

# Neutrino Physics and Dark Matter

**LPHYS2223**

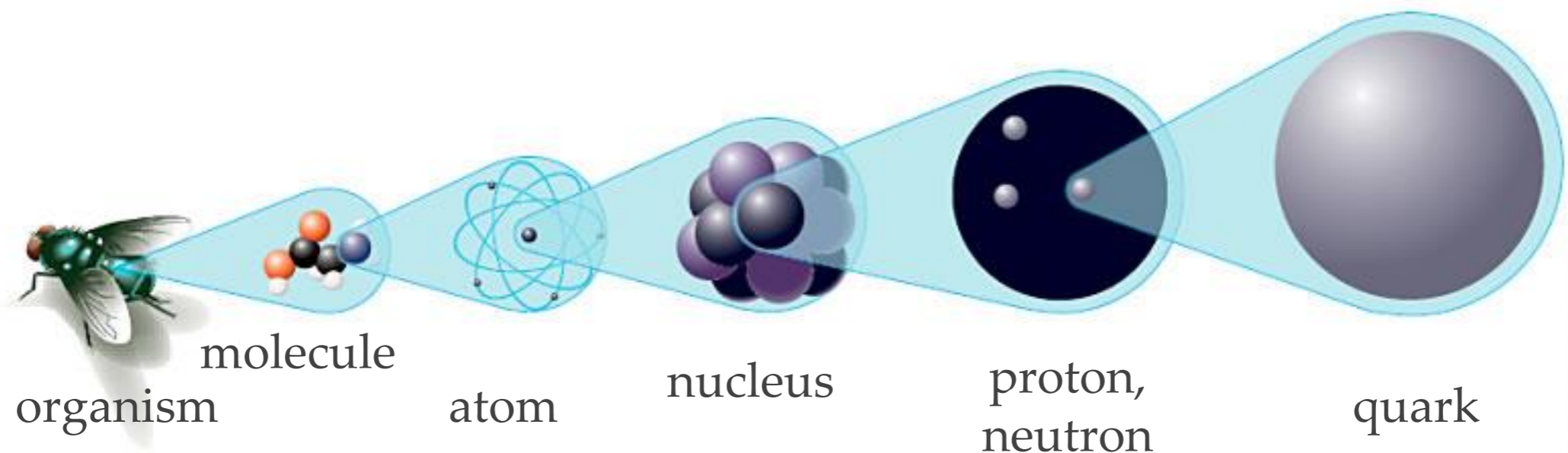
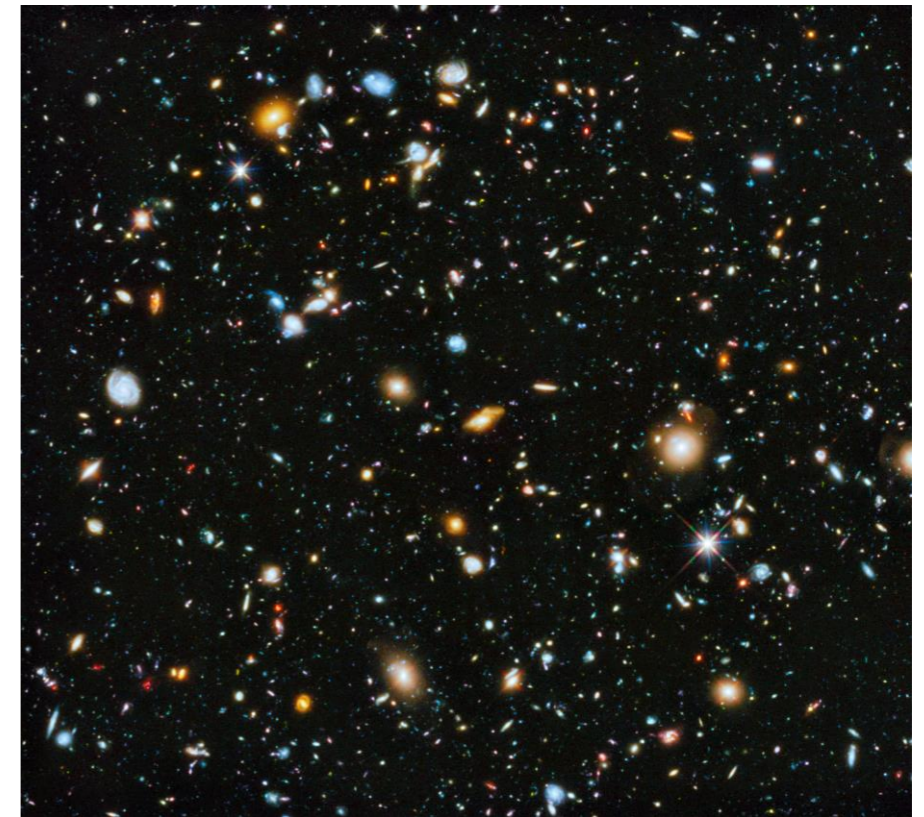
**Marco Drewes**

**Université catholique de Louvain**

# The Standard Model of Particle Physics

# The elementary constituents of matter...

What are the fundamental laws of Nature, and how do they shape the universe that we live in?



# ...and the forces between them

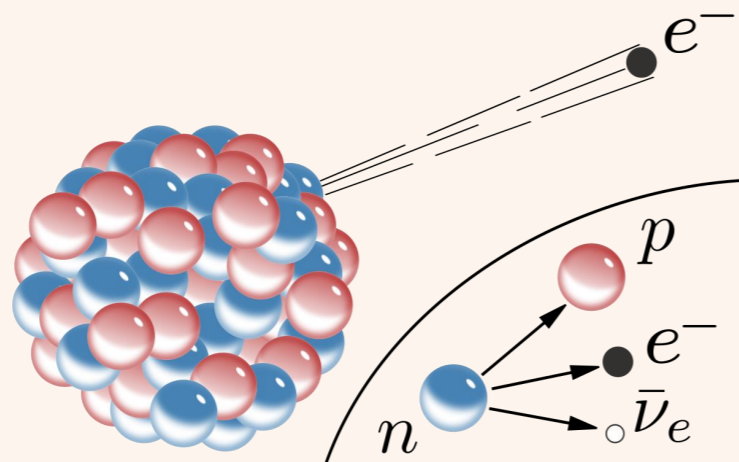
Gravity



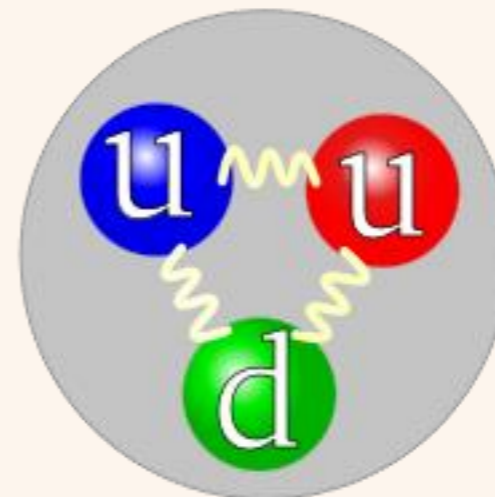
Electromagnetism



Weak Nuclear Force



Strong Nuclear Force





# The Standard Model of Particle Physics

"fermions" = matter particles				"bosons" = force carriers		Brout Englert Higgs mechanism	
	I	II	III				
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0	0	125 GeV	
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0		Higgs boson
name →	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon			
	Left Right	Left Right	Left Right				
				0	0	spin 0	
				<b><math>\gamma</math></b> photon			
				91.2 GeV	0		
				<b>Z<sup>0</sup></b> weak force			
<b>Quarks</b>							
	4.8 MeV	104 MeV	4.2 GeV				
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$				
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom				
	Left Right	Left Right	Left Right				
	0 eV	0 eV	0 eV				
	0	0	0				
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino				
	Left Right	Left Right	Left Right				
	0.511 MeV	105.7 MeV	1.777 GeV				
	-1	-1	-1				
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau				
	Left Right	Left Right	Left Right				
<b>Leptons</b>							
				80.4 GeV	$\pm 1$		
				<b>W<sup>±</sup></b> weak force			

The "periodic table" of elementary particles

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# What the SM cannot explain

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# What the SM cannot explain

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## What is the Dark Matter made of?

- About 80% of the mass in the observable universe are made of an unknown non-luminous (“dark”) substance (which is actually transparent, not dark...)

## Neutrino Flavour oscillations / Neutrino masses

- Where do neutrino masses come from, and why are they so small? Why is the neutrino mixing matrix so different from the quark mixing matrix?

## Baryon asymmetry of the universe

- Why was more matter than antimatter created in the “big bang”?

## Big bang initial conditions

- Why is the universe so homogeneous and isotropic and spatially flat?

## Theoretical consistency

- What about quantum gravity?

## Various anomalies

- Hubble tension, muon magnetic moment...

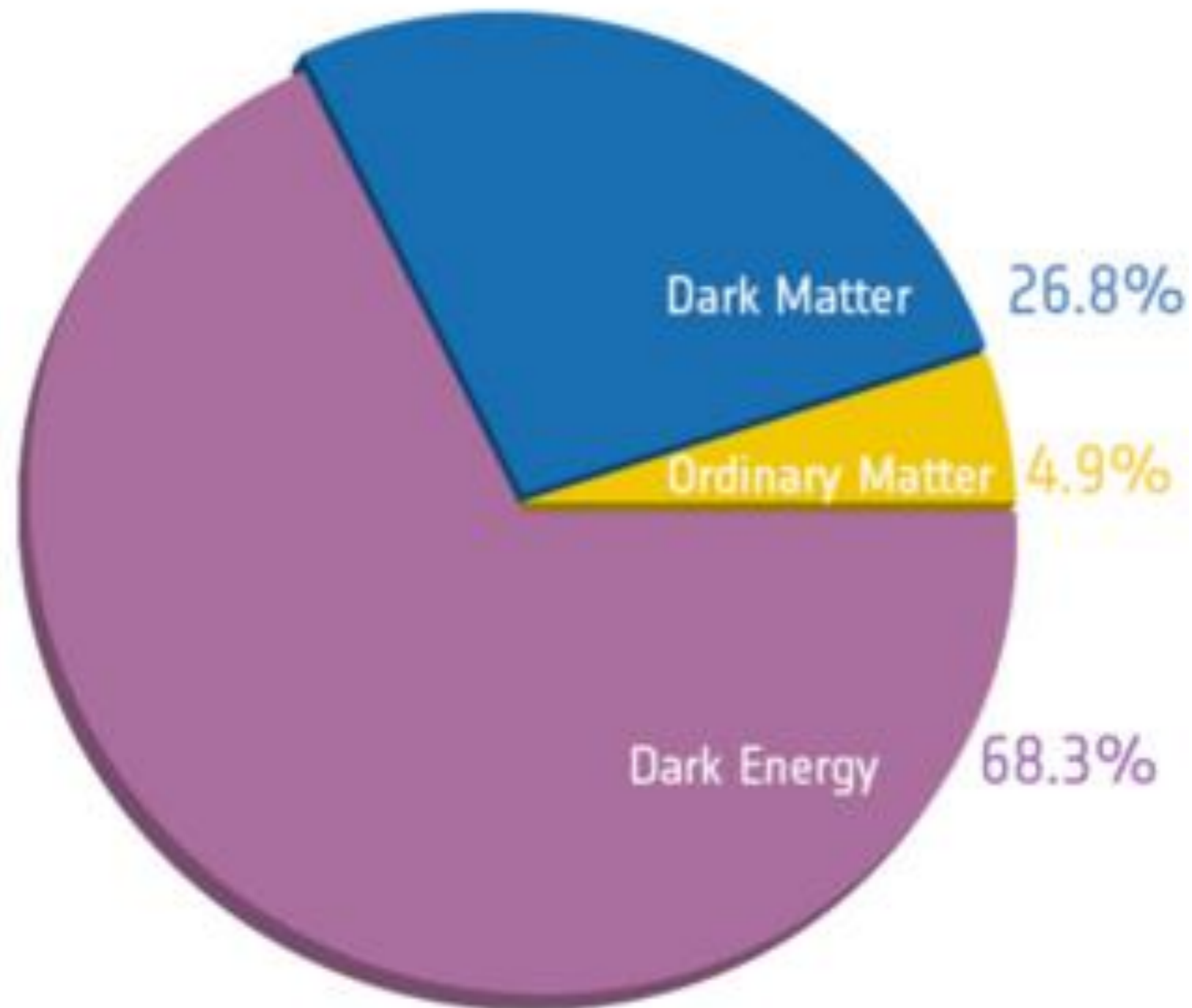
## Tuning problems and “naturalness”

- Why are parameters so vastly different (e.g. Planck mass vs Higgs mass, value of cosmological constant...)

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# The Dark Matter Puzzle

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Neutrinos



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# Why are neutrinos interesting?

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# Why are neutrinos interesting?

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## Neutrino oscillations and neutrino masses

- Neutrino masses are the only firmly established piece of evidence for physics beyond the SM seen in the laboratory.
- They may provide a key how to embed the SM in a more fundamental theory of nature

## Neutrino masses and global symmetries in nature: Majorana or Dirac?

- Neutrino masses violate lepton flavour, a global  $U(1) \times U(1) \times U(1)$  in the SM
- Neutrinos could be their own antiparticles and thereby violate the total lepton number conservation in the SM!

## Neutrino masses and discrete symmetries in nature: Origin of matter?

- Neutrino oscillations appear to violate CP – related to matter-antimatter asymmetry? (“leptogenesis”)
- Neutrinos are the only elementary fermions that are known only with LH parity – key to understanding parity violation in nature?

## Neutrino masses and Dark Matter

- “Sterile neutrinos” could be the Dark Matter
- Neutrinos could be a “portal” to an extended “Dark Sector”

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# References

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## Solar neutrino problem (and neutrino physics in general)

- Raffelt - Stars as a Laboratory for Fundamental Physics  
<http://wwwth.mpp.mpg.de/members/raffelt/mypapers/199613.pdf>
- Giunti - Neutrino Physics and Astrophysics  
[http://theor.jinr.ru/~vnaumov/Eng/JINR\\_Lectures/books/Giunti2007.pdf](http://theor.jinr.ru/~vnaumov/Eng/JINR_Lectures/books/Giunti2007.pdf)

## Massive neutrinos in cosmology

- Lesgourgues/Pastor - Massive Neutrinos and Cosmology, [astro-ph/0603494](https://arxiv.org/abs/astro-ph/0603494)

## Sterile neutrino Dark Matter

- Boyarsky et al - Sterile Neutrino Dark Matter, [arXiv:1807.07938](https://arxiv.org/abs/1807.07938) [hep-ph]

## Leptogenesis , right handed neutrinos

- Buchmuller/Peccei/Yanagida, Leptogenesis as the Origin of Matter, [hep-ph/0502169](https://arxiv.org/abs/hep-ph/0502169)
- Canetti et al, Matter and antimatter in the universe, [arXiv:1204.4186](https://arxiv.org/abs/1204.4186) [hep-ph]
- Drewes, The phenomenology of right handed neutrinos, [arXiv:1303.6912](https://arxiv.org/abs/1303.6912) [hep-ph]

# Neutrino Detection

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# Neutrino Experiments

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**One can distinguish them by neutrino source:**

reactor, particle accelerator, sun, cosmic rays

**One can distinguish them by detection method:**

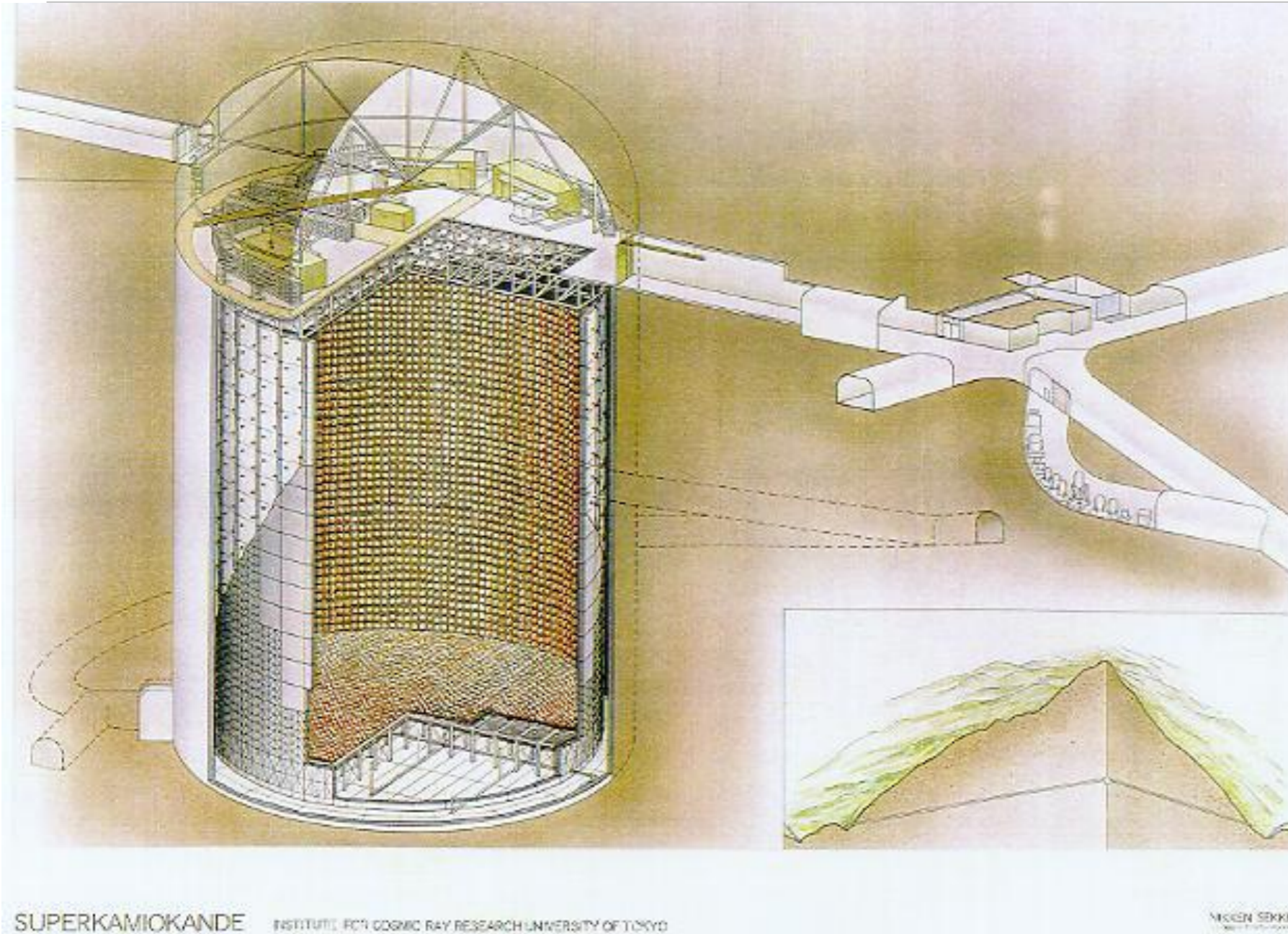
Cherenkov light, nuclear reaction

**One can distinguish them by what they observe:**

appearance or disappearance



# Super Kamiokande



Water Cherenkov  
detector

Sees:

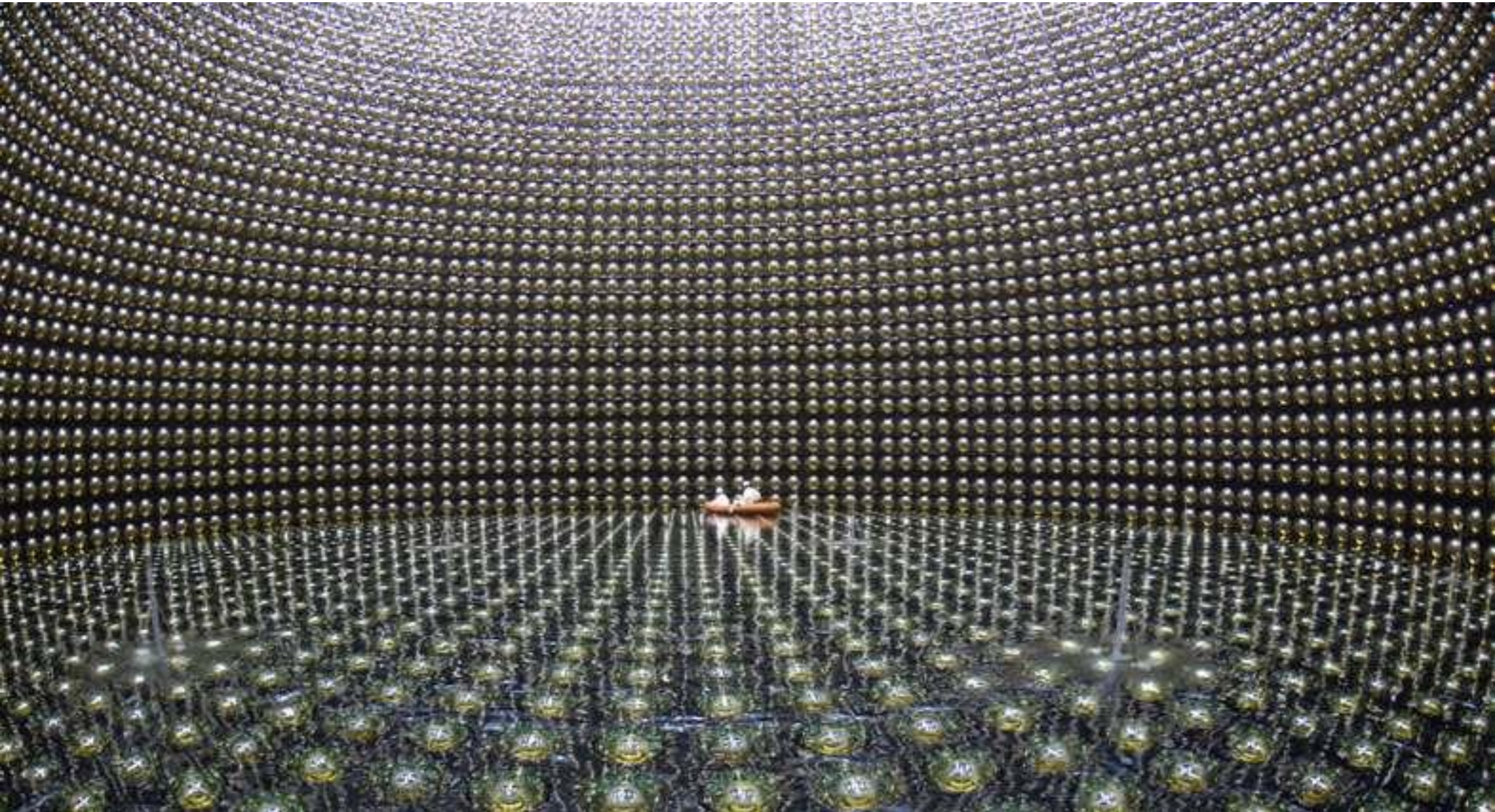
- solar neutrinos
- atmospheric neutrinos
- neutrinos from T2K beam
- reactor neutrinos



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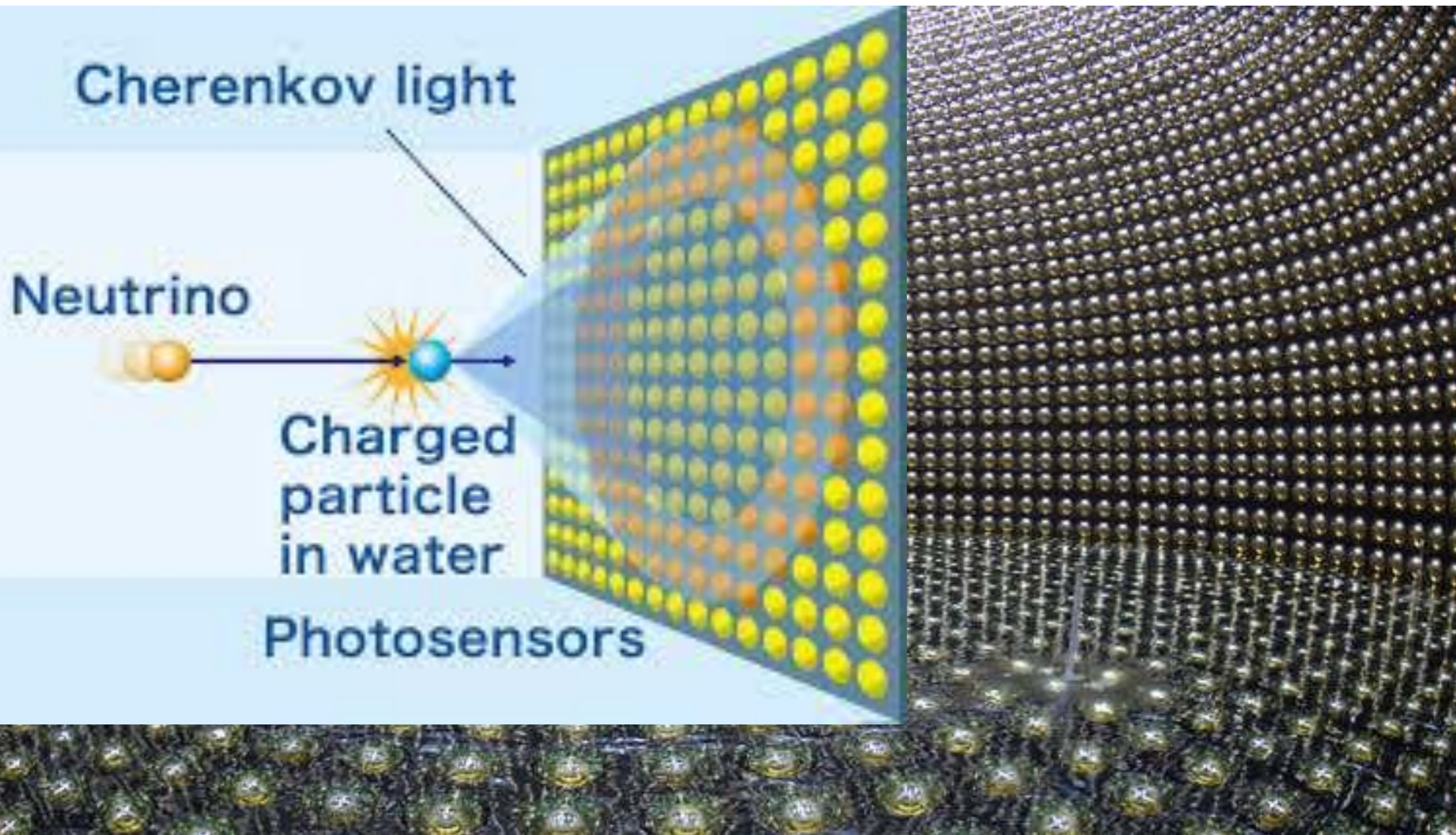
# Super Kamiokande

