural series do the following heavy radionuclides belong: $^{213}_{83}$Bi, $^{215}_{84}$Po, $^{230}_{90}$Th, $^{233}_{92}$U, and $^{224}_{88}$Ra?

28. The average mass of potassium in the human body is about 140 g. From the abundance and half-life given in Appendix D, estimate the average activity of ^{40}K in the body.

29. An atmosphere contains radon and its short-lived daughters in secular equilibrium at a concentration of 52 Bq m^{-3}.

(a) What is the PAEC?
(b) The equilibrium factor?

30. An air sample taken from a room shows the nuclide activity concentrations given in Table 4.4. Calculate
(a) the potential alpha-energy concentration
(b) the EEDC
(c) the equilibrium factor.

Table 4.4 Problem 30

Nuclide	Concentration ($Bq\,L^{-1}$)
^{222}Rn	9.2
^{218}Po	4.6
^{214}Pb	2.7
^{214}Bi	2.0
^{214}Po	2.0

31. A 5-L sample of air contains the activities (disintegrations per minute, dpm) of radon daughters shown in Table 4.5. Calculate
(a) the potential alpha-energy concentration
(b) the equilibrium-equivalent decay-product concentration.

Table 4.5 Problem 31

Nuclide	dpm
^{218}Po	1690
^{214}Pb	1500
^{214}Bi	1320
^{214}Po	1320

32. A room contains ^{222}Rn at a concentration of 370 $Bq\,m^{-3}$. The PAEC is 7.8×10^6 $MeV\,m^{-3}$. What is the equilibrium factor?

33. **(a)** Show that the EEDC for the short-lived ^{222}Rn daughters is given by

$$EEDC = 0.104C(^{218}Po) + 0.516C(^{214}Pb) + 0.380C(^{214}Bi),$$

where $C(^{218}Po)$, $C(^{214}Pb)$, and $C(^{214}Bi)$ are the concentrations of the daughters indicated.
(b) What units are implied in this expression?
(c) Why is the expression independent of $C(^{214}Po)$?

34. If the radon concentration (^{222}Rn) inside a building is 0.85 pCi L^{-1} and the equilibrium factor is 0.6, what is the rate of release of alpha-particle energy in MeV L^{-1} h^{-1}?

35. Show that
- **(a)** 1 WL = 2.1×10^{-5} J m^{-3}
- **(b)** 1 WLM = 0.0036 J h m^{-3}.

36. A 3-L air sample contains the following radon-daughter activities: ^{218}Po, 16.2 Bq; ^{214}Pb, 15.0 Bq; ^{214}Bi, 12.2 Bq; and ^{214}Po, 12.2 Bq. Calculate the WL concentration.

37. A person spends an average of 10 hours per day, 5 days per week, in an atmosphere where the average radon-daughter concentration is 0.68 WL. What is his exposure in WLM after one year of this activity?

38. A basement measures 12 m × 10 m × 2.5 m. The air inside contains the nuclide inventory shown in Table 4.6.
- **(a)** Calculate the WL concentration.
- **(b)** If the given activities are average and a person occupies the basement 10 hours per day, 7 days per week, for 12 months, what will be his exposure in WLM?

Table 4.6 Problem 38

Nuclide	Activity (μCi)
^{222}Rn	0.81
^{218}Po	0.69
^{214}Pb	0.44
^{214}Bi	0.25
^{214}Po	0.25

39. A room measures 10 m × 8 m × 3 m. It contains 80 pCi L^{-1} of ^{218}Po, 60 pCi L^{-1} of ^{214}Pb, and 25 pCi L^{-1} each of ^{214}Bi and ^{214}Po.
- **(a)** Calculate the WL concentration in the room.
- **(b)** Calculate the total potential alpha-particle energy in the room.
- **(c)** What is the concentration of ^{214}Po atoms in the air?
- **(d)** If secular equilibrium existed at this working-level concentration, what would be the activity concentration of ^{214}Pb atoms?
- **(e)** What would be the exposure in WLM of an individual who occupied the room 12 hours per day, 6 days per week, for one year?

40. An atmosphere contains the numbers of atoms per liter shown in Table 4.7.
- **(a)** Calculate the PAEC in J m^{-3}.
- **(b)** Calculate the EEDC.
- **(c)** Calculate the equilibrium factor.

Table 4.7 Problem 40

Nuclide	Atoms L^{-1}
^{222}Rn	2.34×10^5
^{218}Po	52
^{214}Pb	407
^{214}Bi	214
^{214}Po	2

4.9 Answers

2. 0.0564 y^{-1}
5. **(a)** 6.90 h
(b) 9.94 h
6. **(a)** 4.00×10^{12}
(b) 1.62×10^{12}
7. 72.5 d
10. 1.75×10^{-8} g
12. **(a)** 1.30×10^5 Ci
(b) 28.9 g
(c) 1280 d
13. **(a)** 0.575 W g^{-1}
(b) 18.8 kW
14. 120 MBq m^{-3}
15. **(a)** 3.13×10^{10} Bq
(b) 3.13×10^{10} Bq
(c) 1.56 μg
(d) 2.90×10^9 Bq
16. **(a)** 1.97 GBq
(b) 1.00 GBq
21. **(a)** 0.0506 d^{-1}
(b) 2.59×10^{18} Bq kg^{-1}
(c) 2.5×10^{10}
30. **(a)** 9.15×10^7 MeV m^{-3}
(b) 2630 Bq m^{-3}
(c) 0.29
31. **(a)** 1.68×10^8 MeV m^{-3}
(b) 4830 Bq m^{-3}
36. 1.25 WL
37. 10.4 WLM
39. **(a)** 0.481 WL
(b) 1.50×10^{10} MeV
(c) 2.19×10^{-4} L^{-1}
(d) 48.1 pCi L^{-1}
(e) 10.6 WLM