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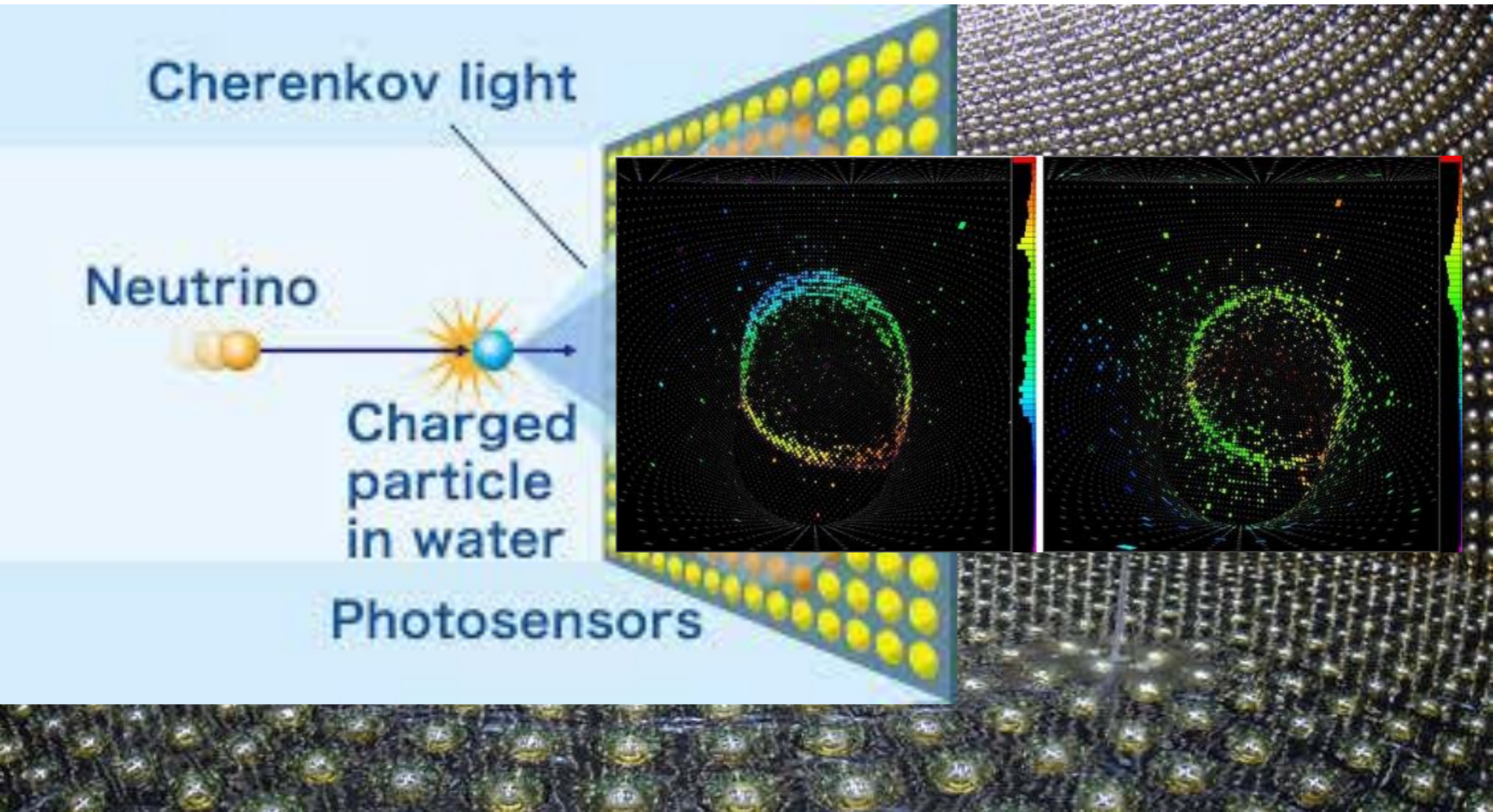


# Super Kamiokande





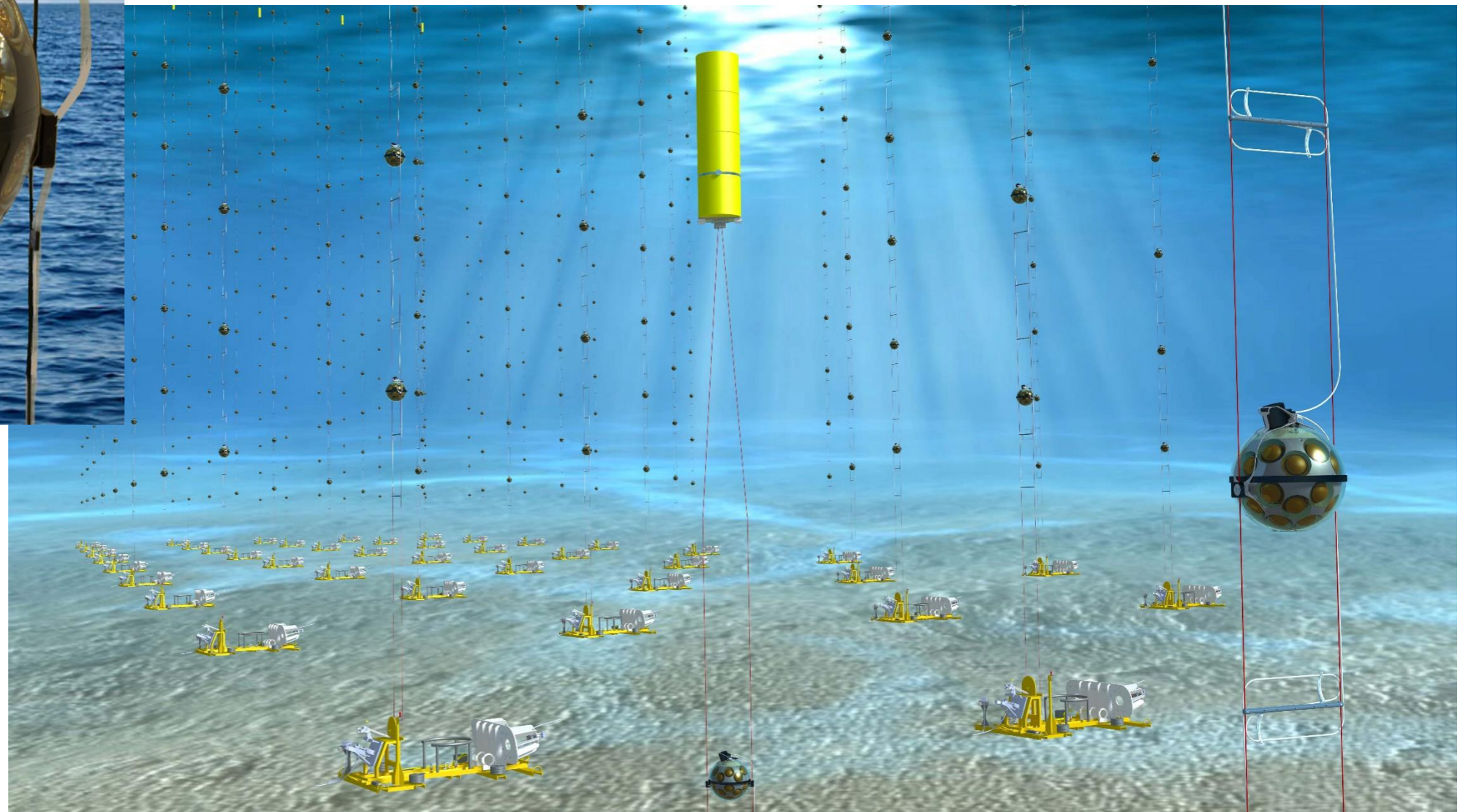
# Super Kamiokande





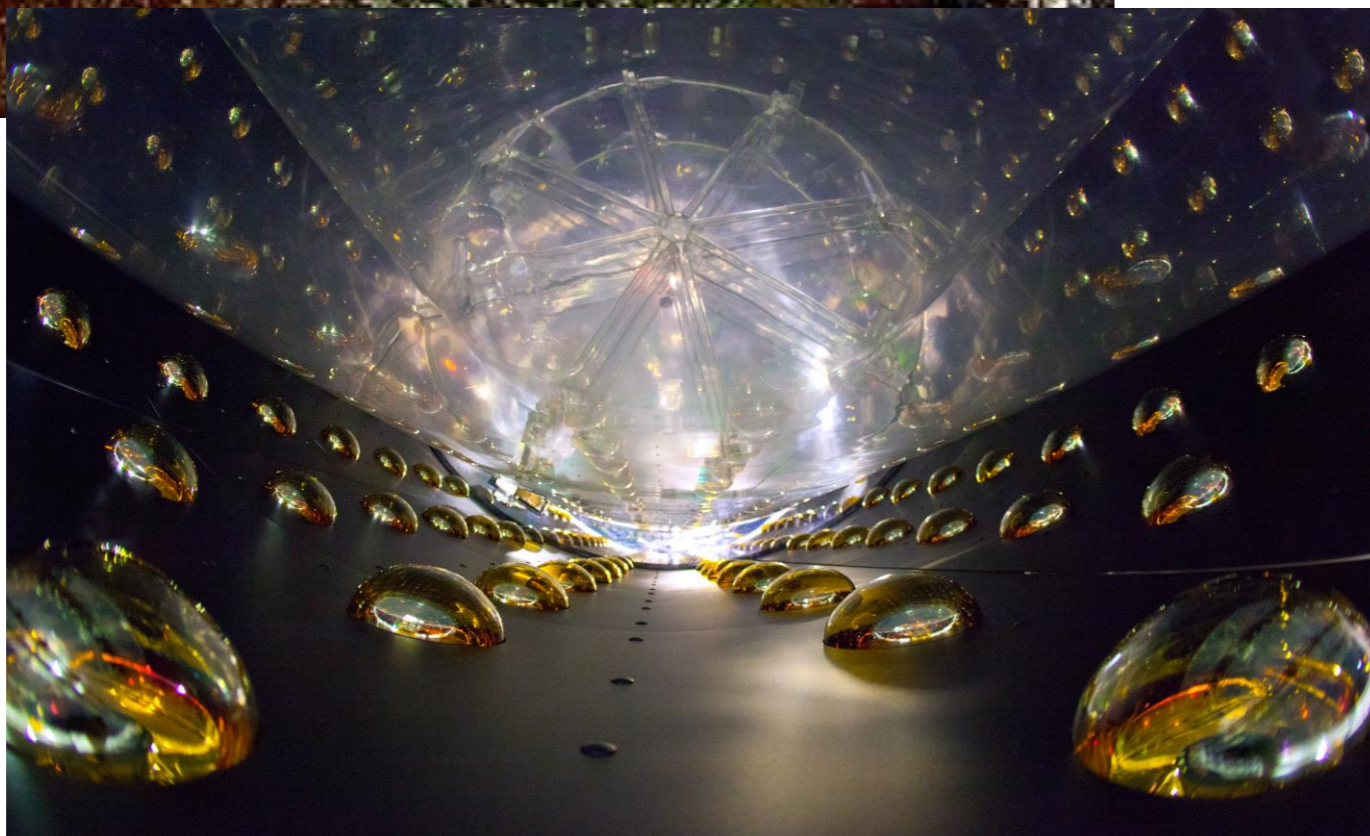
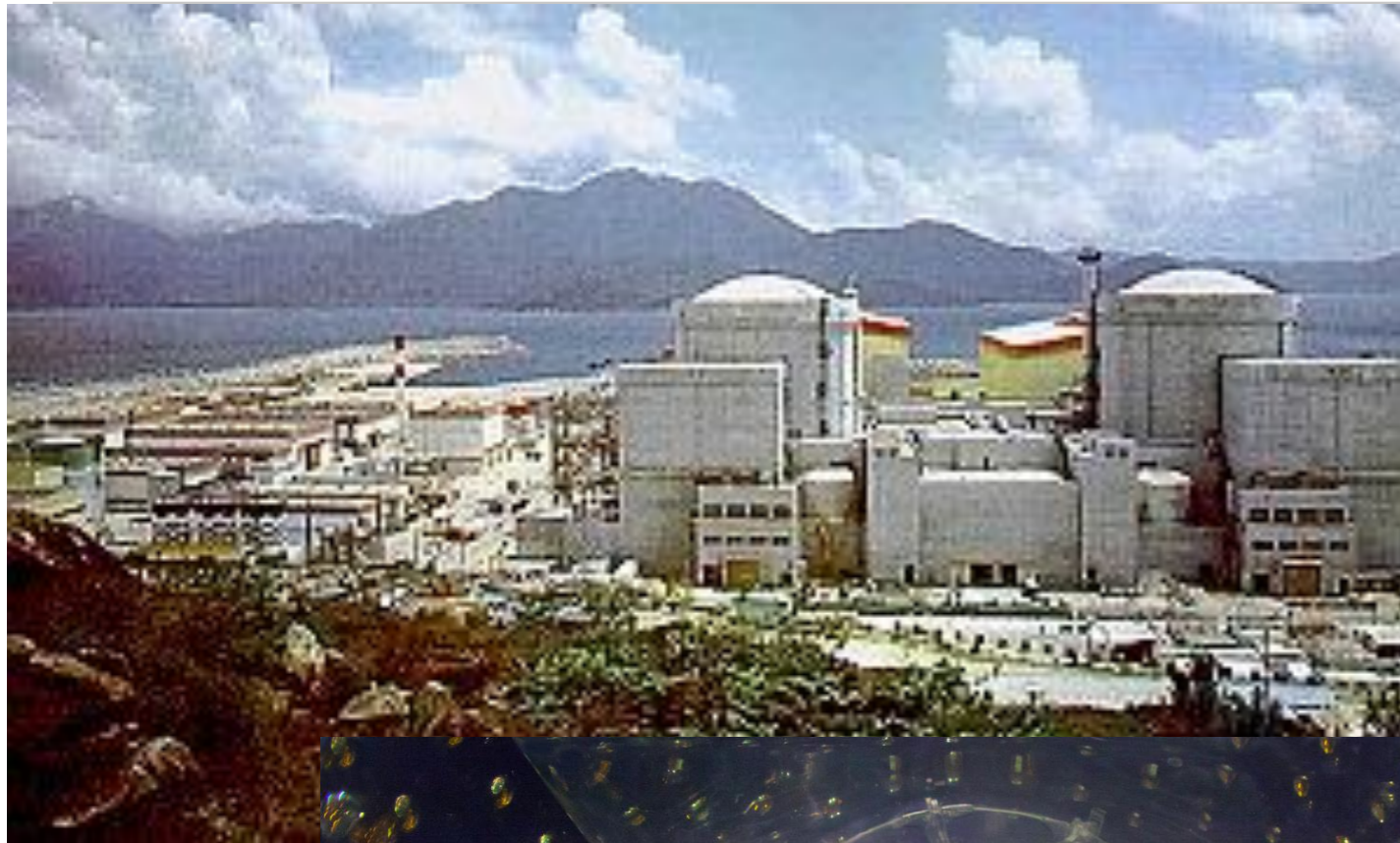
# KM3NeT

- Water Cherenkov detector in the Mediterranean Sea
- Three locations: M3NeT-Fr (off Toulon, France), KM3NeT-It (off Portopalo di Capo Passero, Sicily, Italy) and KM3NeT-Gr (off Pylos, Peloponnese, Greece)
- Sub-detectors ARCA (TeV-PeV energies) and ORCA (GeV energies)





# Daya Bay



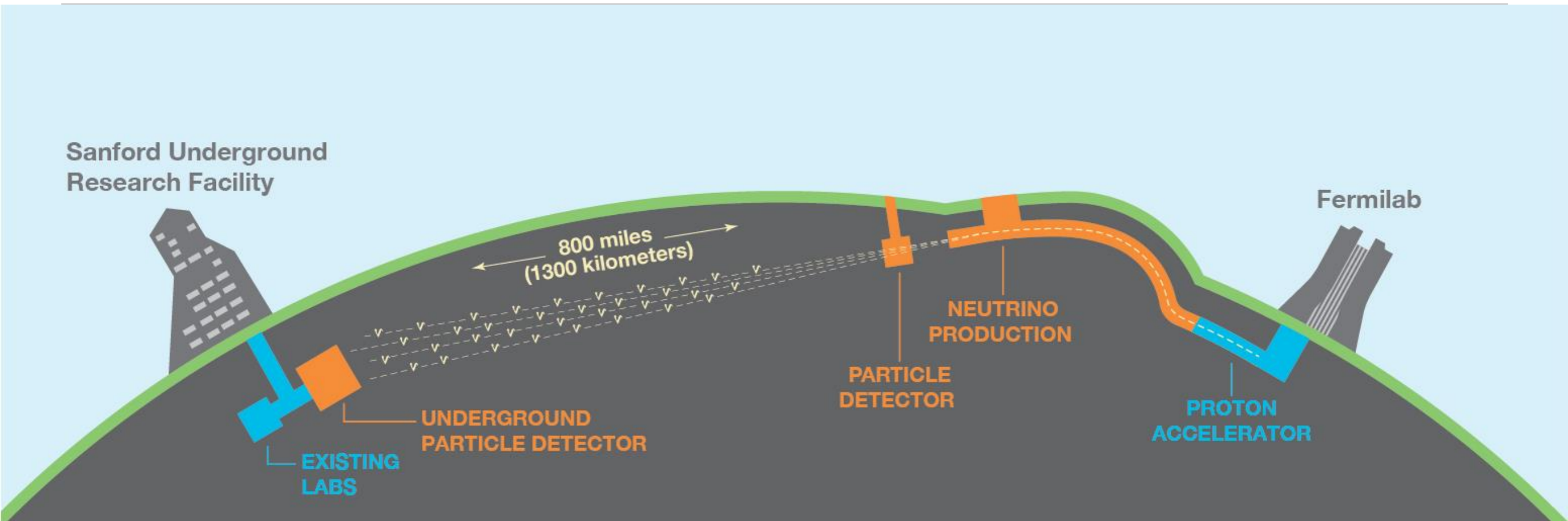
**liquid scintillator**

**built next to reactor**

**discovered  $\theta_{13}$**



# DUNE



## Liquid Argon Time-Projection Chamber

[https://www.youtube.com/watch?v=R5G1\\_hW0ZUA#action=share](https://www.youtube.com/watch?v=R5G1_hW0ZUA#action=share)

catches beam for Fermilab

[https://www.youtube.com/watch?v=U\\_xWDWKq1CM](https://www.youtube.com/watch?v=U_xWDWKq1CM)

is expected to discover CP violation in electron neutrino appearance

# IceCube



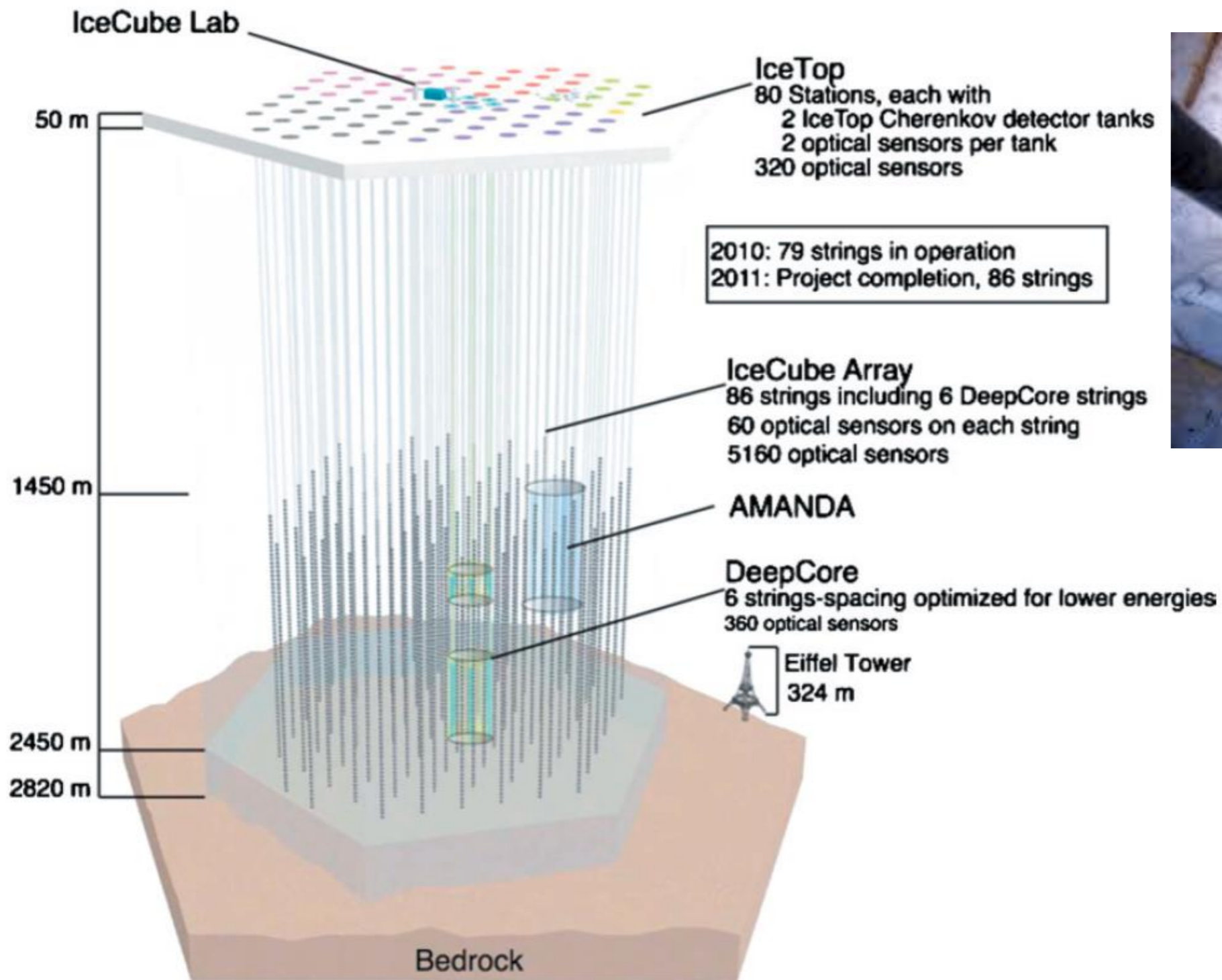


# IceCube

Represented at UCLouvain by  
Gwenhael de Wasseige



# IceCube





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# Neutrinos as Messengers

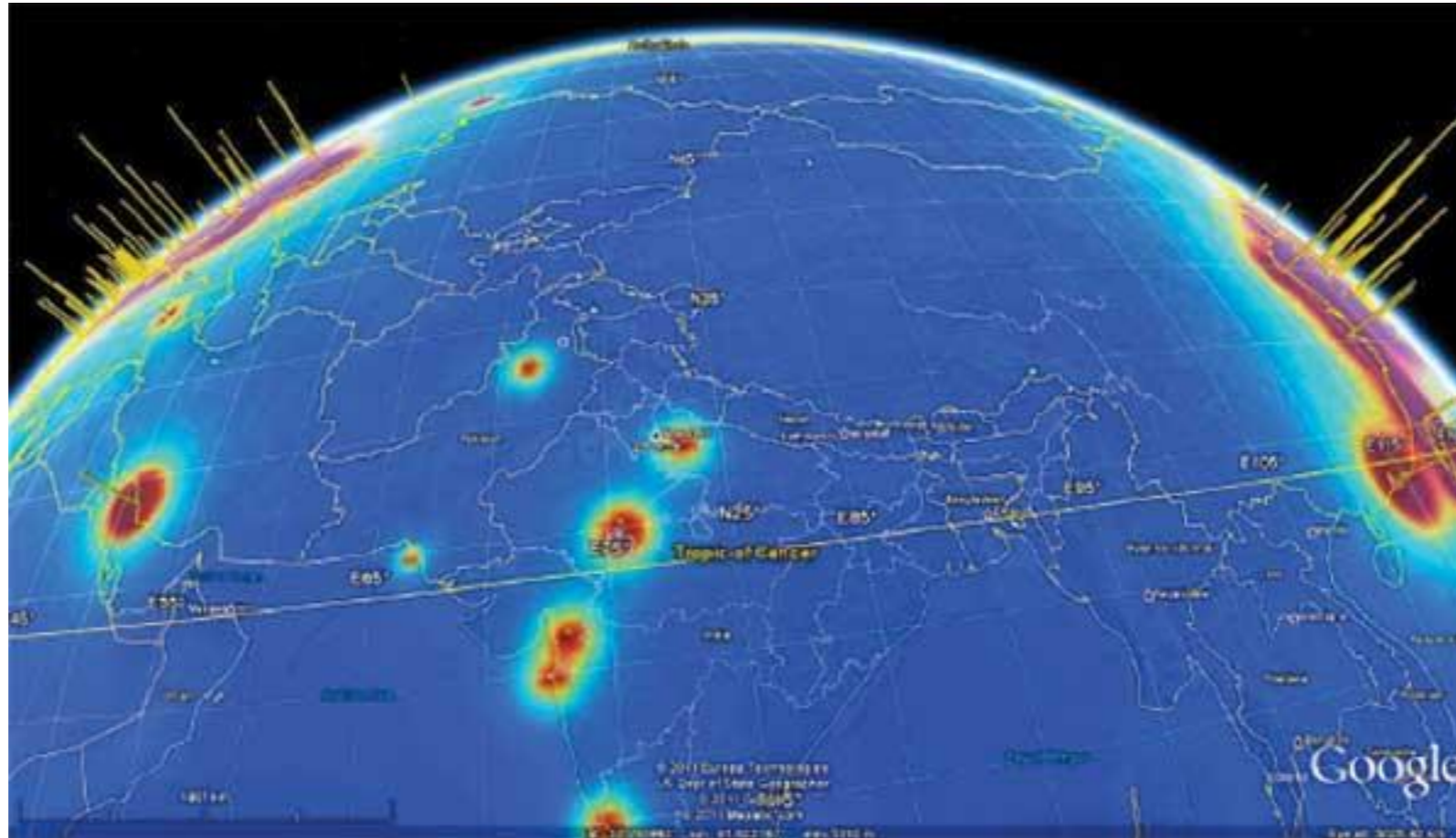
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- **monitor sources on earth**
- **probe the inner structure of earth**
- **monitor nuclear fusion in stars**
- **provide insight into supernova explosions**
- **provide information about the early universe**

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# Sources on Earth

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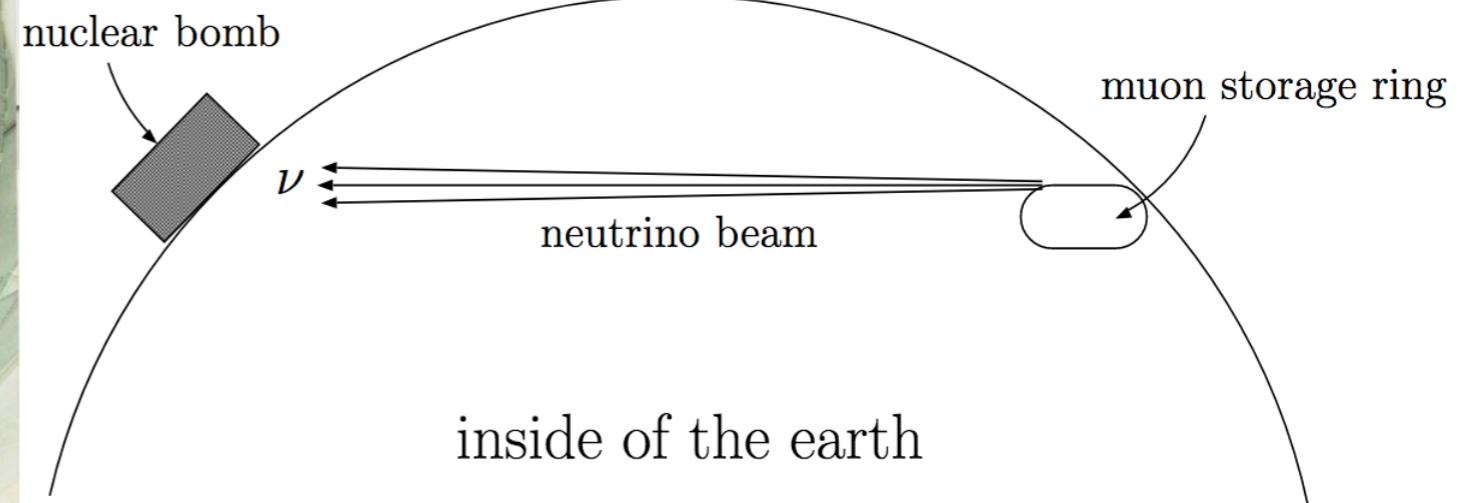
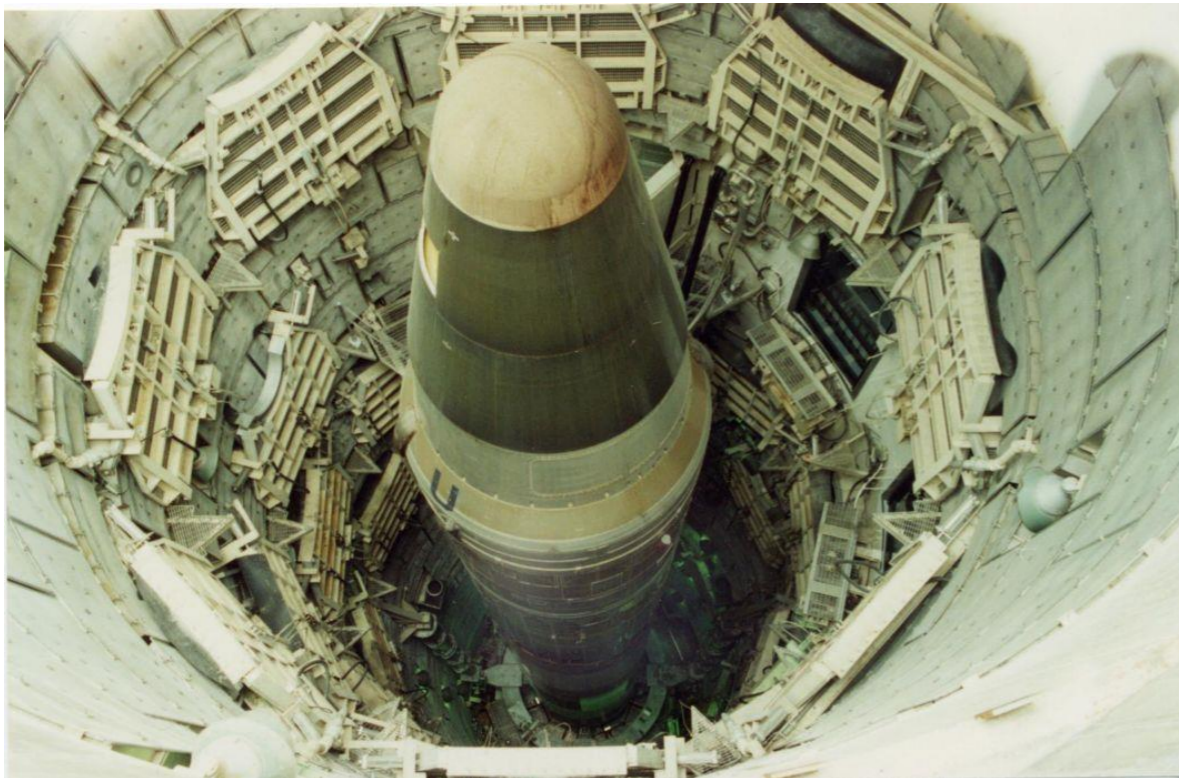


## **WATER CHerenkov Monitor of AntiNeutrinos (WATCHMAN):**

aims to monitor enrichment of nuclear fuel from a distance  
part of nuclear non proliferation control

# Can one destroy nuclear weapons with Neutrinos?

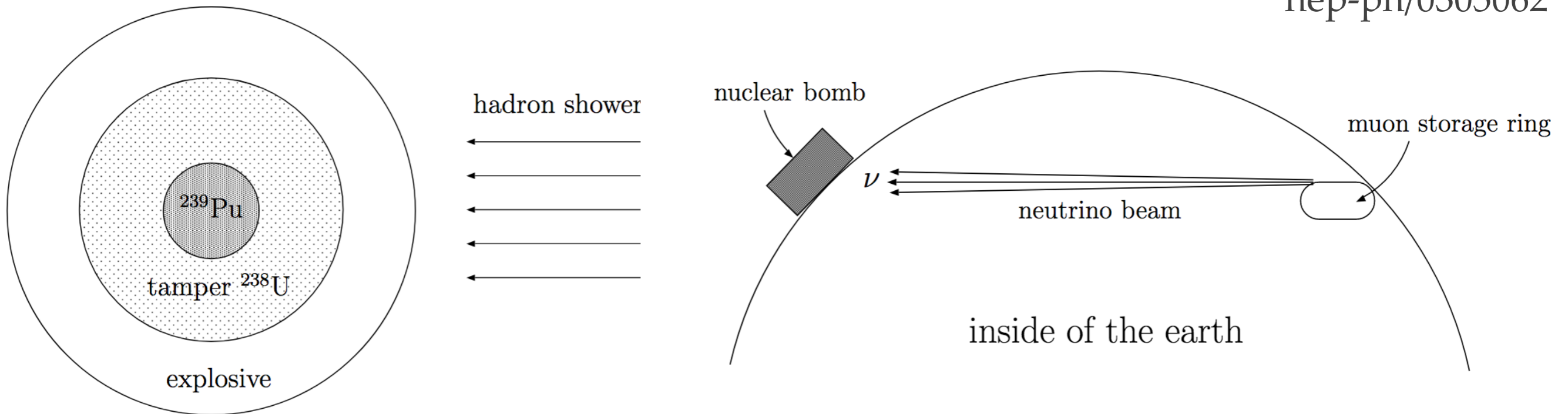
hep-ph/0305062



Can one detonate the enemy's weapons inside their own silo?

# Can one destroy nuclear weapons with Neutrinos?

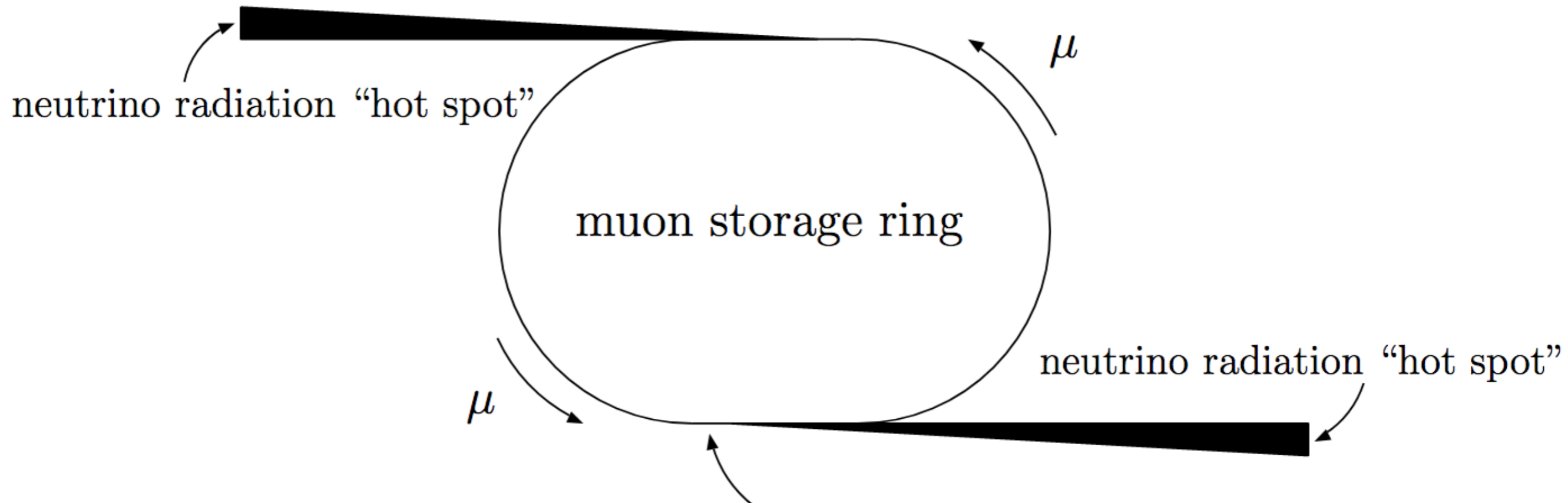
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Can one detonate the enemy's weapons inside their own silo?

Would need 1000 TeV neutrinos... if it works at all...

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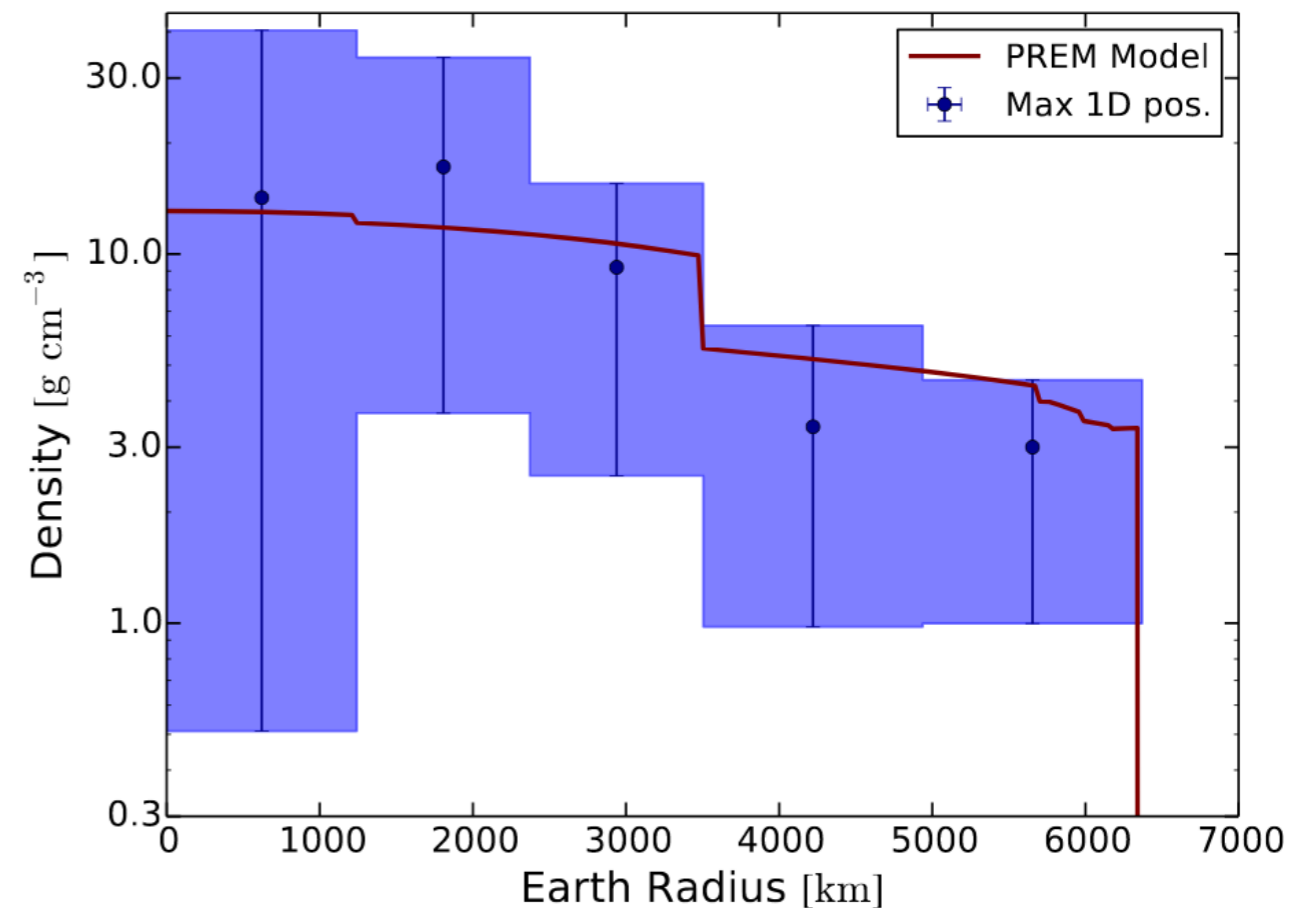
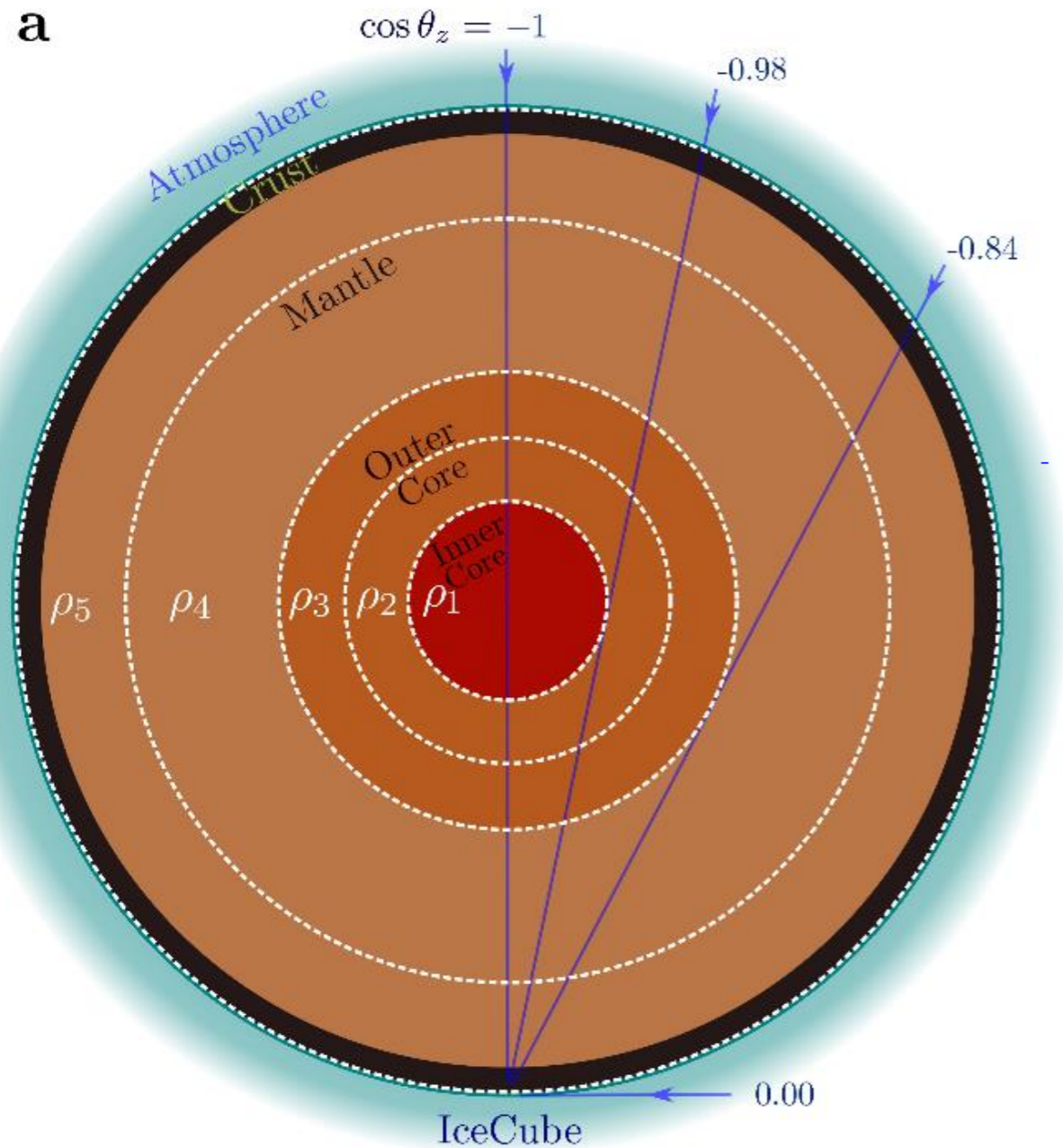


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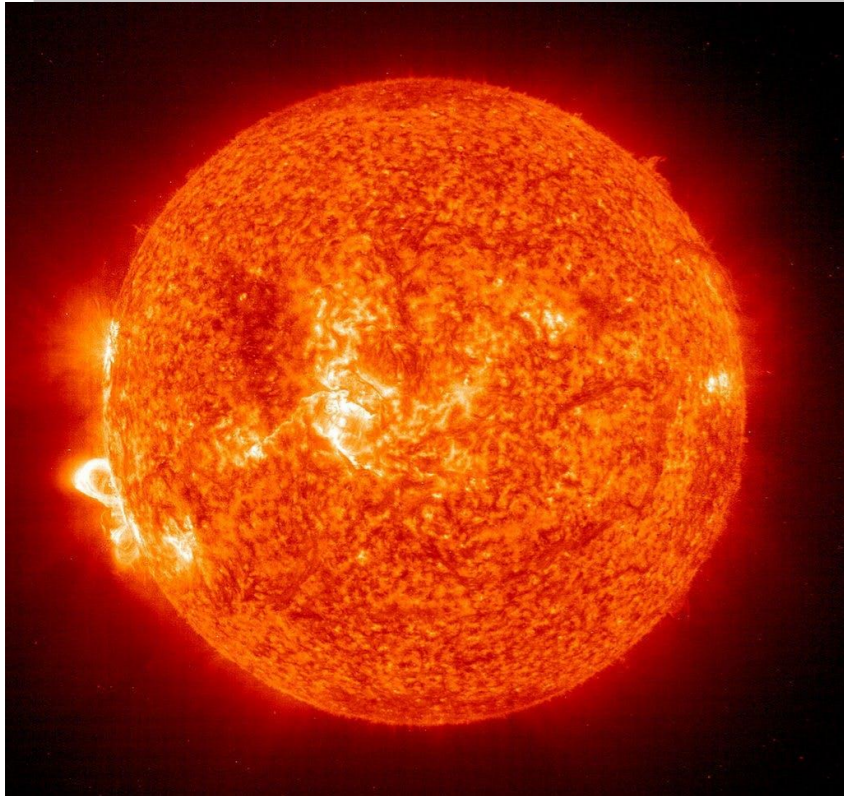


# Looking inside the Earth...

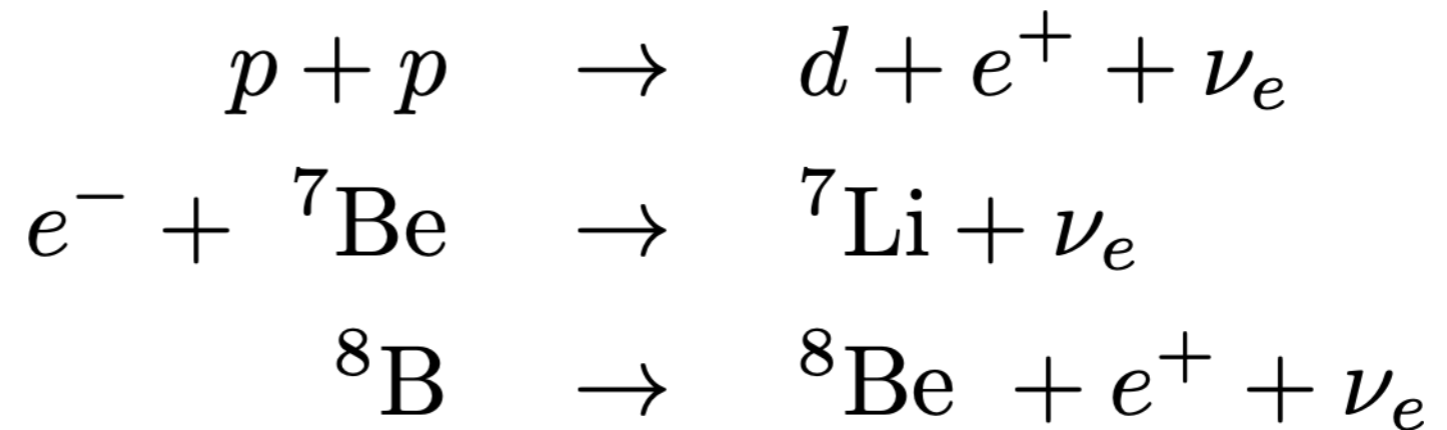




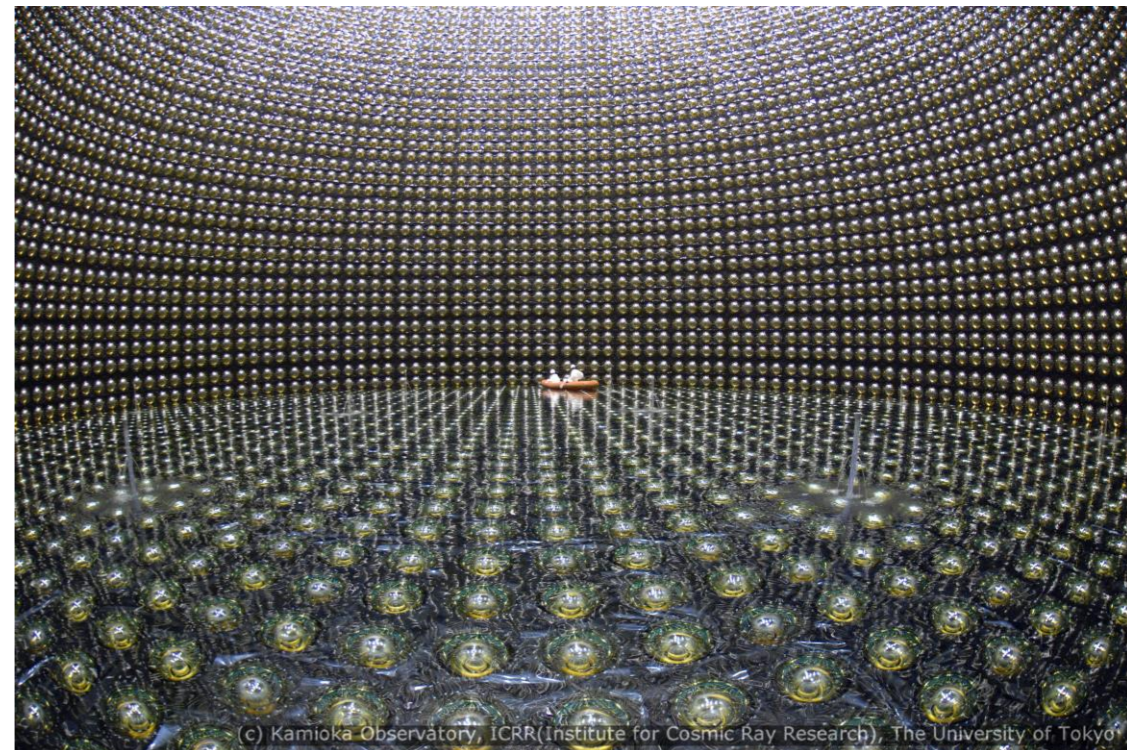
# Fussion in Stars



Nuclear fusion creates electron neutrinos



Solar neutrinos provide an important consistency check for the solar model



# Core Collapse Supernova

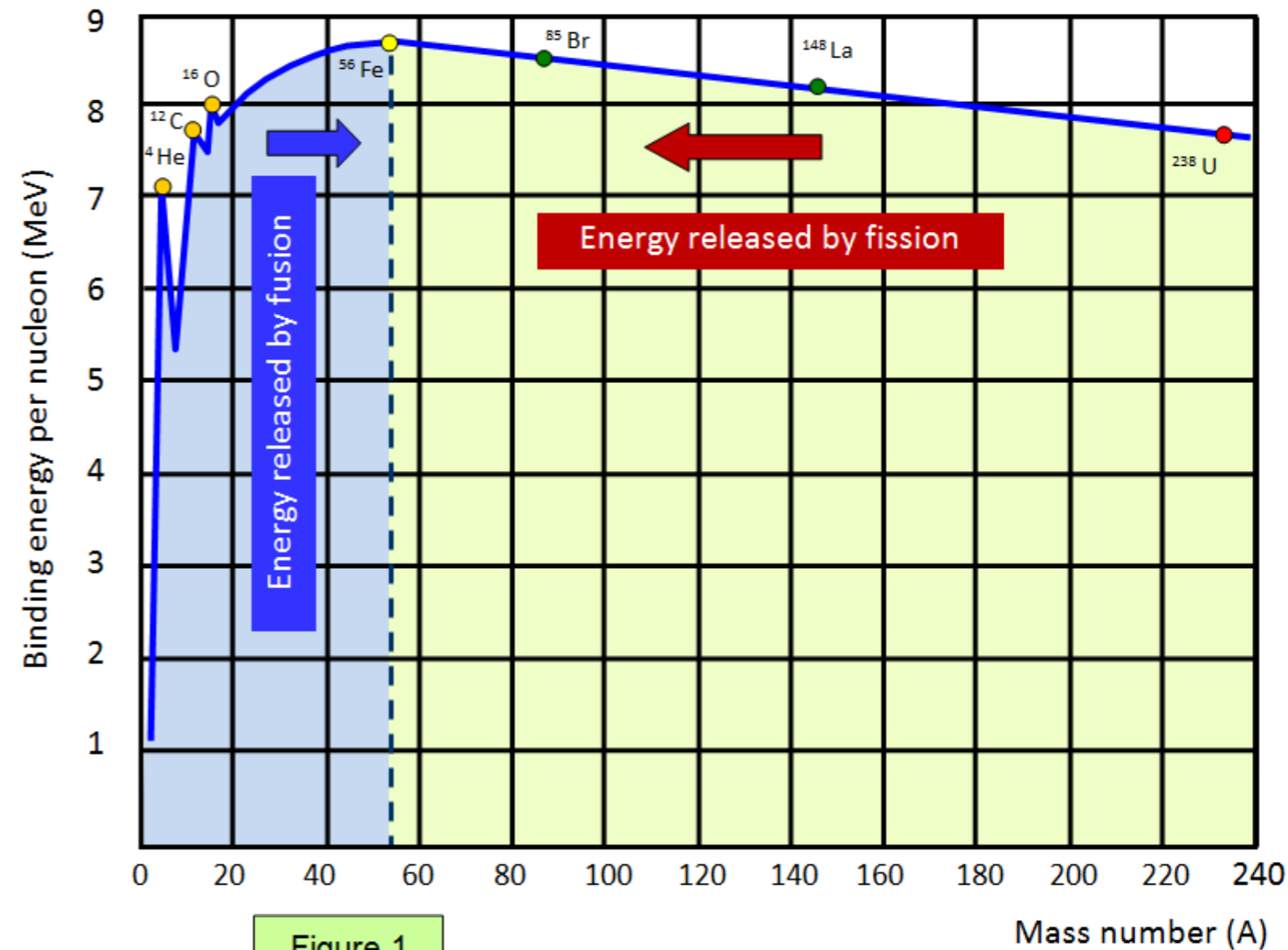
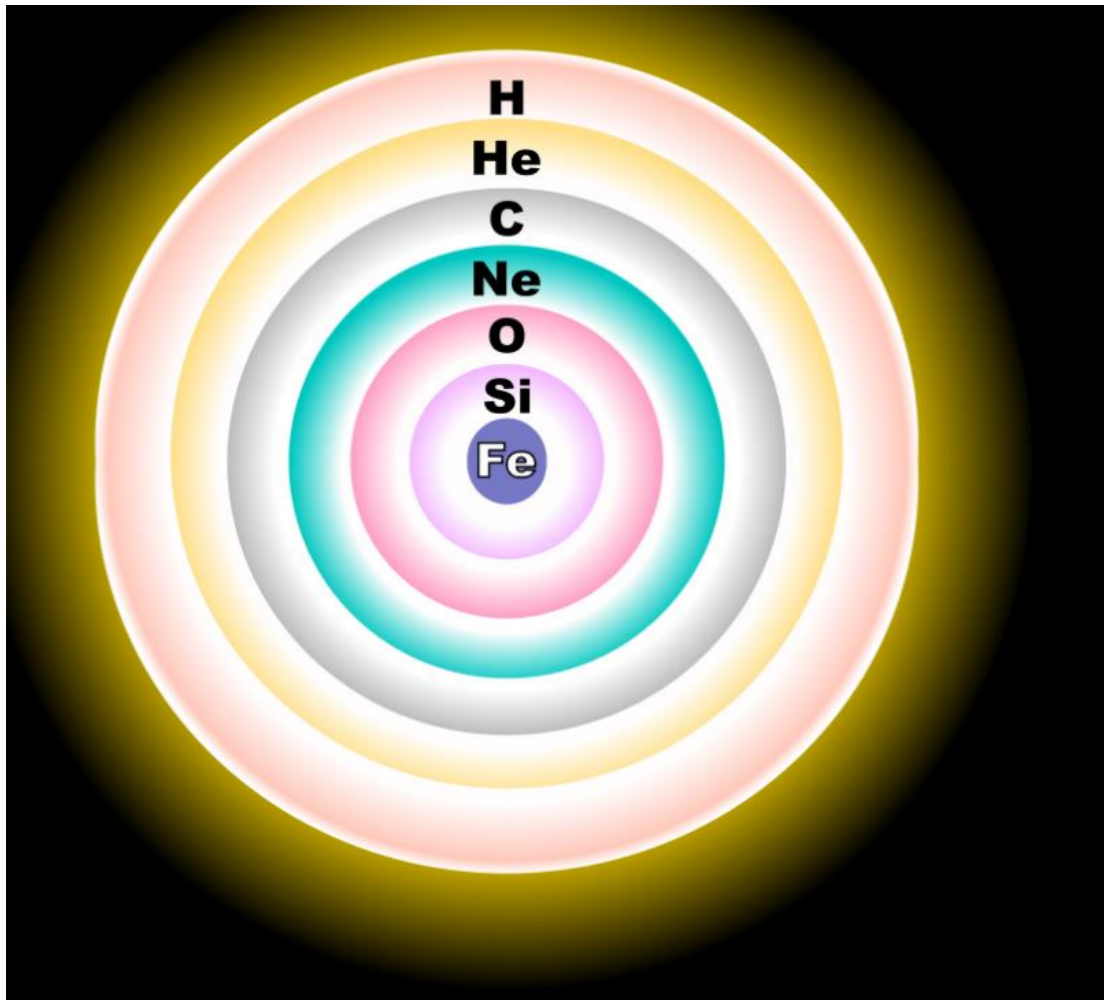
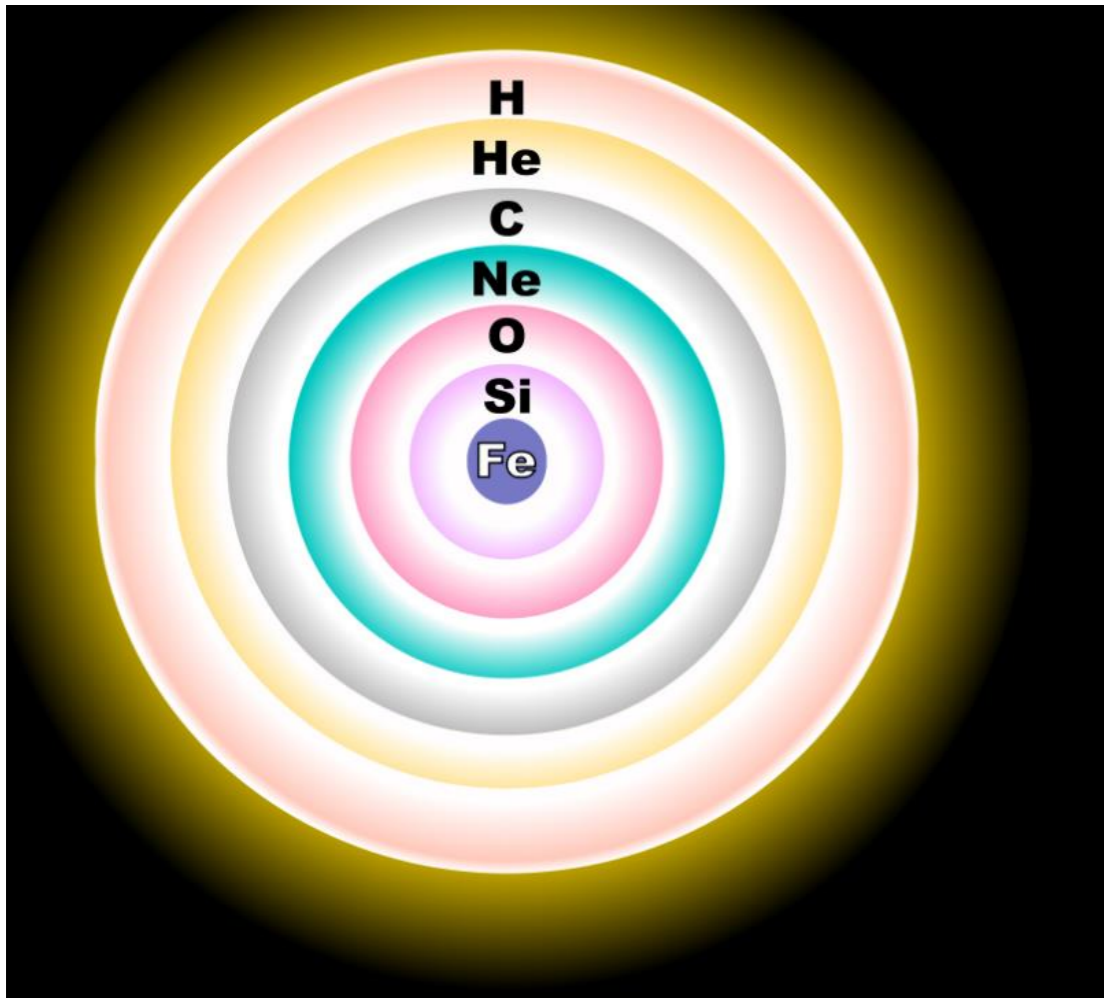


Figure 1

Stars produce heavier and heavier elements through nuclear fusion in their core... up to iron (highest binding energy per nucleon)



# Core Collapse Supernova



Star is stabilised by equilibrium between thermal and gravitational pressure. When fuel runs out, it is stabilised by the **Fermi pressure**:

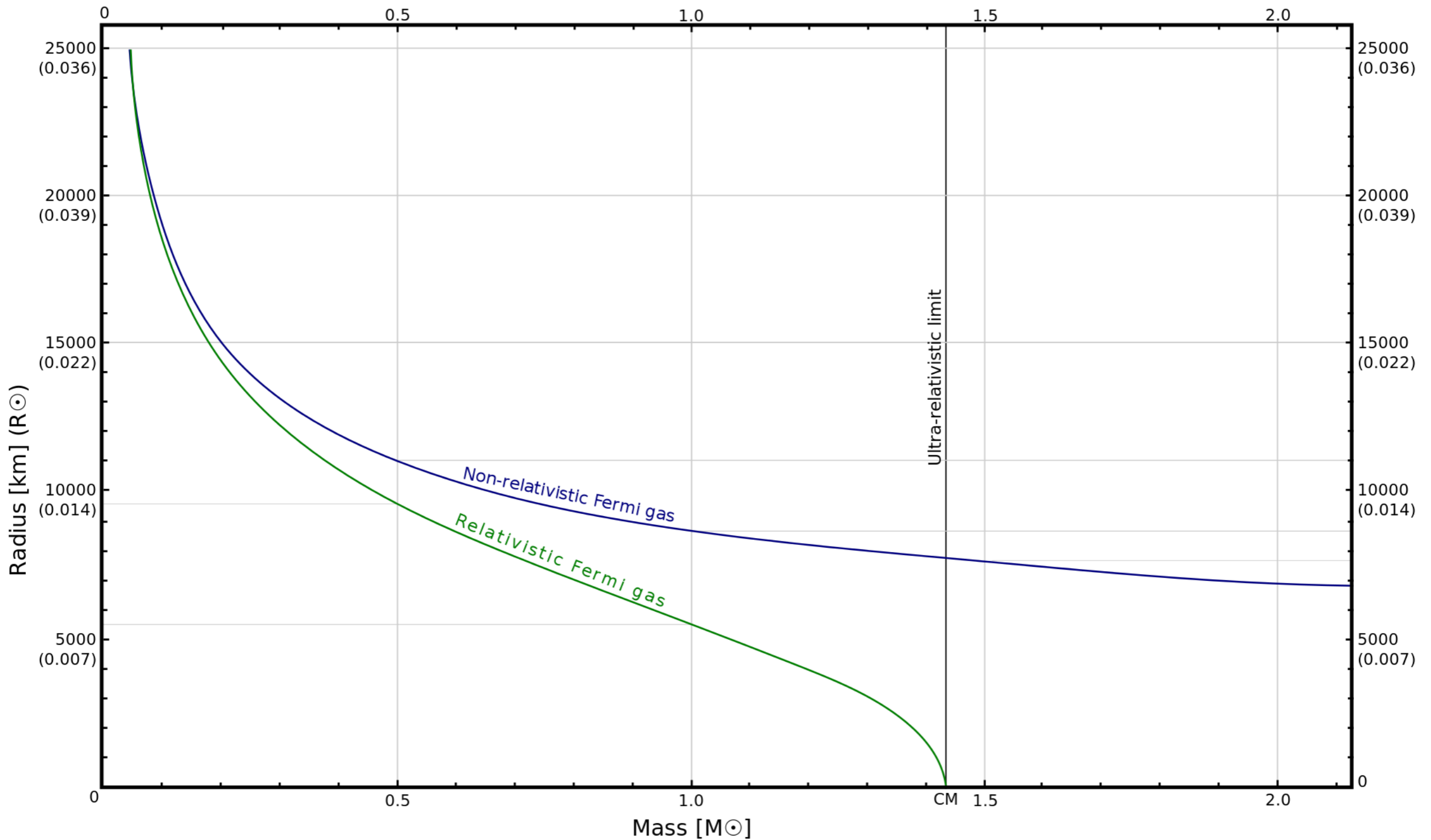
$$P = \frac{g}{(2\pi)^3} \int d^3\mathbf{p} \frac{\mathbf{p}^2}{3\omega_p} f(\mathbf{p}) \simeq \frac{g}{24\pi^2} \mu^4$$

$$n = \frac{g}{(2\pi)^3} \int d^3\mathbf{p} f(\mathbf{p}) \simeq \frac{g}{6\pi^2} \mu^3$$

$$\Rightarrow P \propto n^{4/3}$$

This is to balance the gravitational pressure. Solving for the radius yields the **Chandrasekhar limit** of 1.44 solar masses

# Core Collapse Supernova



# Core Collapse Supernova

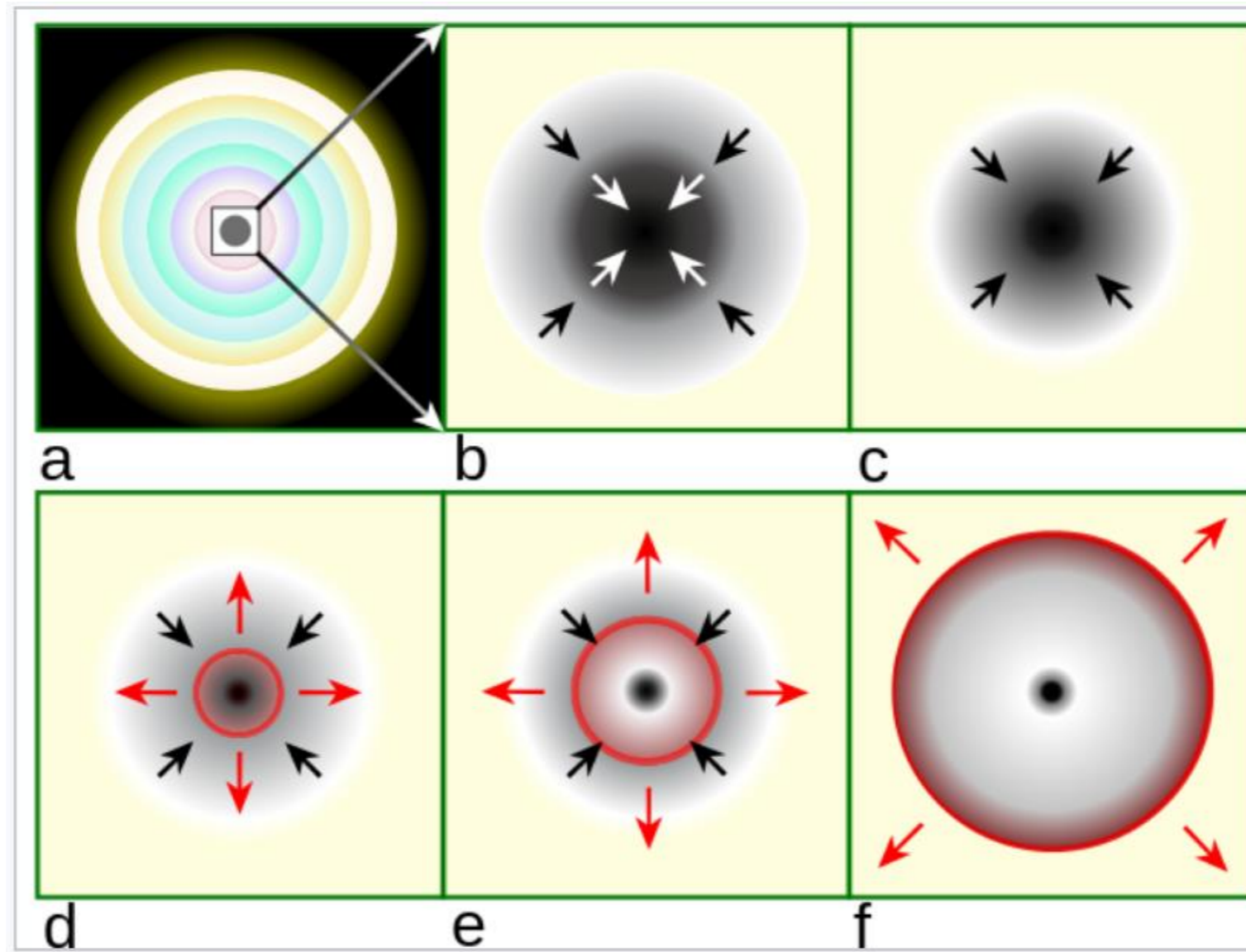
a) fusion in shells forms iron core, collapses when Chandrasekhar-mass is reached

b) electrons and protons in core combine into neutrons

c) inflating matter bounced back from hard core

d) bouncing matter forms shock wave

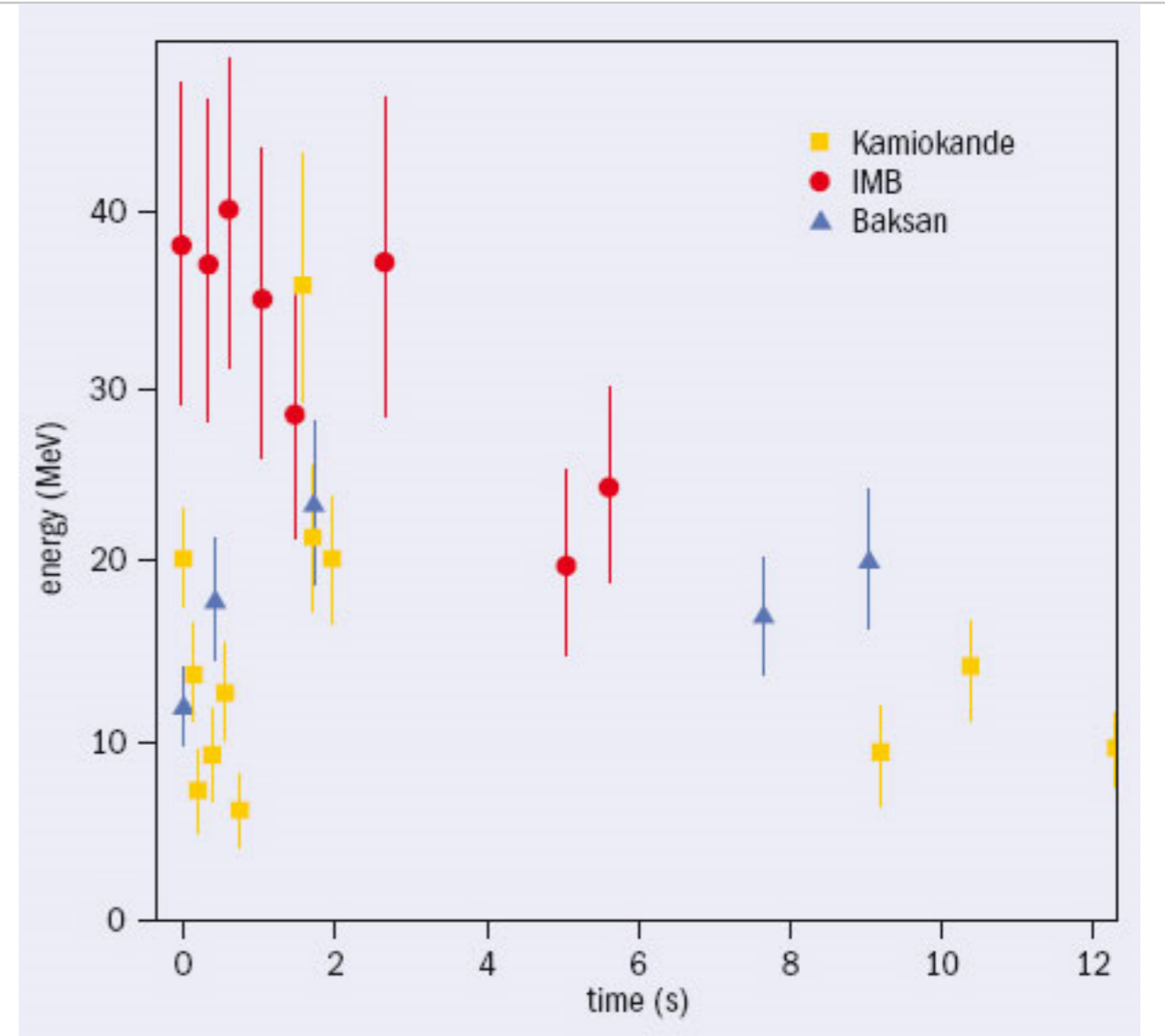
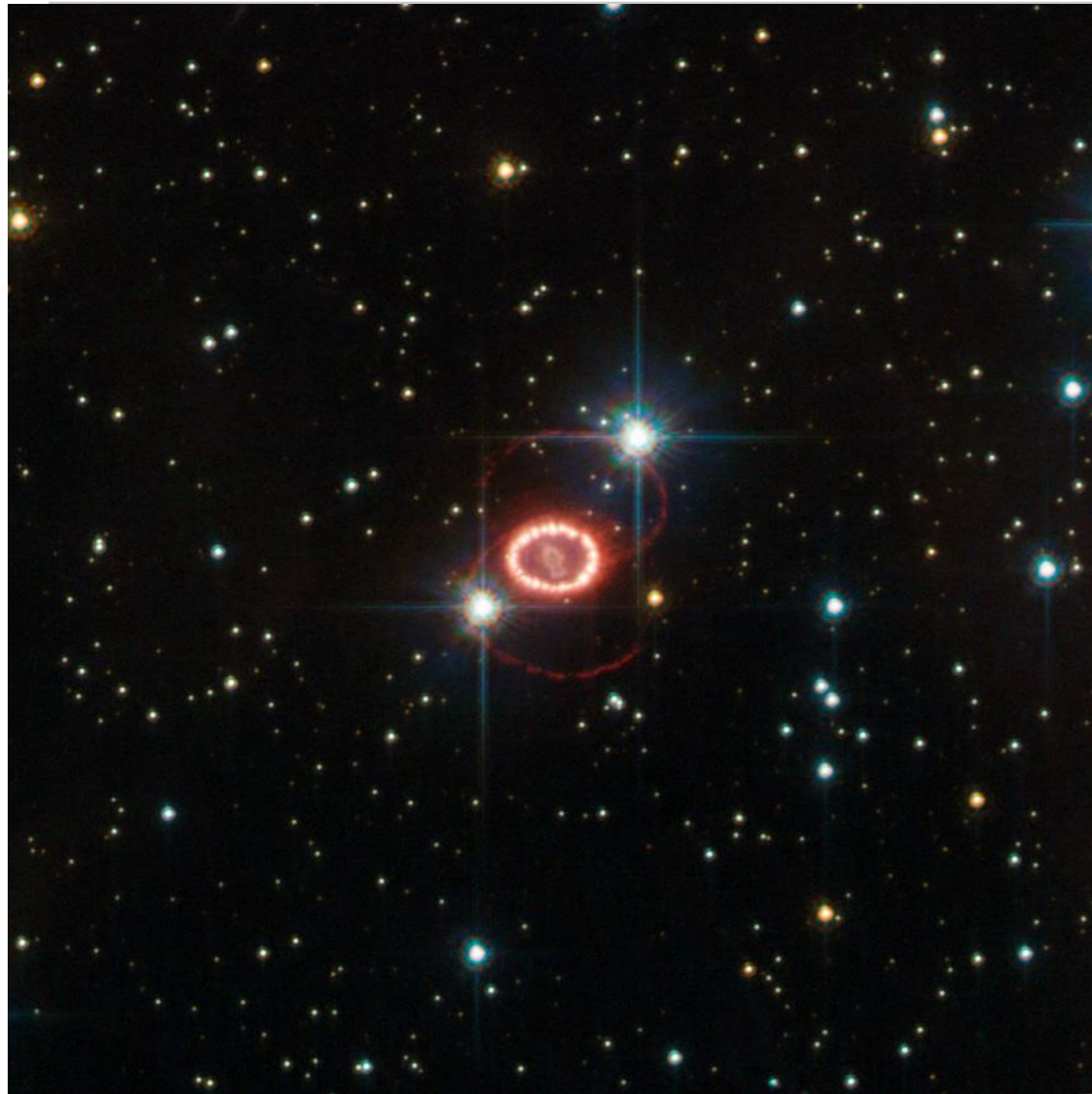
f) shell is blown off



e) complicated dynamics involving neutrino interactions/heating,

**many neutrinos emitted because they can escape the dense medium easier**

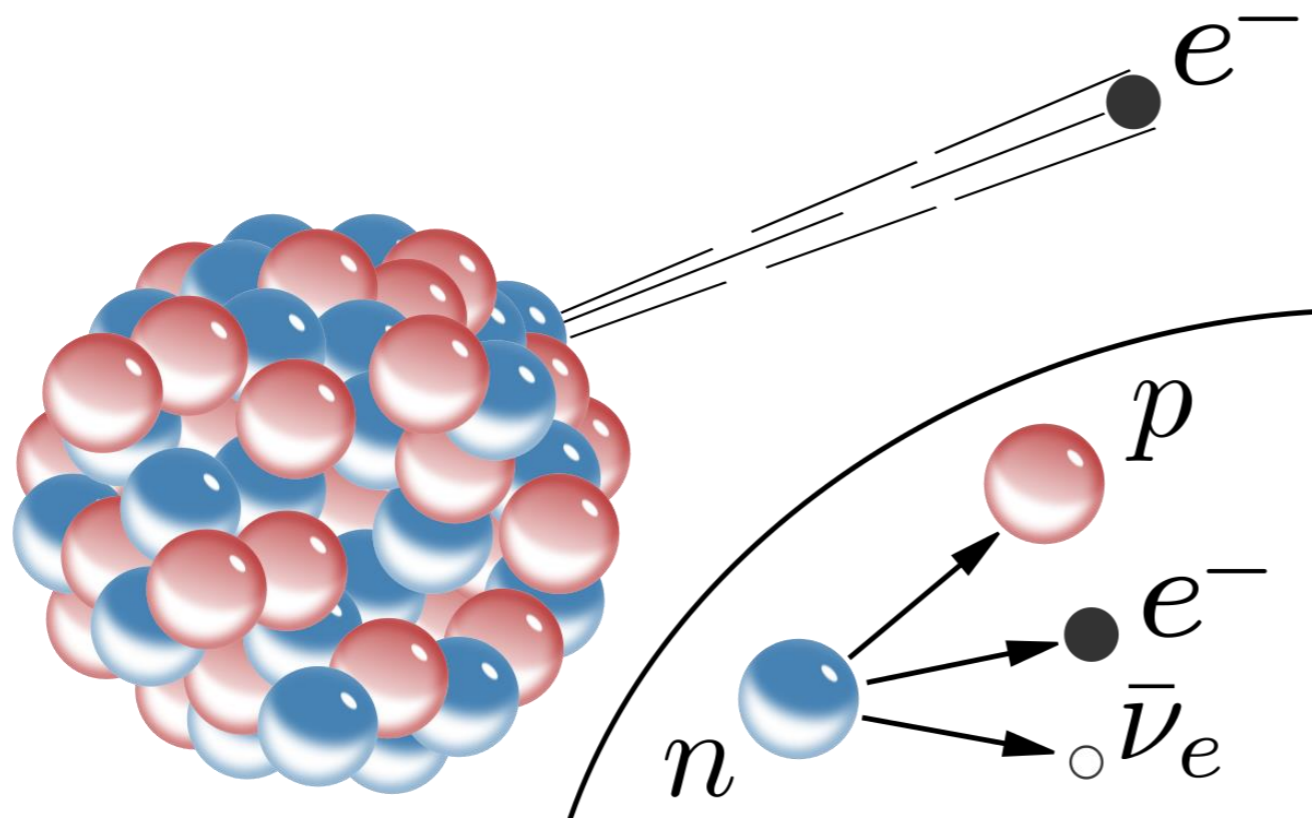
# SN1987a



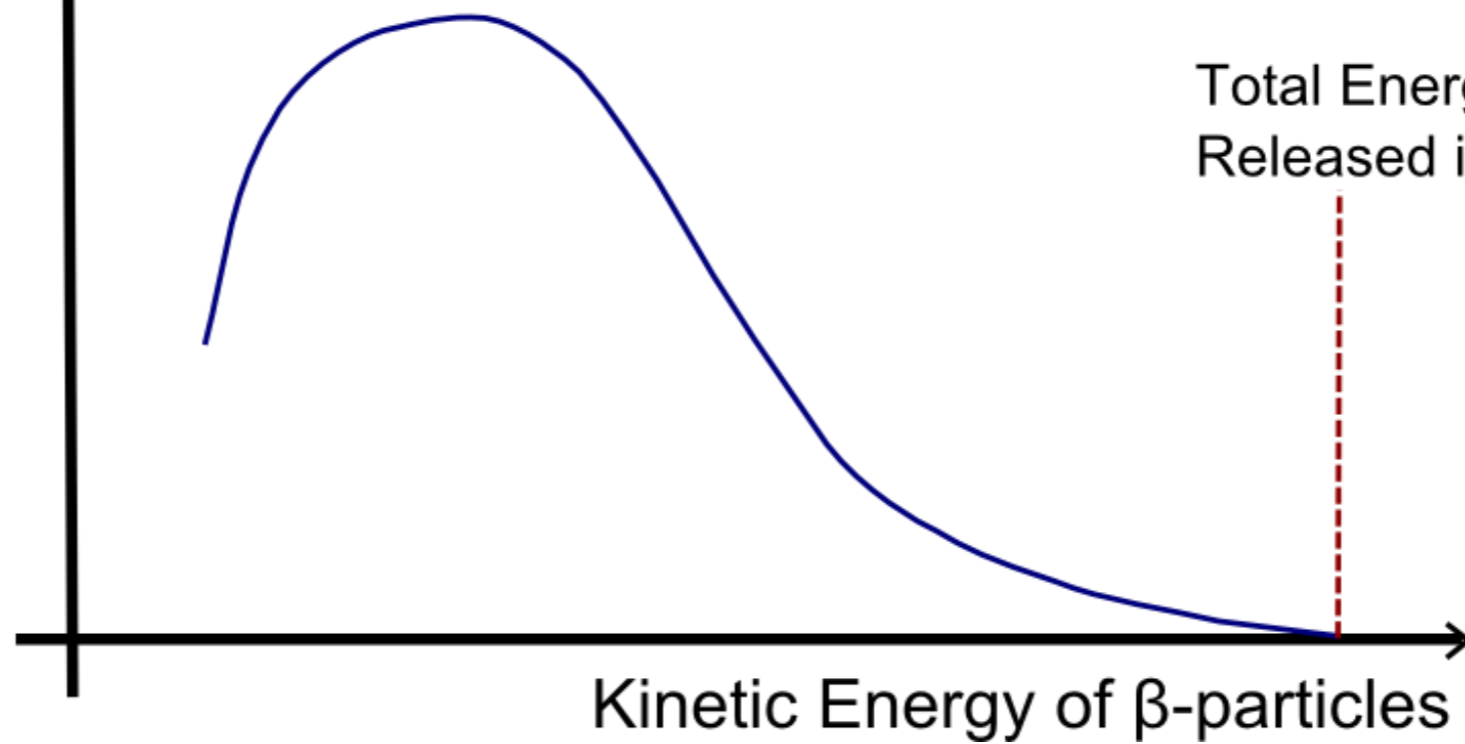
Observation of neutrinos confirms theory that 99% of the energy is emitted in the form of neutrinos!

# Neutrino Oscillations

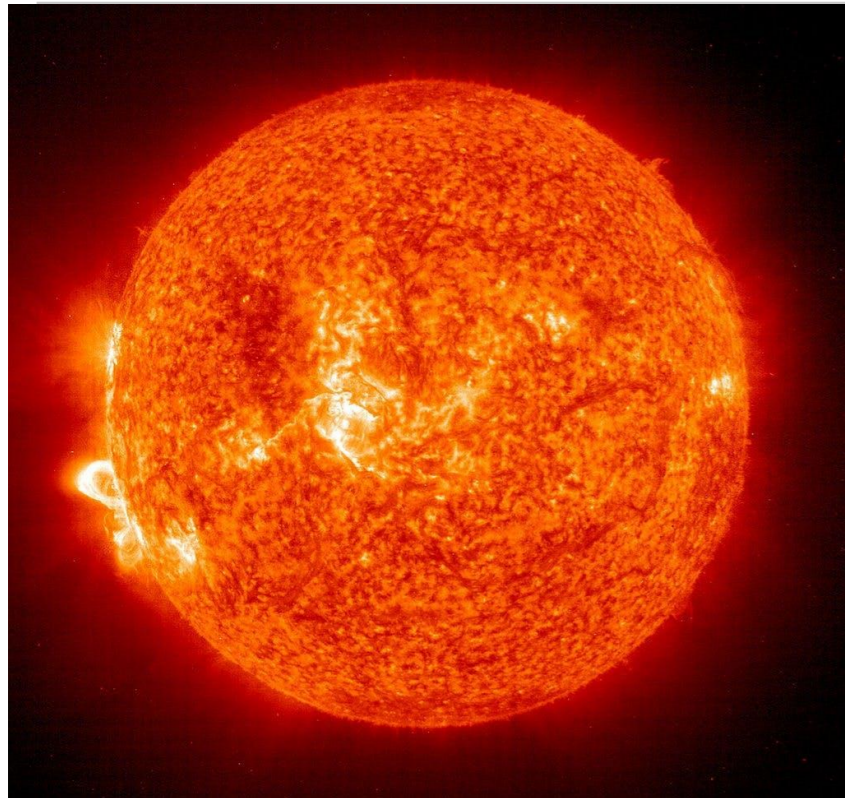




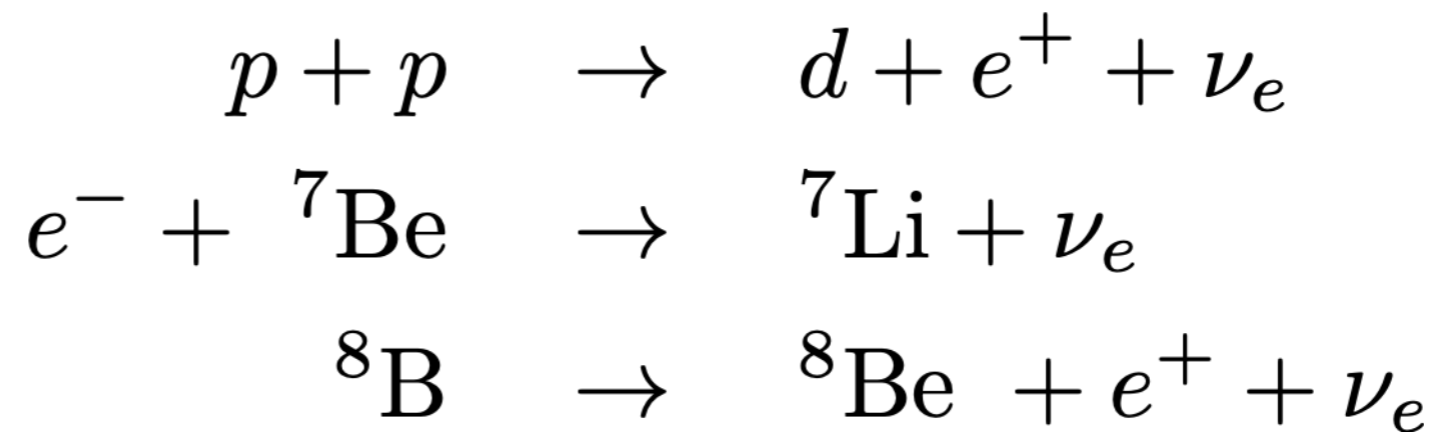
$\beta$ -particle  
count



# The Solar Neutrino Problem



Nuclear fusion creates electron neutrinos

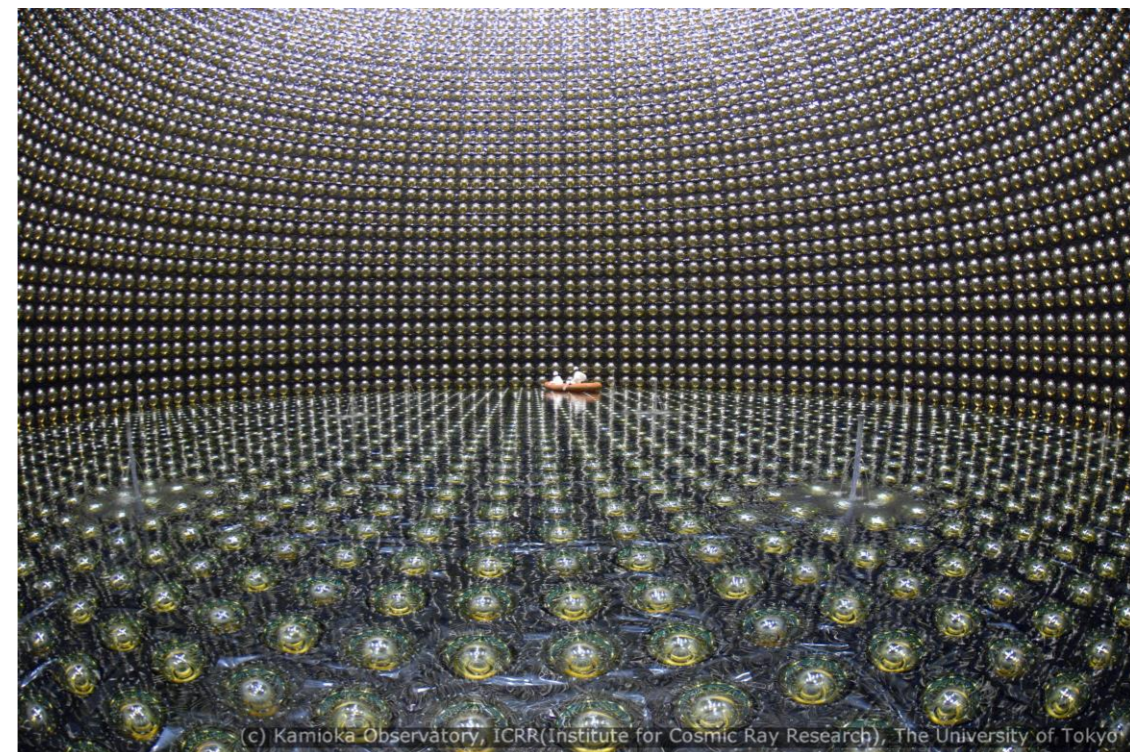


We know how bright the sun is.

We know the nuclear reactions.

So we can predict the neutrino flux.

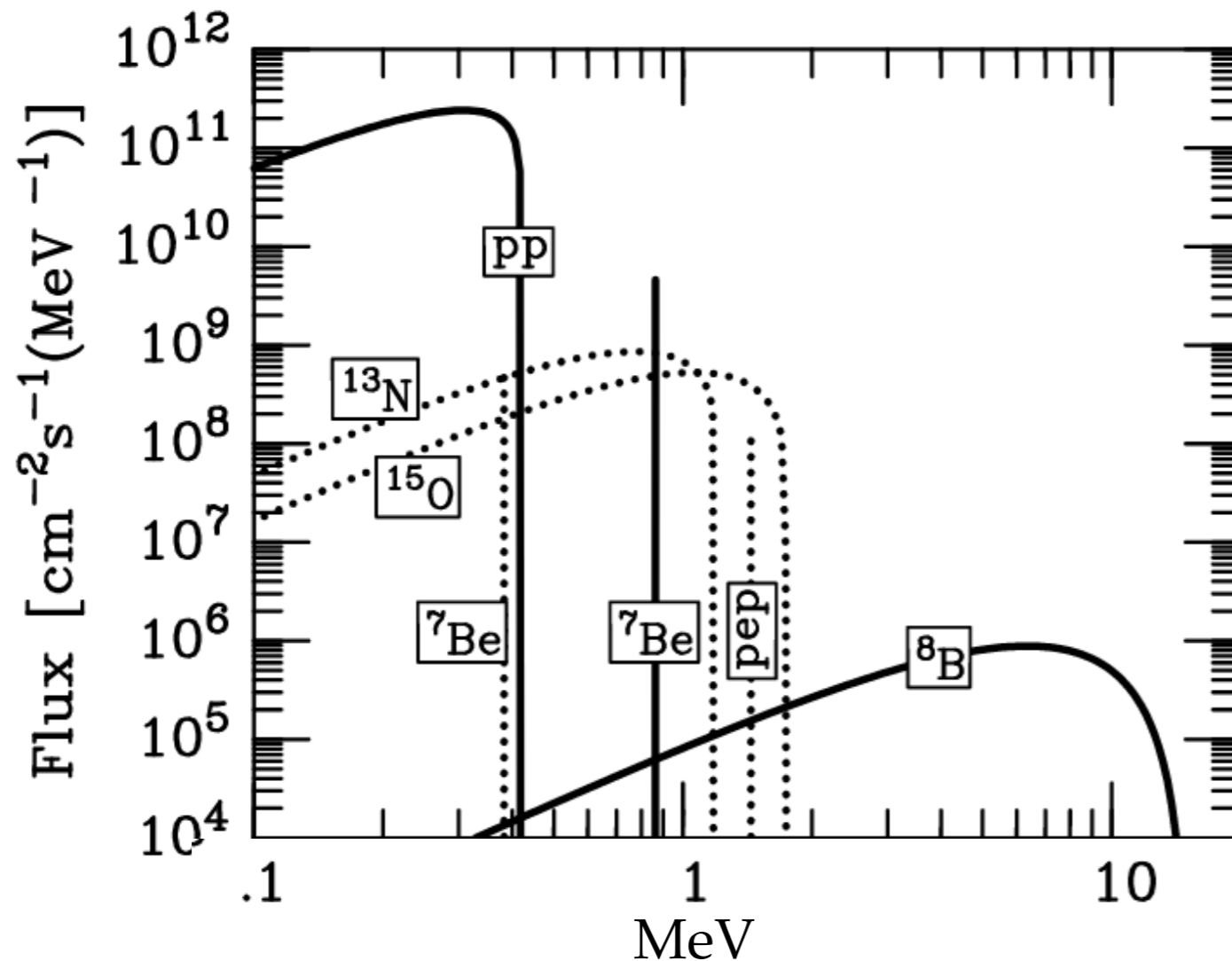
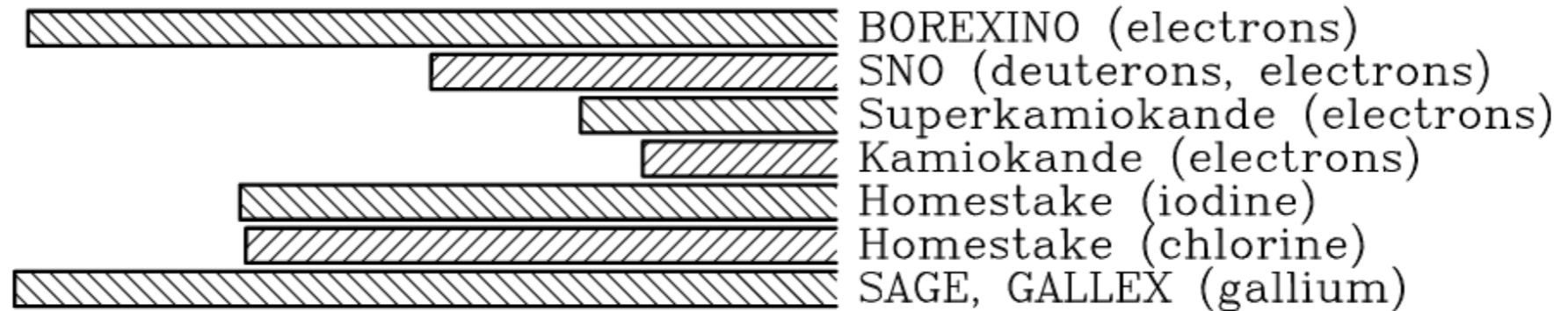
**Experiments see only 1/3 - 1/2 of that!**





# Solar Model

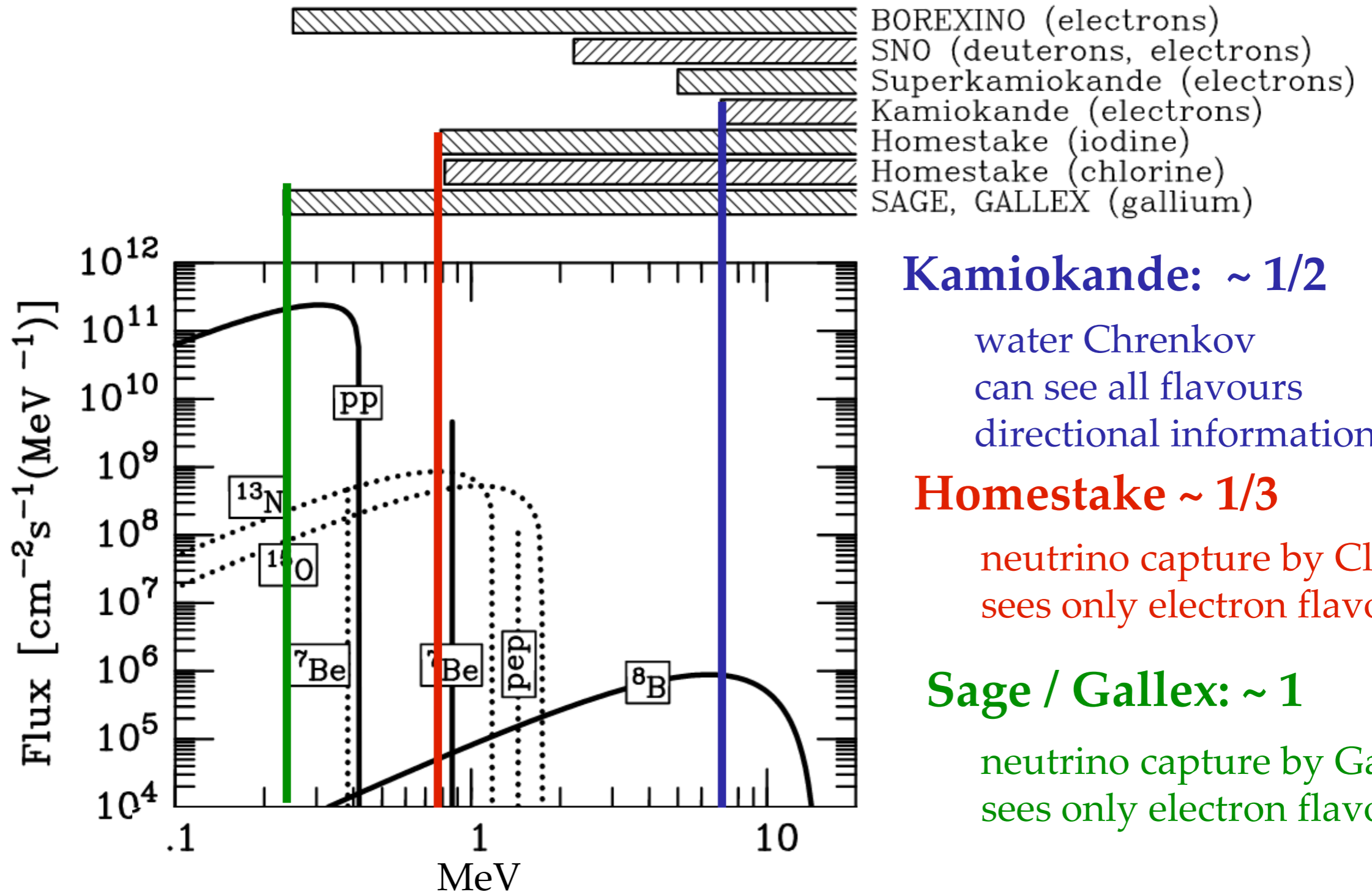
Total flux: 66 billion  $\nu$  / (s cm<sup>2</sup>)





# Solar Model

Total flux: 66 billion  $\nu$  / (s cm<sup>2</sup>)



**Kamiokande: ~ 1/2**

water Chrenkov  
 can see all flavours  
 directional information

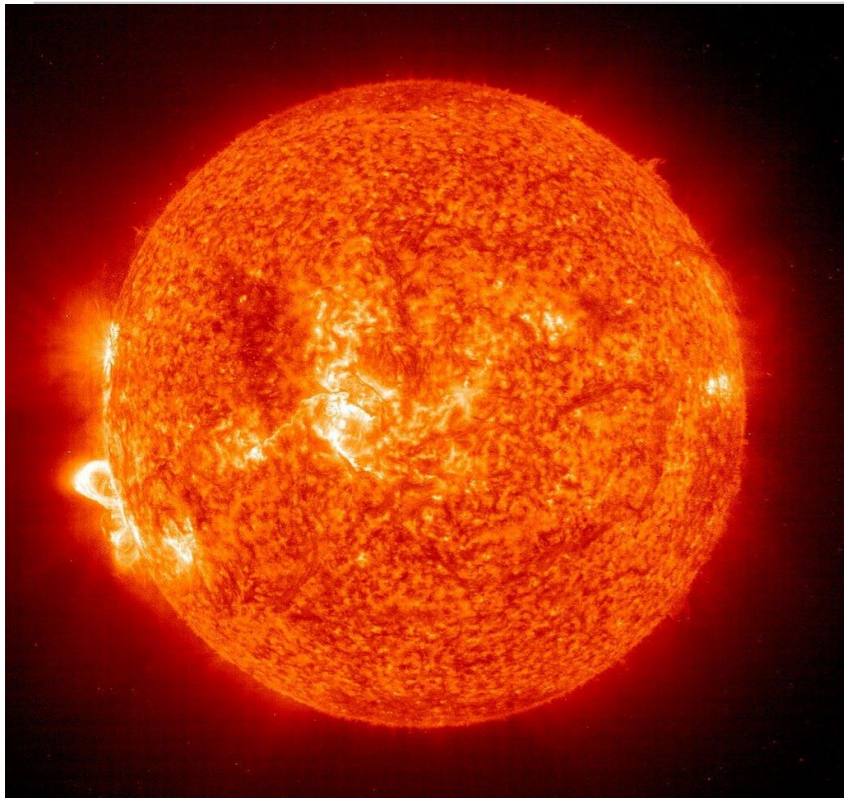
**Homestake ~ 1/3**

neutrino capture by Cl  
 sees only electron flavour

**Sage / Gallex: ~ 1**

neutrino capture by Ga  
 sees only electron flavour

# The Solar Neutrino Problem



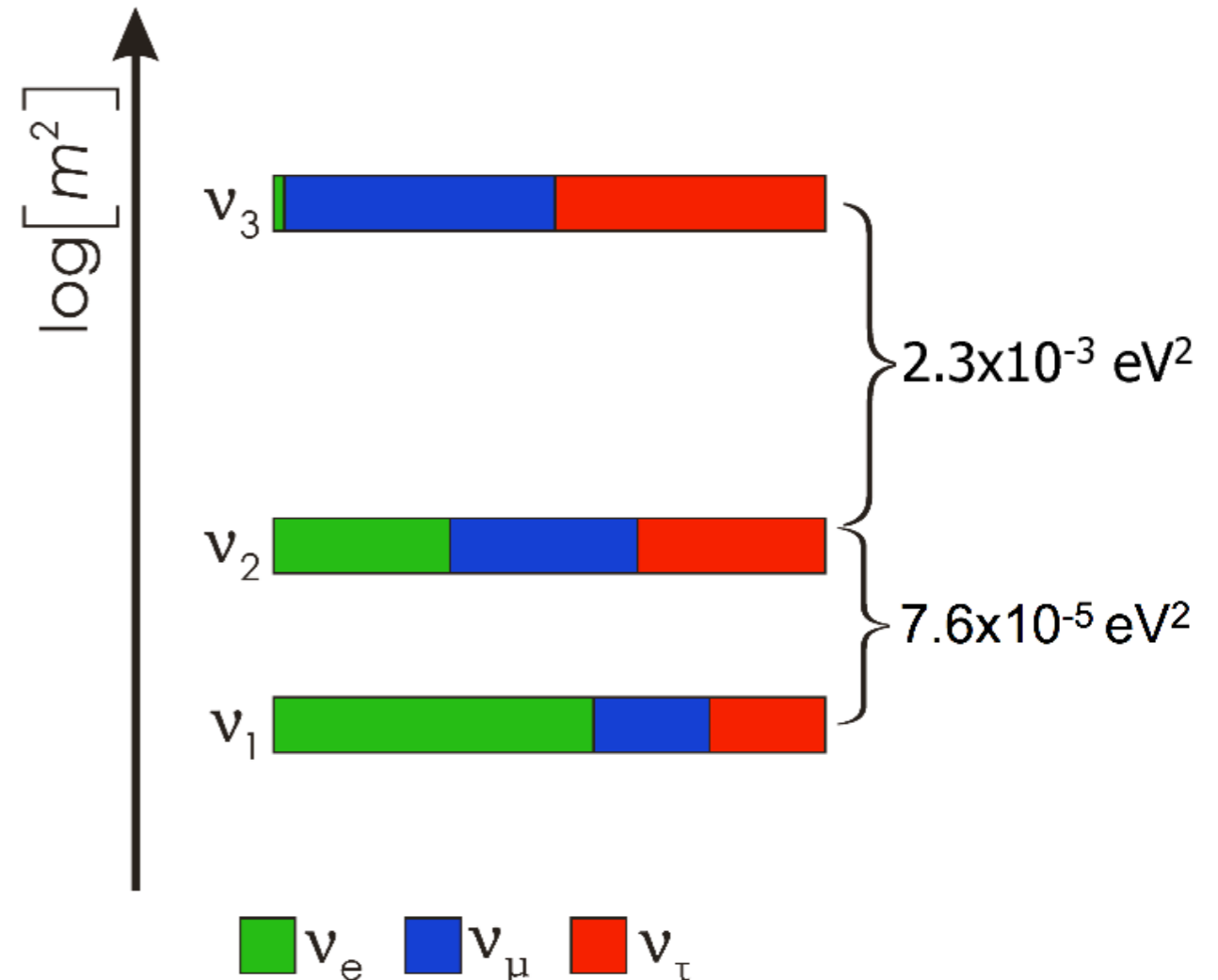
Neutrinos are quantum mechanical objects...

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

The “electron neutrino” quantum state has no well-defined mass.

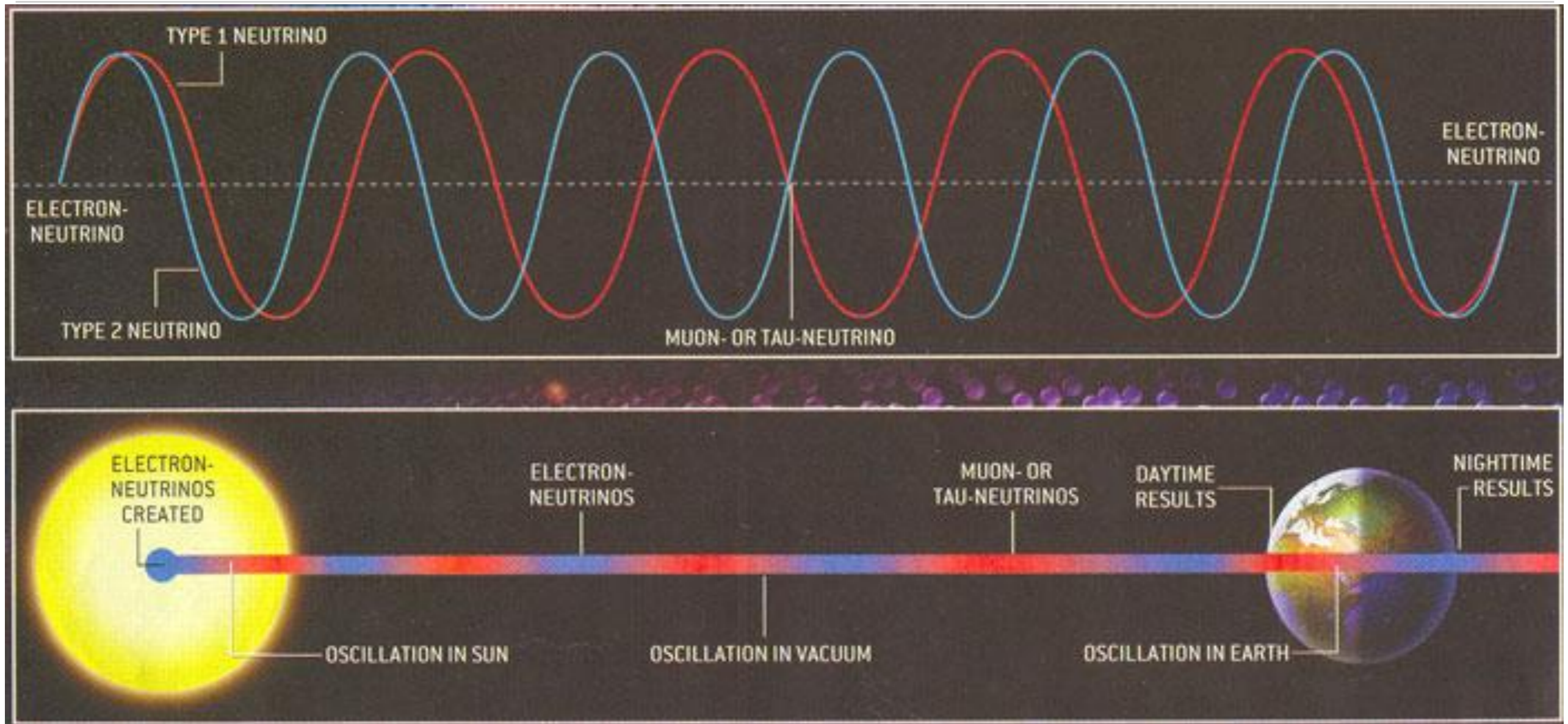
It is a superposition of quantum states with well-defined mass!

Their wave functions oscillate at different frequencies.





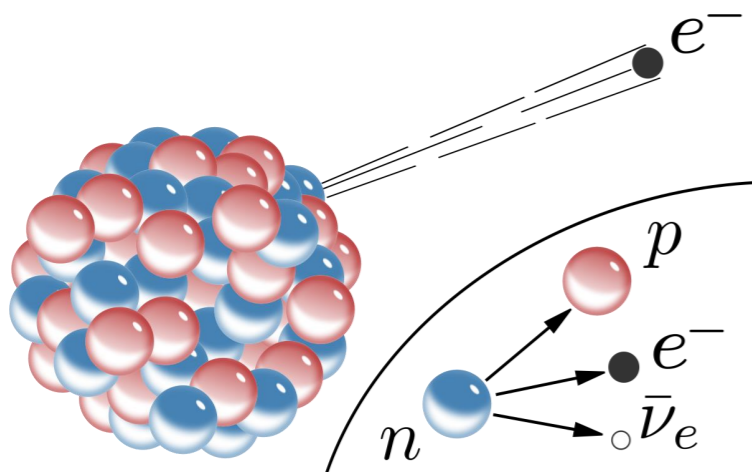
# The Solar Neutrino Problem



Constructive and destructive interference along the path changes the decomposition of the quantum state.

**Electron Neutrinos turn into muon neutrinos!**

# What defines the Neutrino Flavour?



An electron neutrino is the thing that gets produced together with an electron.

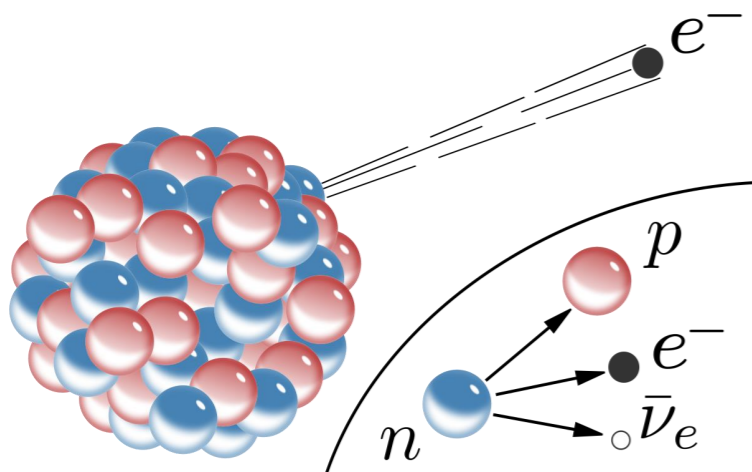
At the level of Lagrangians, the “interaction basis” is the flavour basis where the weak currents are diagonal:

$$-\frac{g}{\sqrt{2}}\bar{\nu}_L\gamma^\mu e_L W_\mu^+ - \frac{g}{\sqrt{2}}\bar{e}_L\gamma^\mu \nu_L W_\mu^- - \frac{g}{2\cos\theta_W}\bar{\nu}_L\gamma^\mu \nu_L Z_\mu,$$

If it related to the “mass basis” by a flavour rotation:

$$\nu_{L\alpha} = (V_\nu)_{\alpha i}\nu_i. \quad V_\nu = (1 + \eta) U_\nu.$$

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If it related to the “mass basis” by a flavour rotation: deviation from unitarity

$$\nu_{L\alpha} = (V_\nu)_{\alpha i}\nu_i. \quad V_\nu = (1 + \eta) U_\nu. \quad \text{unitary part}$$



# Relativistic Wave Equation

$$(\partial_t^2 - \nabla^2 + M^2) \Psi = 0$$

relativistic wave equation  
(for each spinor component)

$$\Psi(t, \mathbf{x}) = \Psi_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$$

decomposition into plane waves

$$(\partial_t^2 + \mathbf{k}^2 + M^2) \Psi_{\mathbf{k}}(t) = 0$$

$$\partial_t^2 + \mathbf{k}^2 = (i\partial_t + k)(-i\partial_t + k)$$

relativistic approximation

$$k = |\mathbf{k}| \gg m_i$$

$$\partial_t^2 + \mathbf{k}^2 \approx 2k(-i\partial_t + k)$$

$$i\partial_t \Psi_{\mathbf{k}} = \Omega_{\mathbf{k}} \Psi_{\mathbf{k}} \quad \text{where} \quad \Omega_{\mathbf{k}} \equiv \left( k + \frac{M^2}{2k} \right)$$

**effective Schrodinger equation in time**

# Neutrino Flux in Space

But in reality we are more interested in the flux as a function of position

$$(\partial_t^2 - \nabla^2 + M^2) \Psi = 0, \quad \Psi_\omega(\mathbf{x}) e^{-i\omega t};$$

$$(-\omega^2 - \nabla^2 + M^2) \Psi_\omega(\mathbf{x}) = 0.$$

$$i\partial_z \Psi_\omega = -K_\omega \Psi_\omega \quad \text{where} \quad K_\omega \equiv \left( \omega - \frac{M^2}{2\omega} \right).$$

**effective Schrodinger equation in space**

**solution:**  $\Psi_\omega(z) = e^{iKz} \Psi_\omega(0)$

**with**  $W(z) \equiv (e^{iKz})_{\text{weak}} = U(e^{iKz})_{\text{mass}} U^\dagger$

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# Two Flavour Oscillation

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$$W(z) \equiv (e^{iKz})_{\text{weak}} = U (e^{iKz})_{\text{mass}} U^\dagger$$

For two flavour case:  $U = \cos \theta I + i \sin \theta \sigma_2$

Parameterise mass as:  $M^2/2\omega = b_0 - \frac{1}{2} \mathbf{B} \cdot \boldsymbol{\sigma},$

with  $\mathbf{B} = \frac{2\pi}{\ell_{\text{osc}}} \begin{pmatrix} \sin 2\theta \\ 0 \\ \cos 2\theta \end{pmatrix},$   $b_0 = (m_1^2 + m_2^2)/4\omega.$

$$\ell_{\text{osc}} \equiv \frac{4\pi \omega}{m_2^2 - m_1^2}$$



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# Two Flavour Oscillation

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For two flavour case:  $U = \cos \theta I + i \sin \theta \sigma_2$

Parameterise mass as:  $M^2/2\omega = b_0 - \frac{1}{2} \mathbf{B} \cdot \boldsymbol{\sigma},$

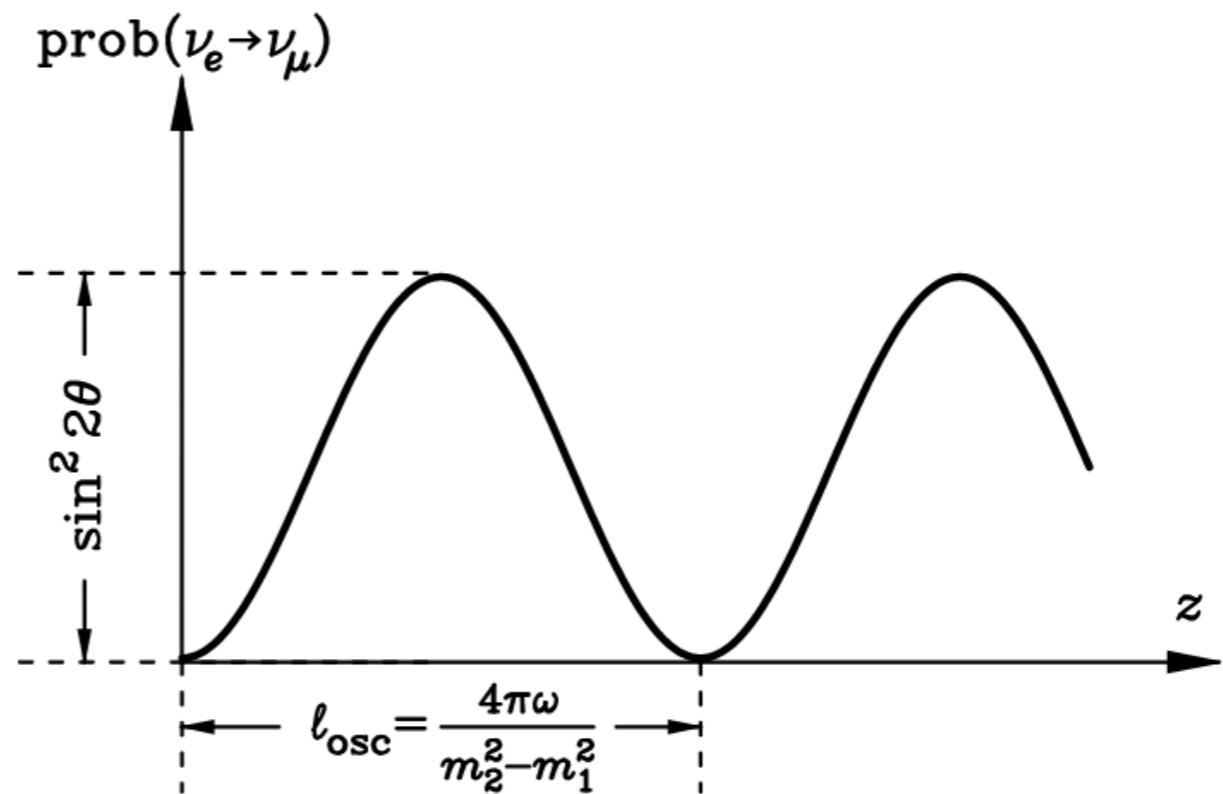
with  $\mathbf{B} = \frac{2\pi}{\ell_{\text{osc}}} \begin{pmatrix} \sin 2\theta \\ 0 \\ \cos 2\theta \end{pmatrix},$   $b_0 = (m_1^2 + m_2^2)/4\omega.$

$\ell_{\text{osc}} \equiv \frac{4\pi \omega}{m_2^2 - m_1^2}$

# Appearance and Disappearance

$$W = e^{i(\omega - b_0)z} \left[ \cos\left(\frac{\pi z}{\ell_{\text{osc}}}\right) - i \sin\left(\frac{\pi z}{\ell_{\text{osc}}}\right) \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \right].$$

$$K = \omega - b_0 + \frac{1}{2} \mathbf{B} \cdot \boldsymbol{\sigma}$$



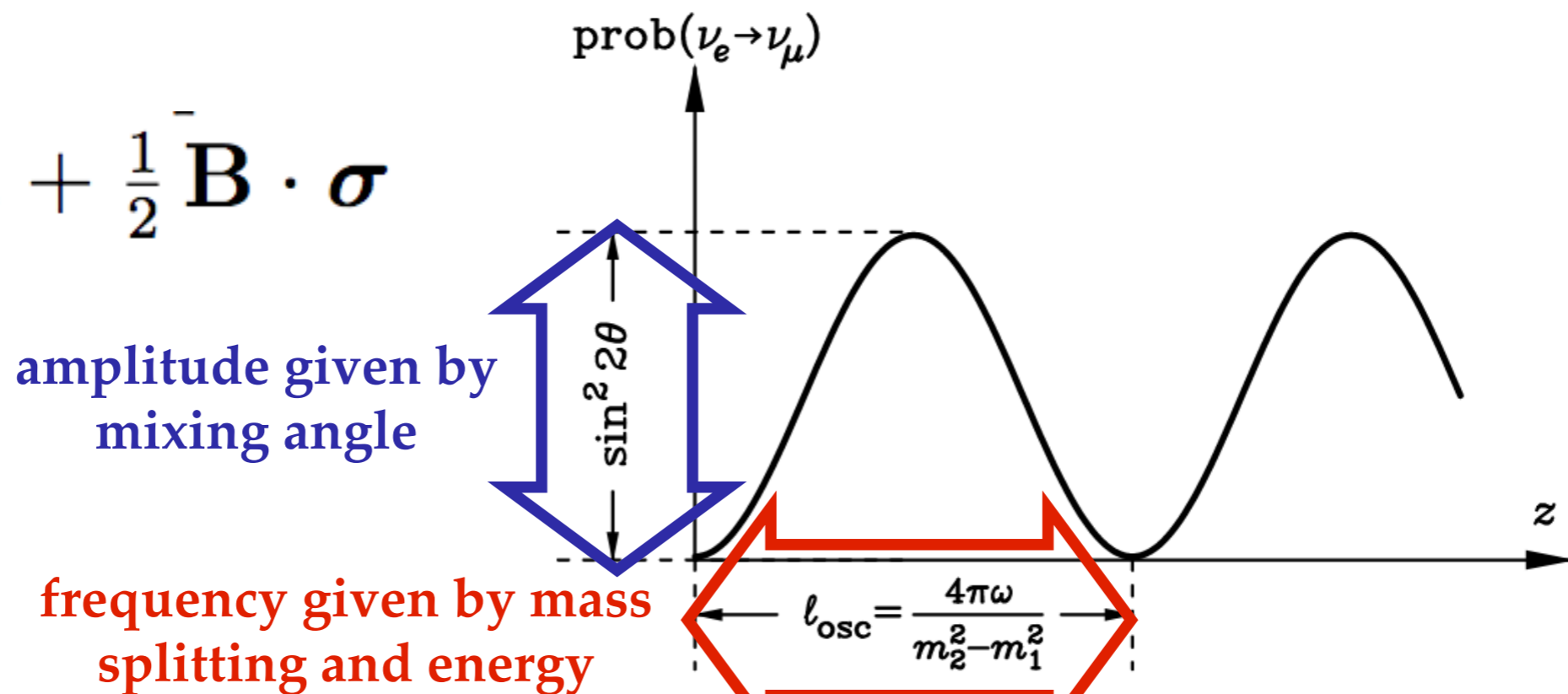
$$\text{prob}(\nu_e \rightarrow \nu_\mu) = |W_{e\mu}|^2 = \sin^2(2\theta) \sin^2(\pi z / \ell_{\text{osc}}),$$

$$\text{prob}(\nu_e \rightarrow \nu_e) = |W_{ee}|^2 = 1 - \text{prob}(\nu_e \rightarrow \nu_\mu).$$

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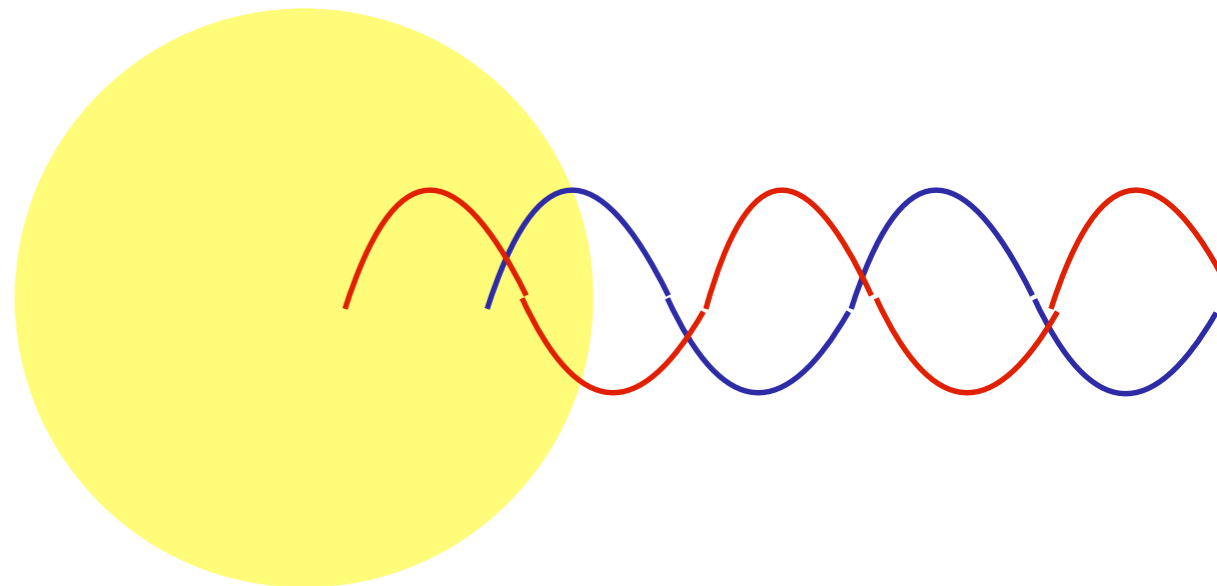
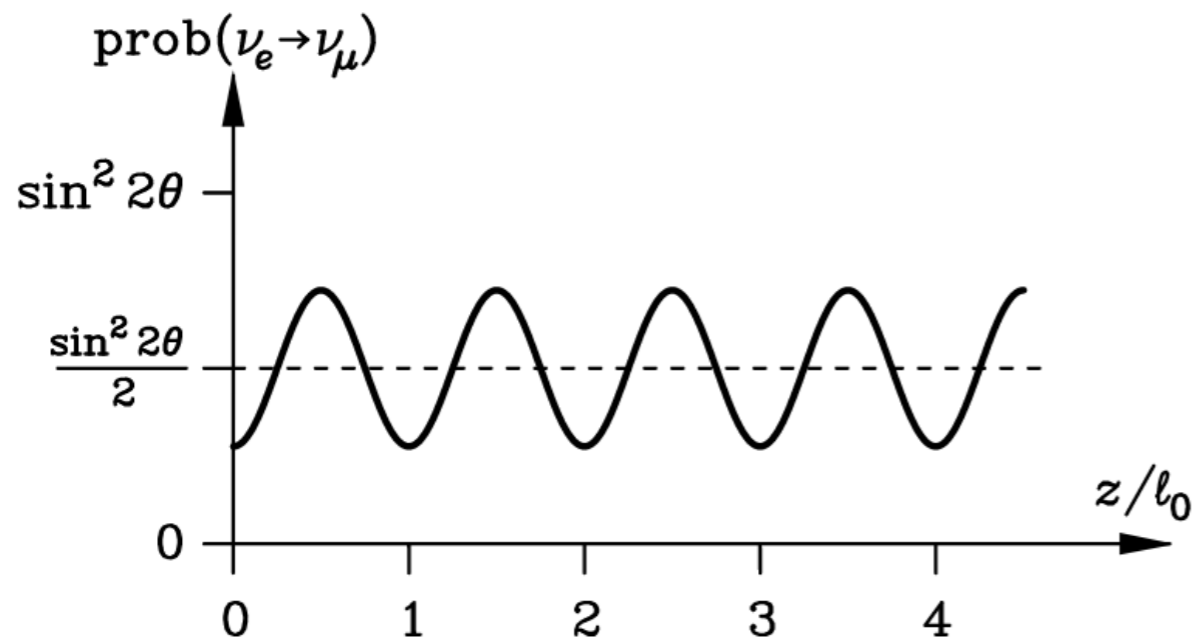


# Spatially Extended Source

$$\text{prob}(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \int dz_0 f(z_0) \sin^2 \frac{\pi(z - z_0)}{\ell_{\text{osc}}}$$

For Gaussian shape:  $f(z_0) = e^{-z_0^2/2s^2} / s\sqrt{2\pi}$

$$\text{prob}(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 2\theta \left[ 1 - e^{-2\pi^2(s/\ell_{\text{osc}})^2} \cos(2\pi z/\ell_{\text{osc}}) \right]$$

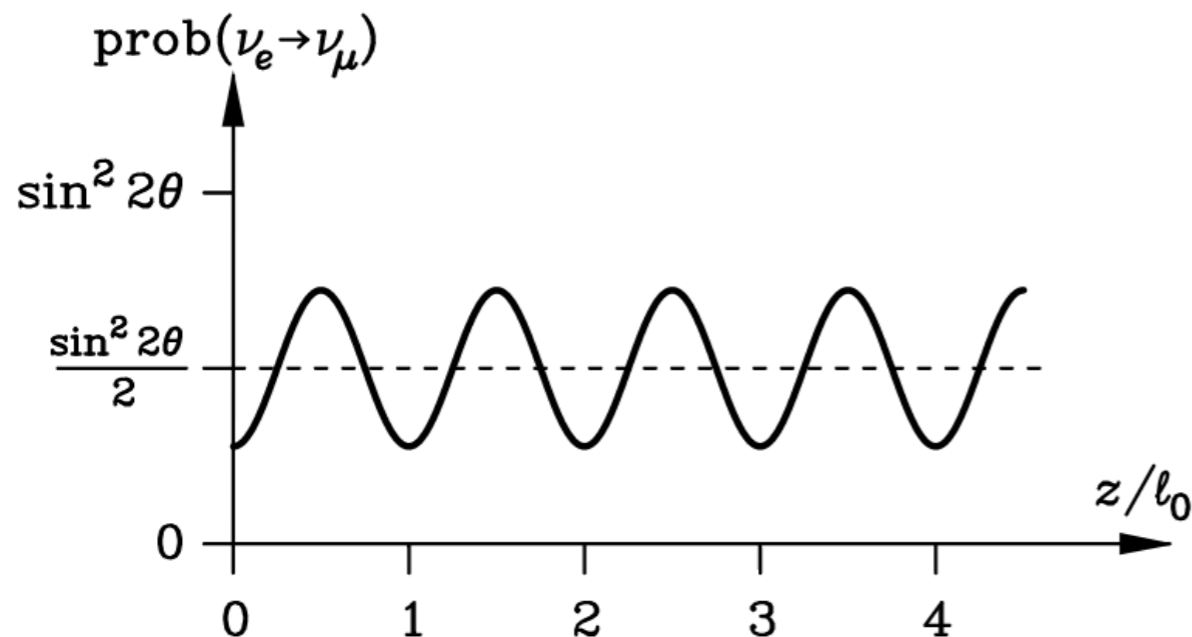


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**large source limit**

$$s \gg \ell_{\text{osc}}$$

$$\frac{1}{2} \sin^2 2\theta$$

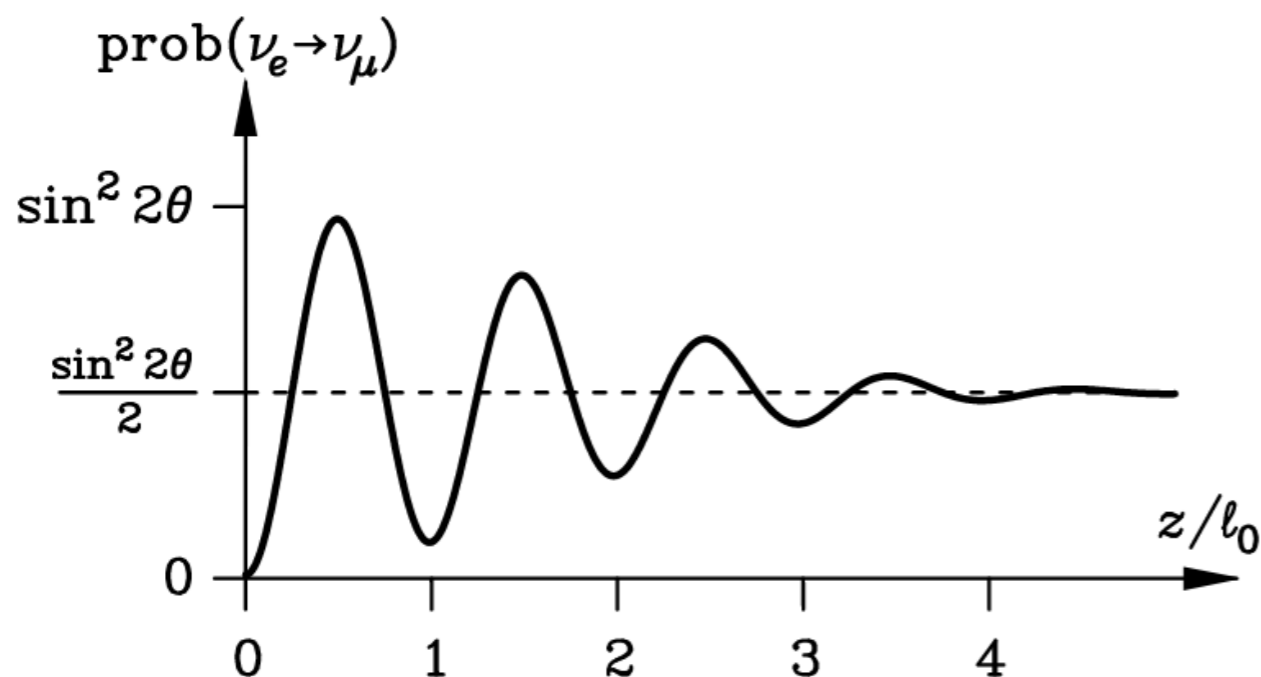
# Non-Monochromatic Source

$$\text{prob}(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \int d\omega g(\omega) \sin^2 \frac{(m_2^2 - m_1^2) z}{4\omega}$$

For Gaussian shape:  $e^{-(\Delta - \Delta_0)^2 / 2\delta^2} / \delta\sqrt{2\pi}$

$$\Delta = 2\pi / \ell_{\text{osc}}$$

$$\text{prob}(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 2\theta \left[ 1 - e^{-\delta^2 z^2 / 2} \cos(2\pi z / \ell_0) \right]$$





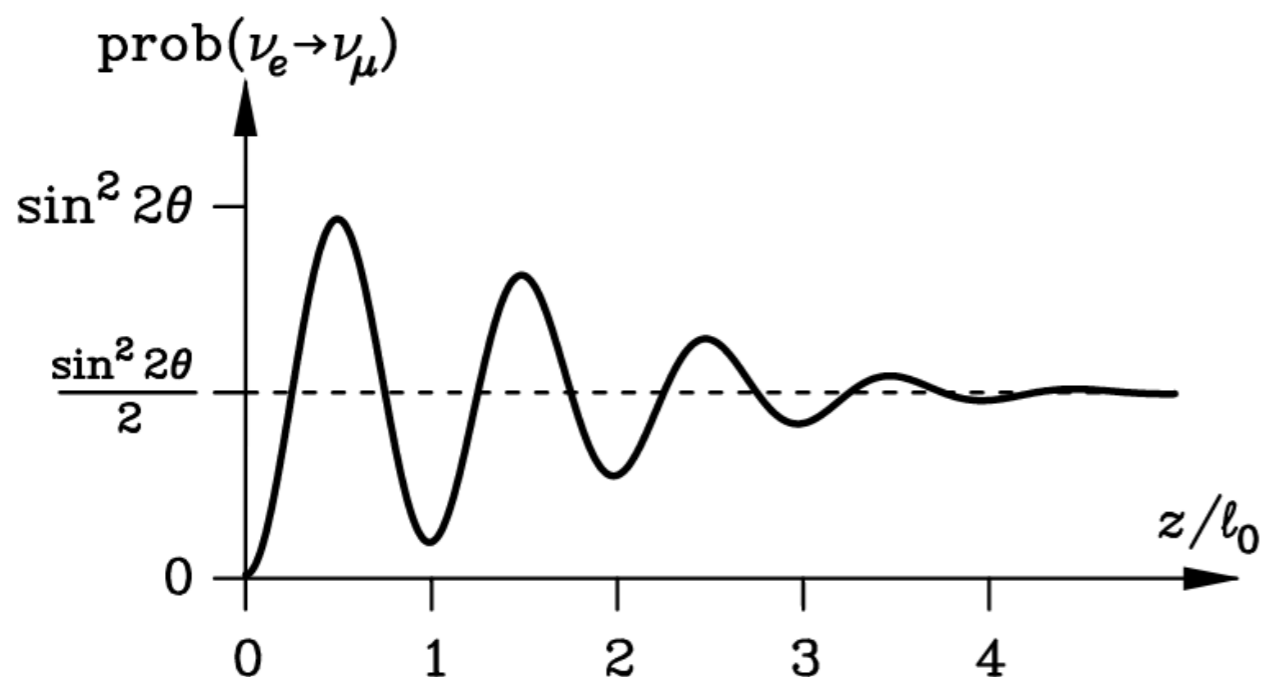
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**continuum spectrum**

$$z \gg \delta^{-1}$$

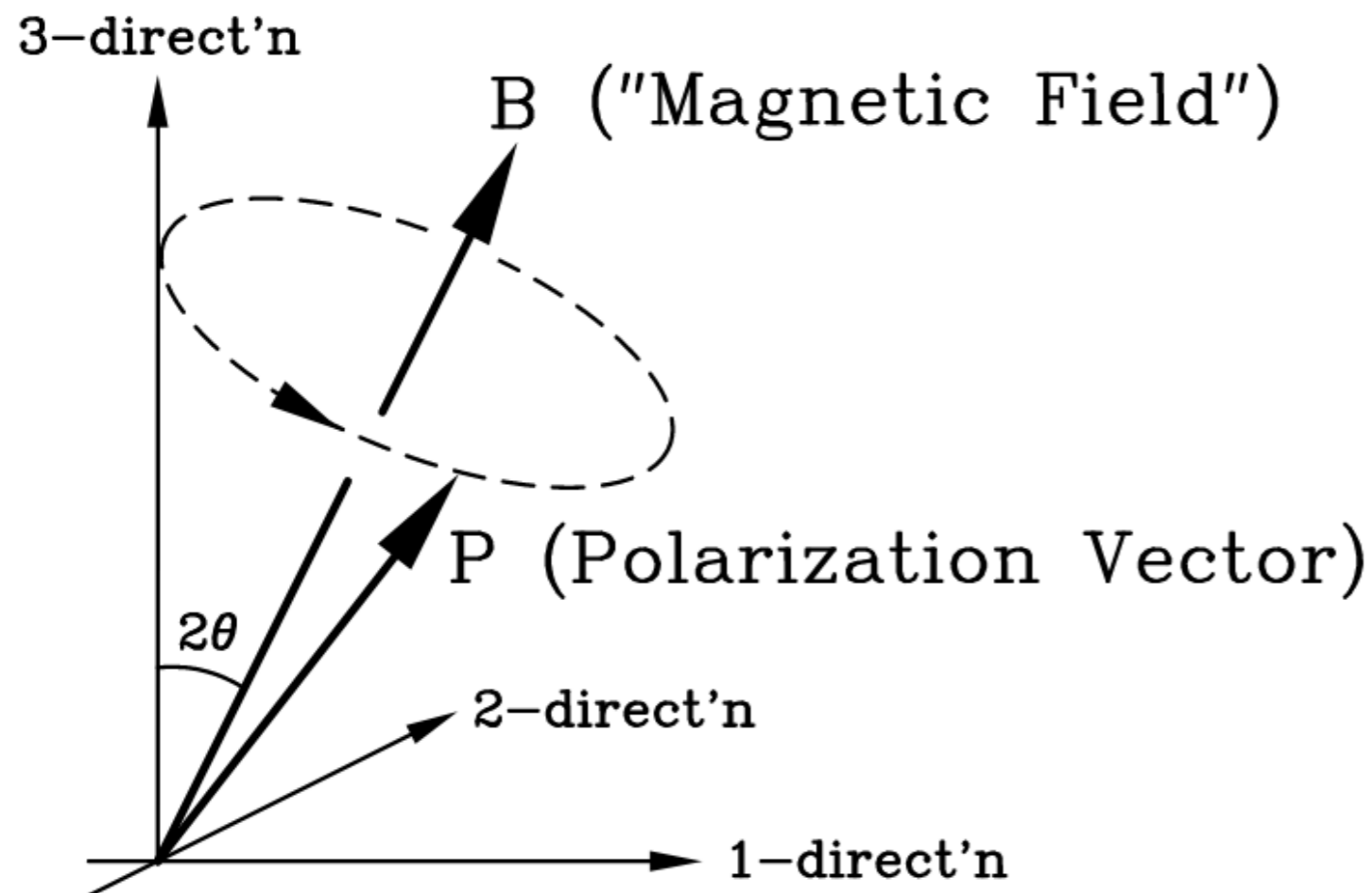
$$\frac{1}{2} \sin^2 2\theta$$

# Flavour Pendulum

$$\rho_{ab} = \Psi_b^* \Psi_a \text{ "density matrix"}$$

$$\rho = \frac{1}{2} (1 + \mathbf{P} \cdot \boldsymbol{\sigma})$$

$$\mathbf{B} = \frac{2\pi}{\ell_{\text{osc}}} \begin{pmatrix} \sin 2\theta \\ 0 \\ \cos 2\theta \end{pmatrix},$$



$$|\Psi_e|^2 = \frac{1}{2} (1 + \mathbf{P}_3)$$

$$|\Psi_\mu|^2 = \frac{1}{2} (1 - \mathbf{P}_3)$$

**flavour evolution mimics  
spin precession in  
magnetic field**

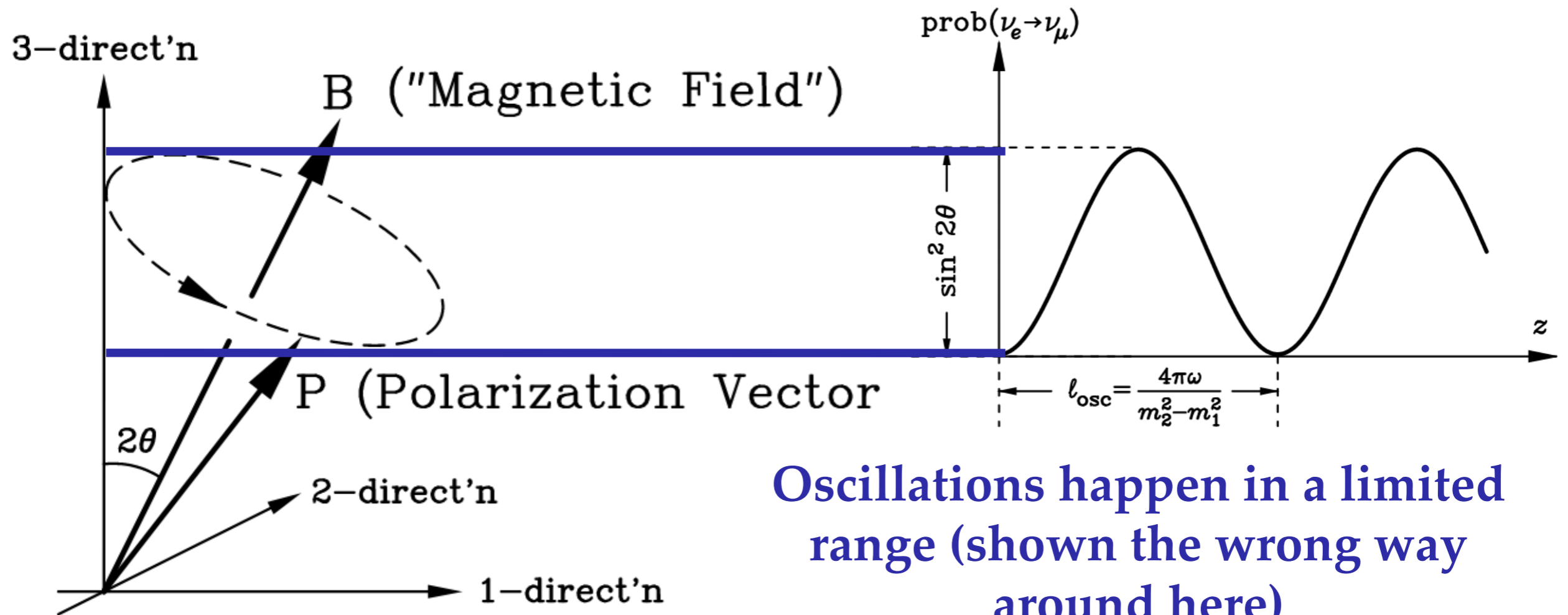
$$\partial_z \mathbf{P} = \mathbf{B} \times \mathbf{P}$$

# Flavour Pendulum

$$|\Psi_e|^2 = \frac{1}{2} (1 + \mathbf{P}_3) ;$$

$$|\Psi_\mu|^2 = \frac{1}{2} (1 - \mathbf{P}_3)$$

$$\mathbf{B} = \frac{2\pi}{l_{\text{osc}}} \begin{pmatrix} \sin 2\theta \\ 0 \\ \cos 2\theta \end{pmatrix},$$

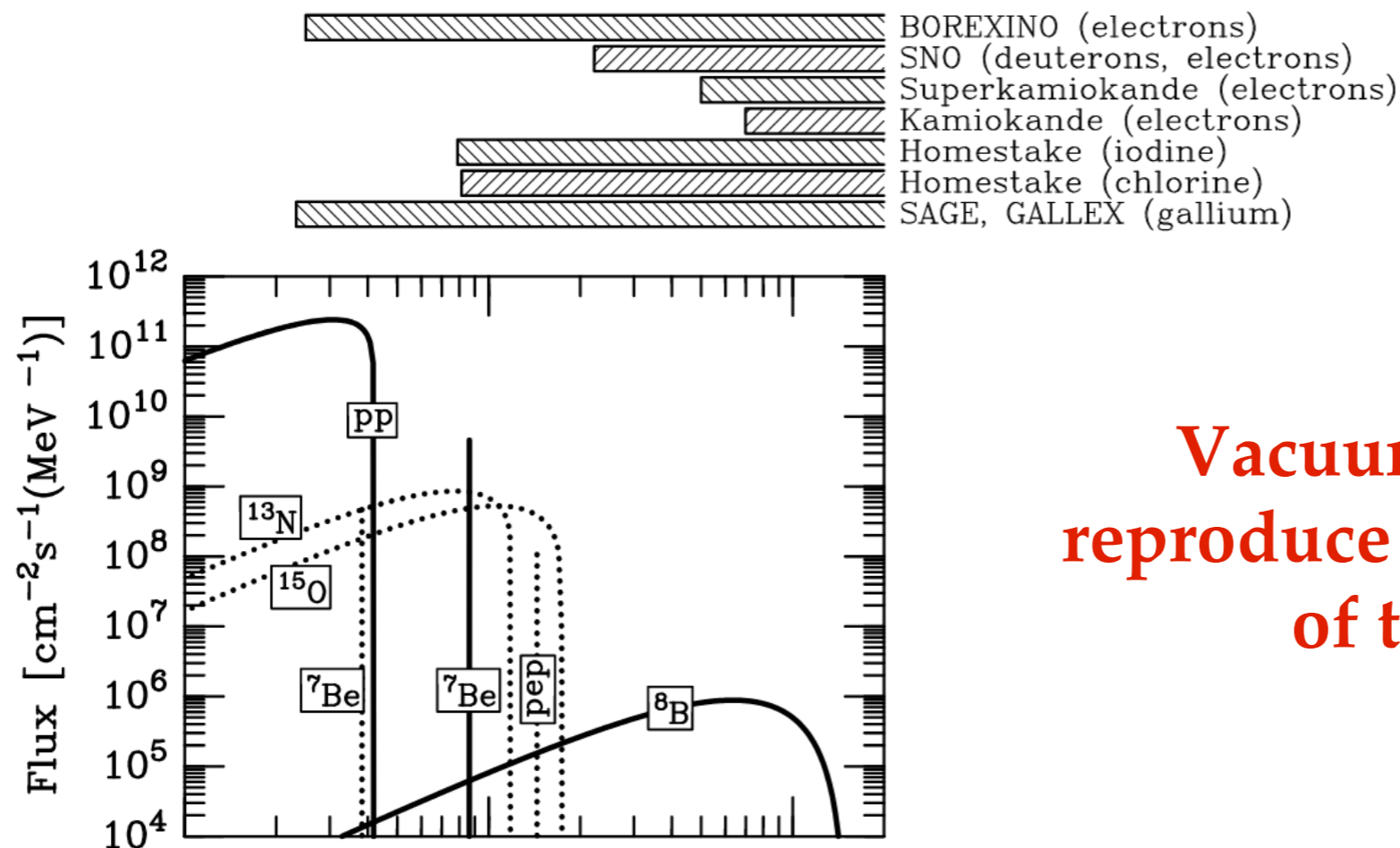




# Can this explain the Solar Neutrino Problem?

The sun is large and not monochromatic....

$$\text{prob} (\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 2\theta$$



**Vacuum oscillations fail to reproduce the energy dependence of the observations!**

---

# Matter Potentials

---

In matter the neutrinos have **effective masses** due to the interaction with the medium, just like electrons in condensed matter systems

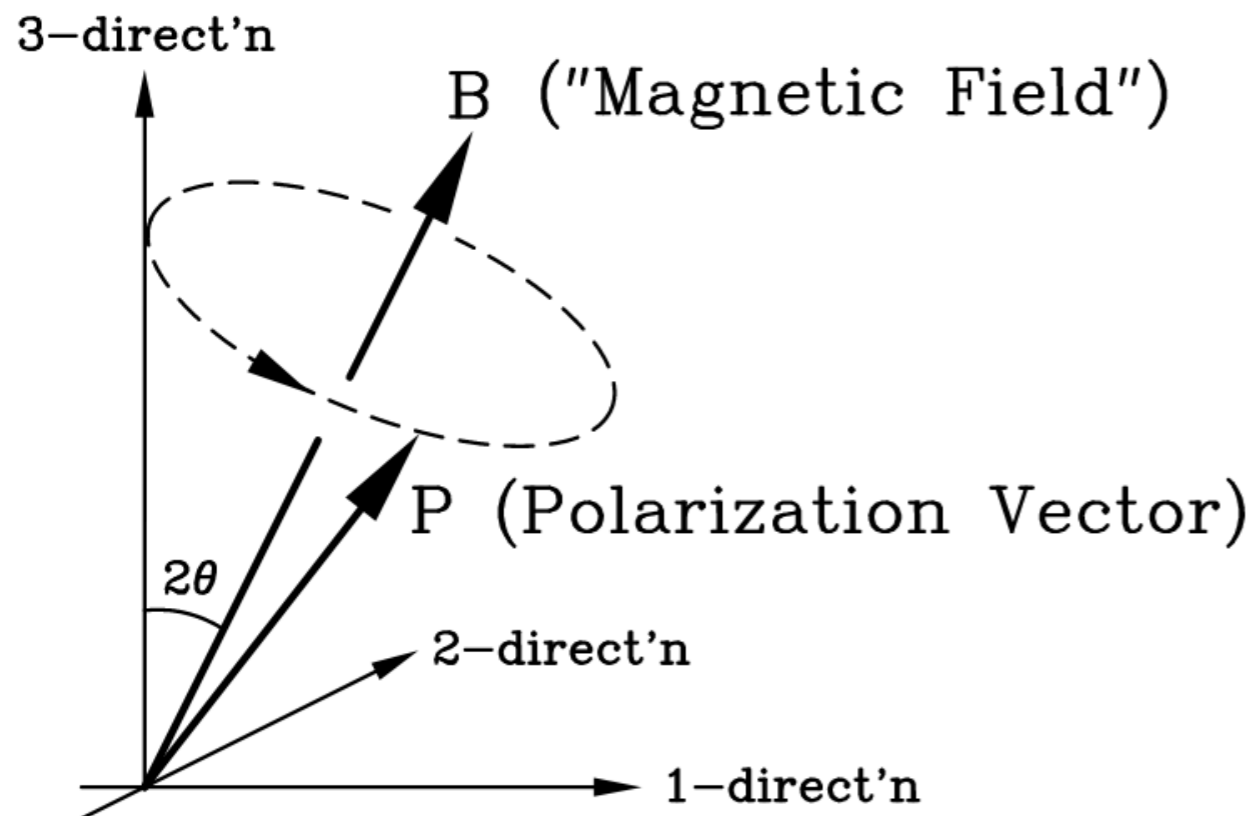
$$\left\{ \left[ \omega - \frac{G_F n_B}{\sqrt{2}} \begin{pmatrix} 3Y_e - 1 & 0 & 0 \\ 0 & Y_e - 1 & 0 \\ 0 & 0 & Y_e - 1 \end{pmatrix} \right]^2 - k^2 - U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger \right\} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = 0$$

The **“matter potentials”** depend on flavour because the sun contains electrons, but no muons and tauons

# Two Flavour Oscillations in Matter

$$b_0 = (m_1^2 + m_2^2)/4k + \sqrt{2} G_F n_B (Y_e - \frac{1}{2})$$

$$\mathbf{B} = \frac{m_2^2 - m_1^2}{2\omega} \begin{pmatrix} \sin 2\theta_0 \\ 0 \\ \cos 2\theta_0 \end{pmatrix} - \sqrt{2} G_F n_e \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



**effective "B field" depends on electron density, and therefore on space and time!**

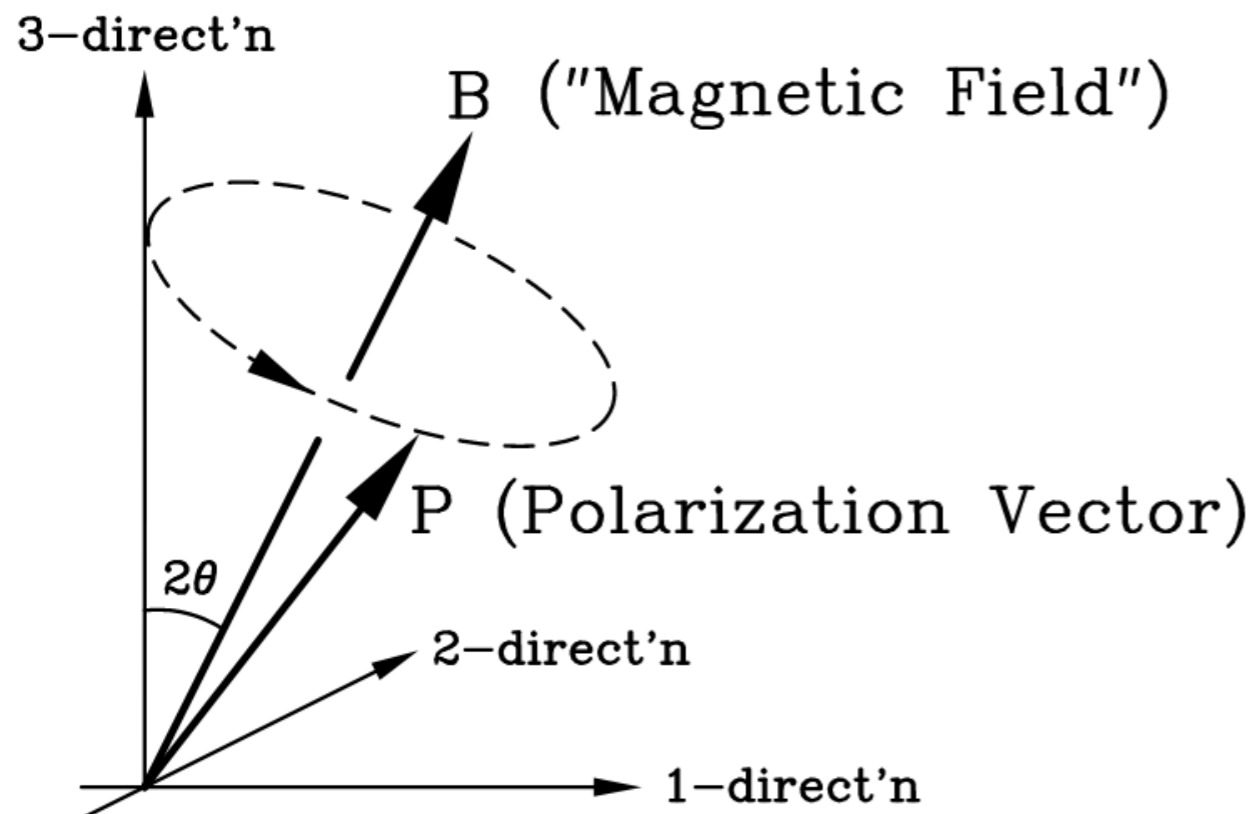
**flavour state can completely be changed by changing  $B$ .**



# Two Flavour Oscillations in Matter

$$b_0 = (m_1^2 + m_2^2)/4k + \sqrt{2} G_F n_B (Y_e - \frac{1}{2})$$

$$\mathbf{B} = \frac{2\pi}{l_{\text{osc}}} \begin{pmatrix} \sin 2\theta \\ 0 \\ \cos 2\theta \end{pmatrix} = \frac{m_2^2 - m_1^2}{2\omega} \begin{pmatrix} \sin 2\theta_0 \\ 0 \\ \cos 2\theta_0 \end{pmatrix} - \sqrt{2} G_F n_e \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



**effective "B field" depends on electron density, and therefore on space and time!**

**flavour state can completely be changed by changing  $B$ .**

---

# Effective Mass and Mixing

---

Effective mixing angle in matter:

$$\sin 2\theta = \frac{\sin 2\theta_0}{[\sin^2 2\theta_0 + (\cos 2\theta_0 - \xi)^2]^{1/2}}$$

Effective oscillation length in matter:

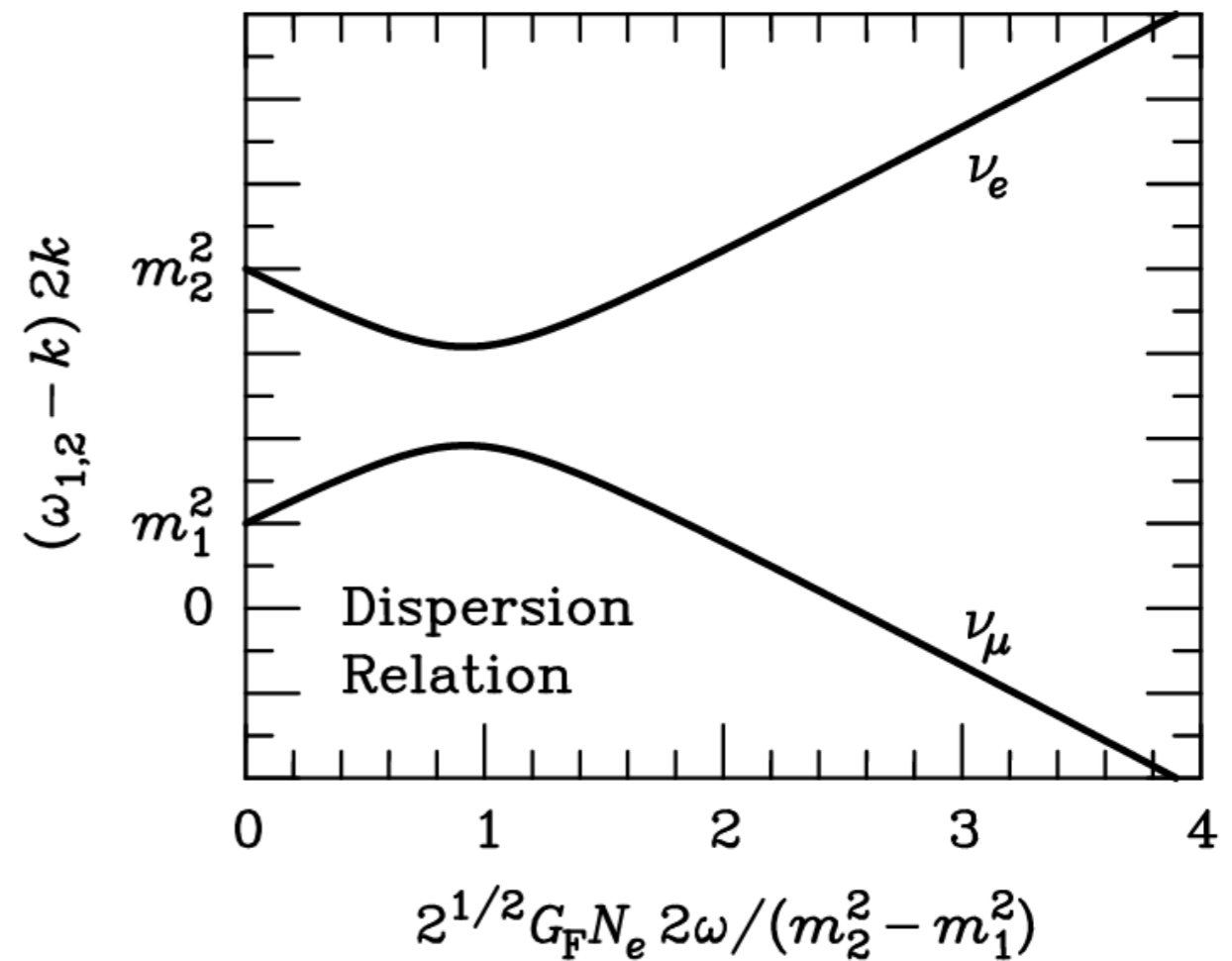
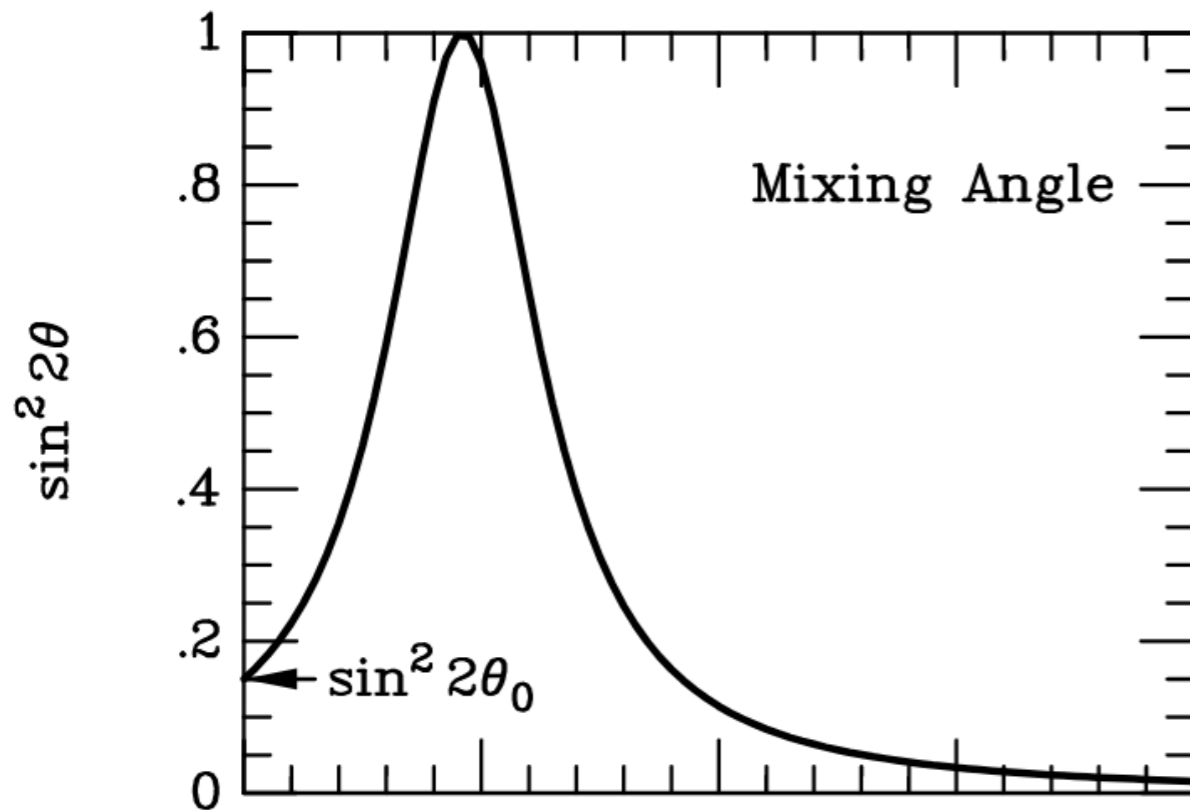
$$l_{\text{osc}} = \frac{4\pi\omega}{m_2^2 - m_1^2} \frac{\sin 2\theta}{\sin 2\theta_0}.$$

The decisive parameter depends on density, masses and energy:

$$\xi \equiv \frac{\sqrt{2} G_F n_e 2\omega}{m_2^2 - m_1^2} = 1.53 \times 10^{-7} \frac{Y_e \rho}{\text{g cm}^{-3}} \frac{\omega}{\text{MeV}} \frac{\text{eV}^2}{m_2^2 - m_1^2},$$

# MSW Resonance

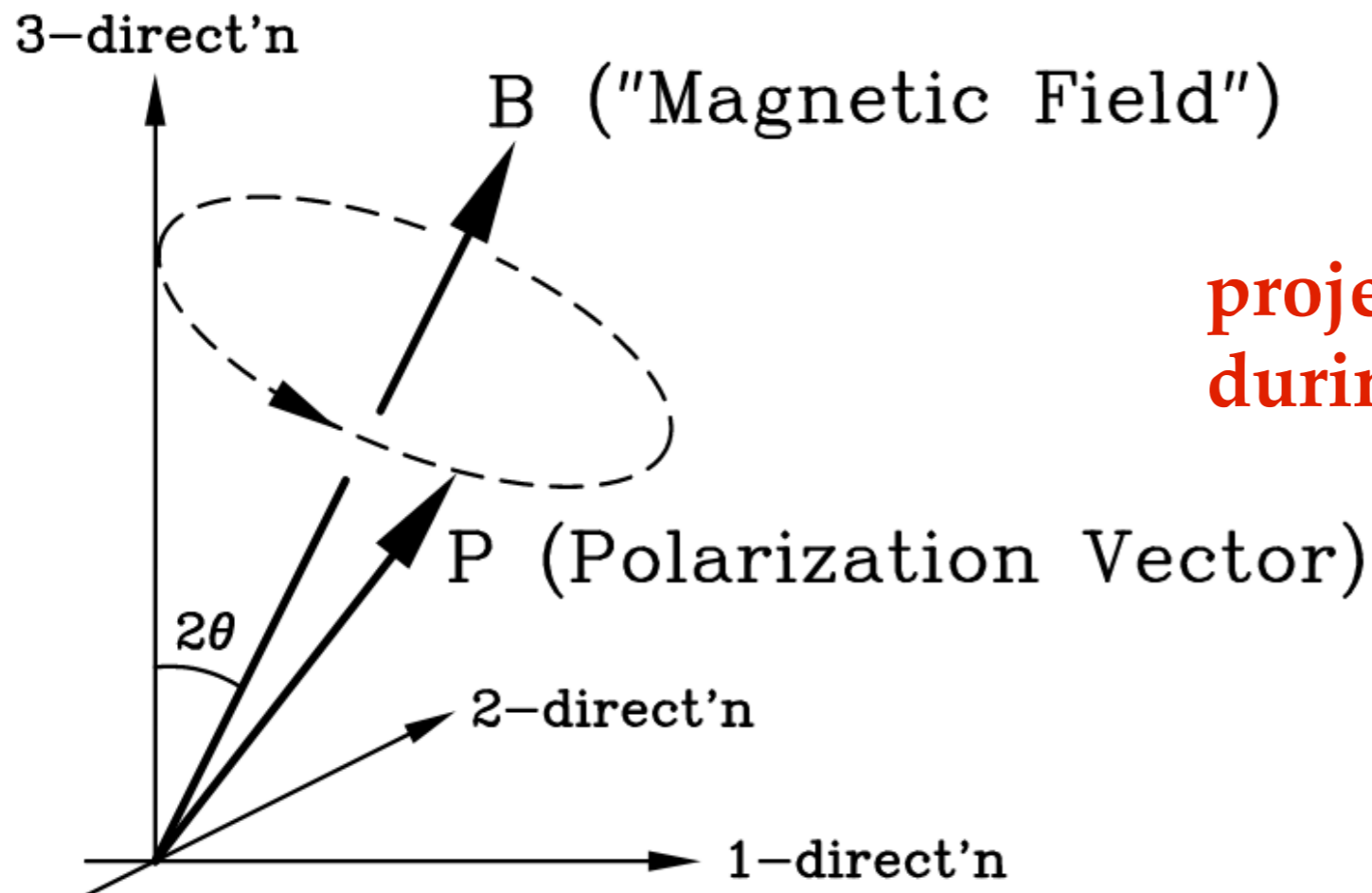
$$\sin 2\theta = \frac{\sin 2\theta_0}{[\sin^2 2\theta_0 + (\cos 2\theta_0 - \xi)^2]^{1/2}}$$





# Adiabatic Limit

$$\text{prob}(\nu_e \rightarrow \nu_e) = \frac{1}{2} (1 + \cos 2\theta_0 \cos 2\theta).$$



**projection of P on B is preserved during adiabatic evolution**

# Non-Adiabatic Limit

$$|\nabla\theta| \ll \pi/\ell_{\text{osc}} \quad \text{adiabaticity condition}$$

$$\xi \sin^3 2\theta |\nabla \ln n_e| \ll \sin^2 2\theta_0 |m_2^2 - m_1^2|/2\omega.$$

probability  $p$  to jump from one dispersion relation to the other:

$$\text{prob}(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - p\right) \cos 2\theta_0 \cos 2\theta$$

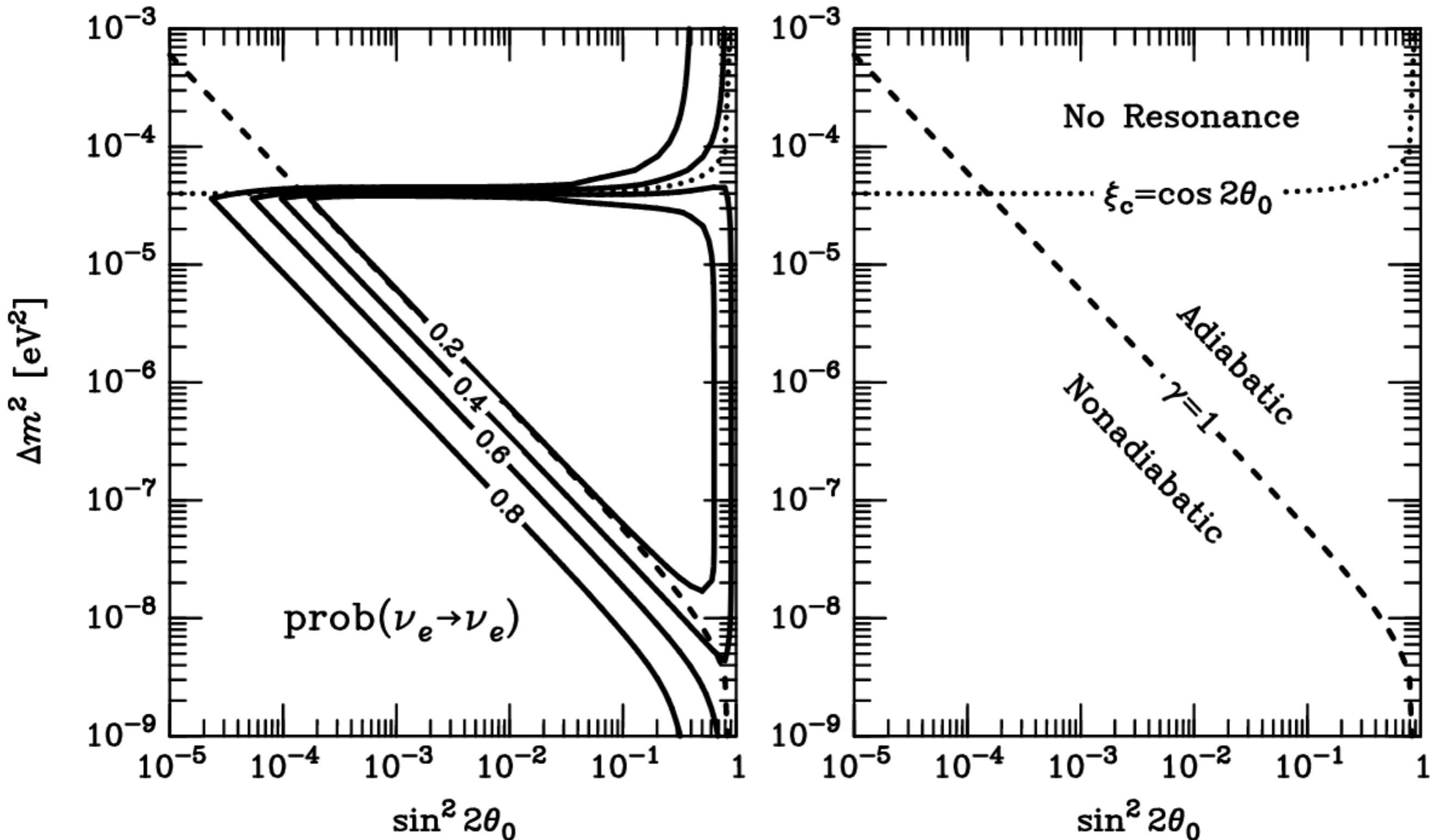
$$p = \frac{e^{-(\pi\gamma/2)F} - e^{-(\pi\gamma/2)F'}}{1 - e^{-(\pi\gamma/2)F'}}$$

$$F = 1 - \tan^2 \theta_0$$

$$\gamma \equiv \frac{m_2^2 - m_1^2}{2\omega} \frac{\sin 2\theta_0 \tan 2\theta_0}{|\nabla \ln n_e|_{\text{res}}}$$

for an exponential profile

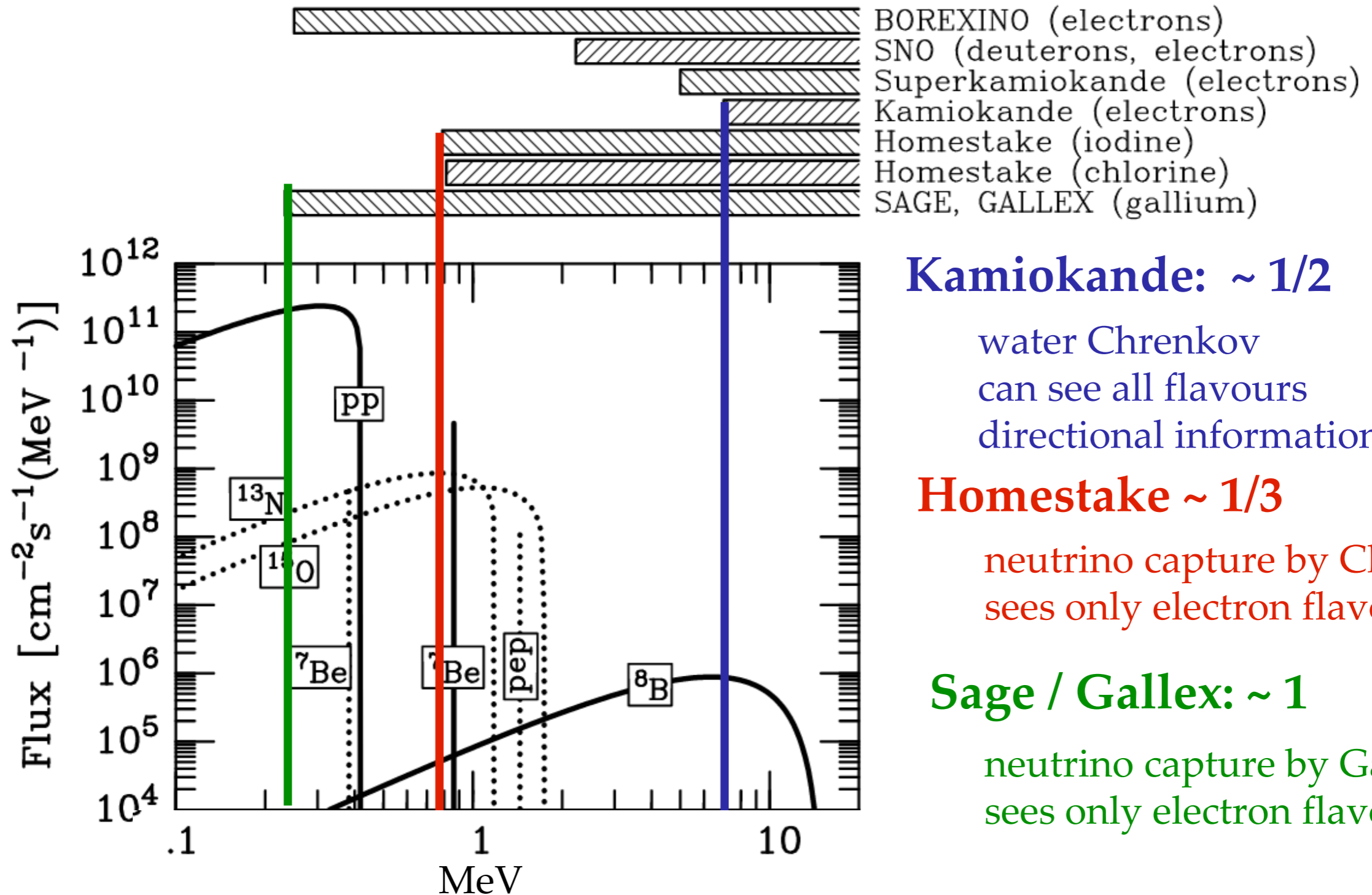
# MSW Resonance in the Sun





# Solar Model

Total flux: 66 billion  $\nu$  / (s cm<sup>2</sup>)



**Kamiokande: ~ 1/2**

water Chrenkov  
can see all flavours  
directional information

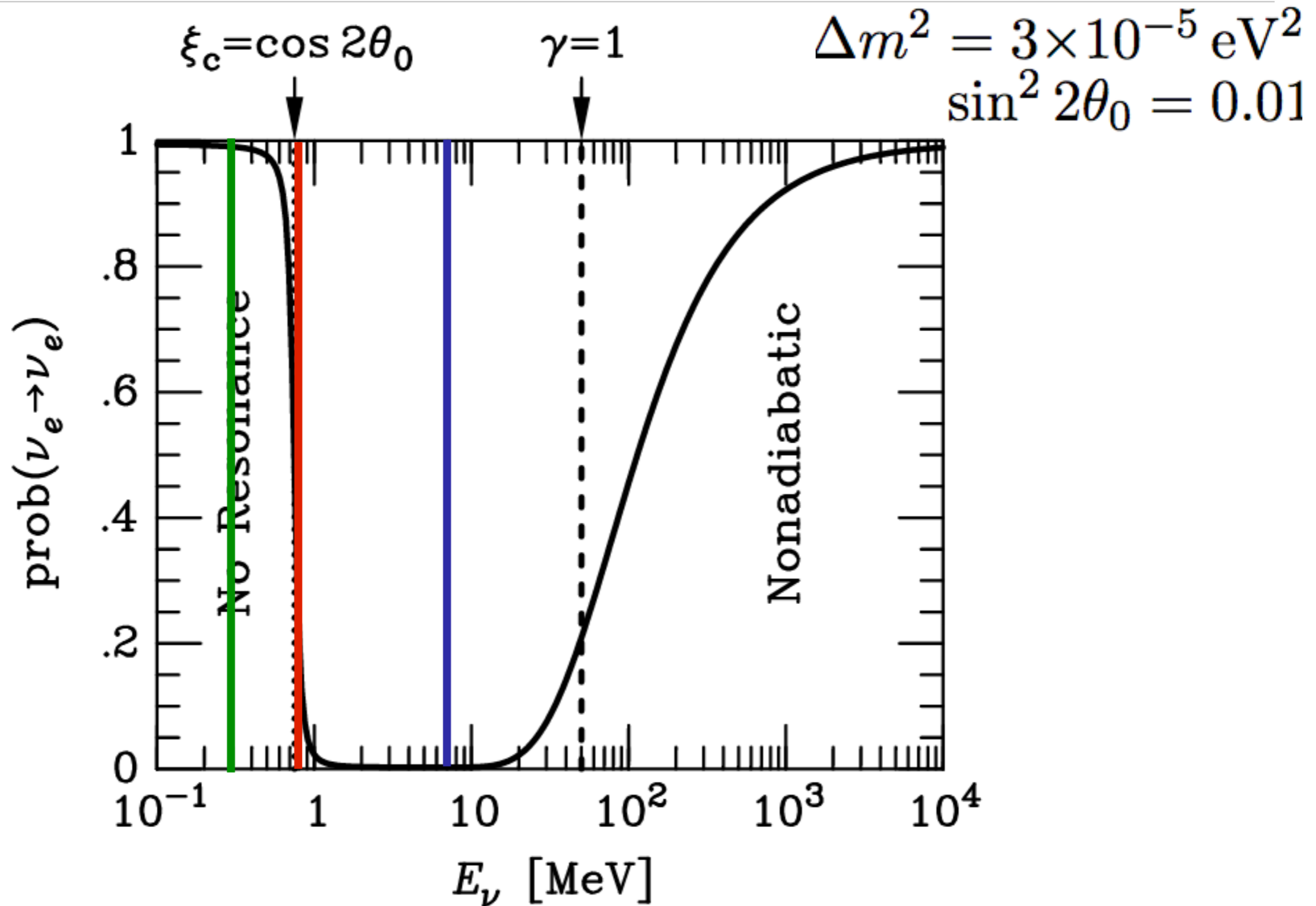
**Homestake ~ 1/3**

neutrino capture by Cl  
sees only electron flavour

**Sage / Gallex: ~ 1**

neutrino capture by Ga  
sees only electron flavour

# The “Bathtub Plot”





# Neutrino Oscillations

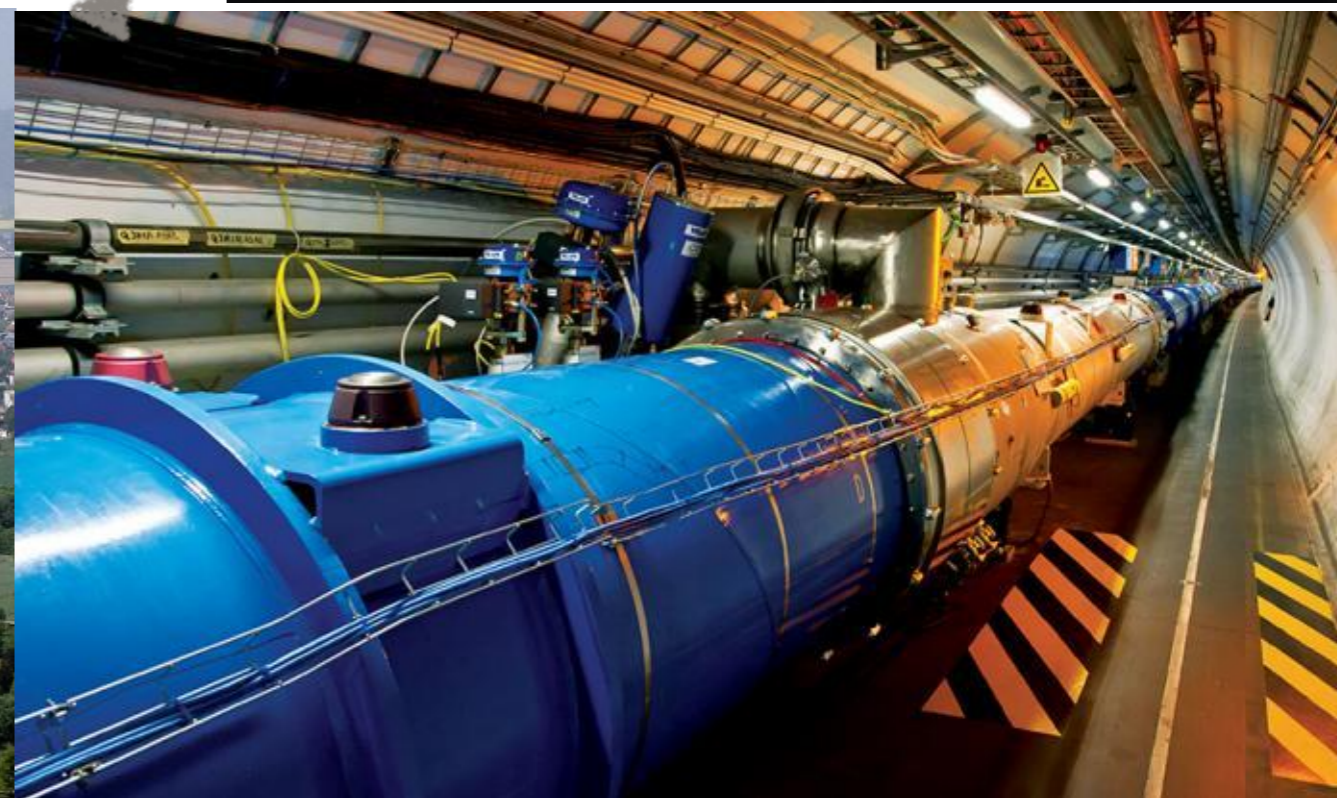
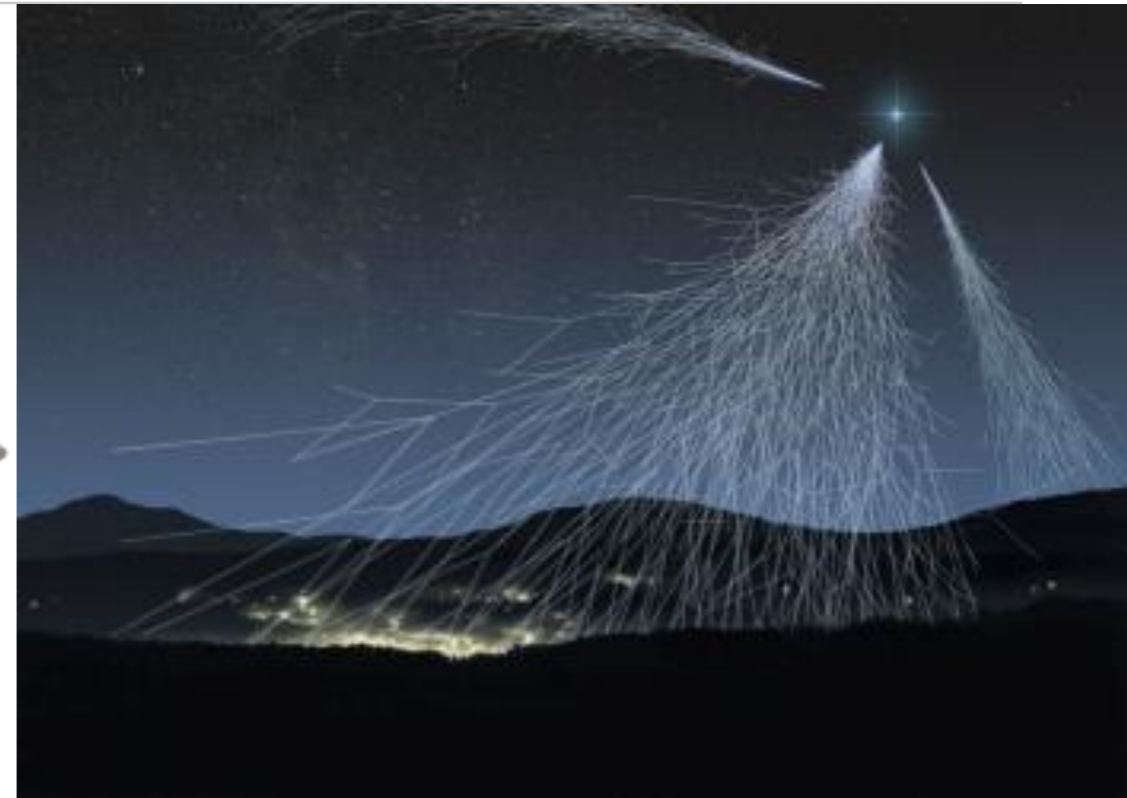
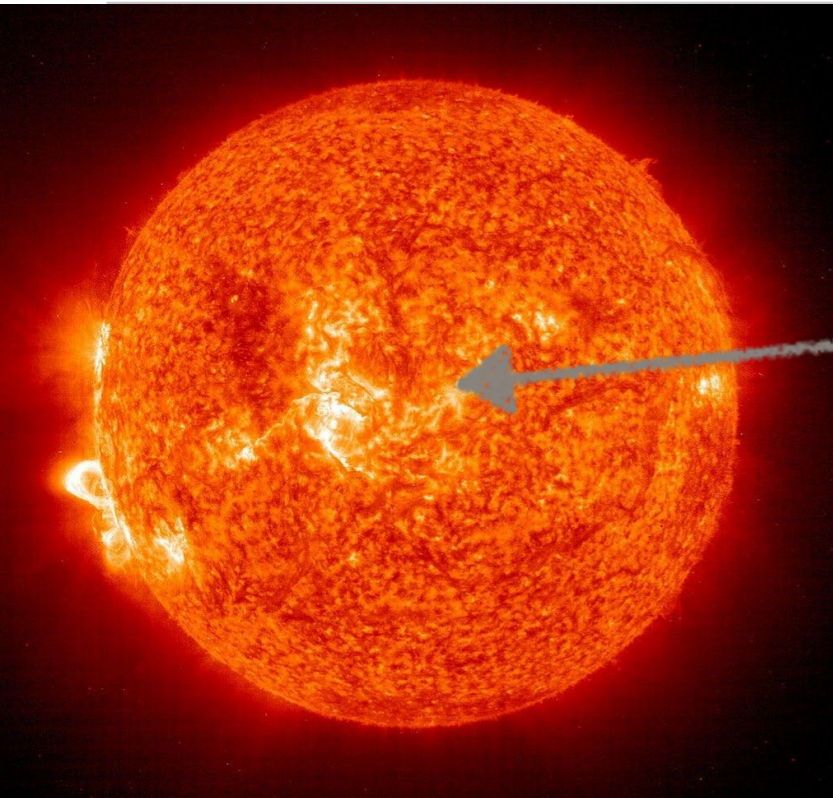
Evidence comes from many sources:

sun

cosmic rays

nuclear reactors

particle colliders





# Neutrino Properties

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# What we know

---

- neutrinos undergo flavour oscillations
- those can be explained by a Dirac or Majorana mass term

$$\bar{\nu}_L m_D \nu_R + h.c.$$

$$\bar{\nu}_L m_M \nu_L^c + h.c.$$

- ...which can be diagonalised as

$$\text{diag}(m_1, m_2, m_3) = U_\nu^\dagger m_D \tilde{U}_\nu \quad \text{diag}(m_1, m_2, m_3) = U_\nu^\dagger m_M U_\nu^*$$

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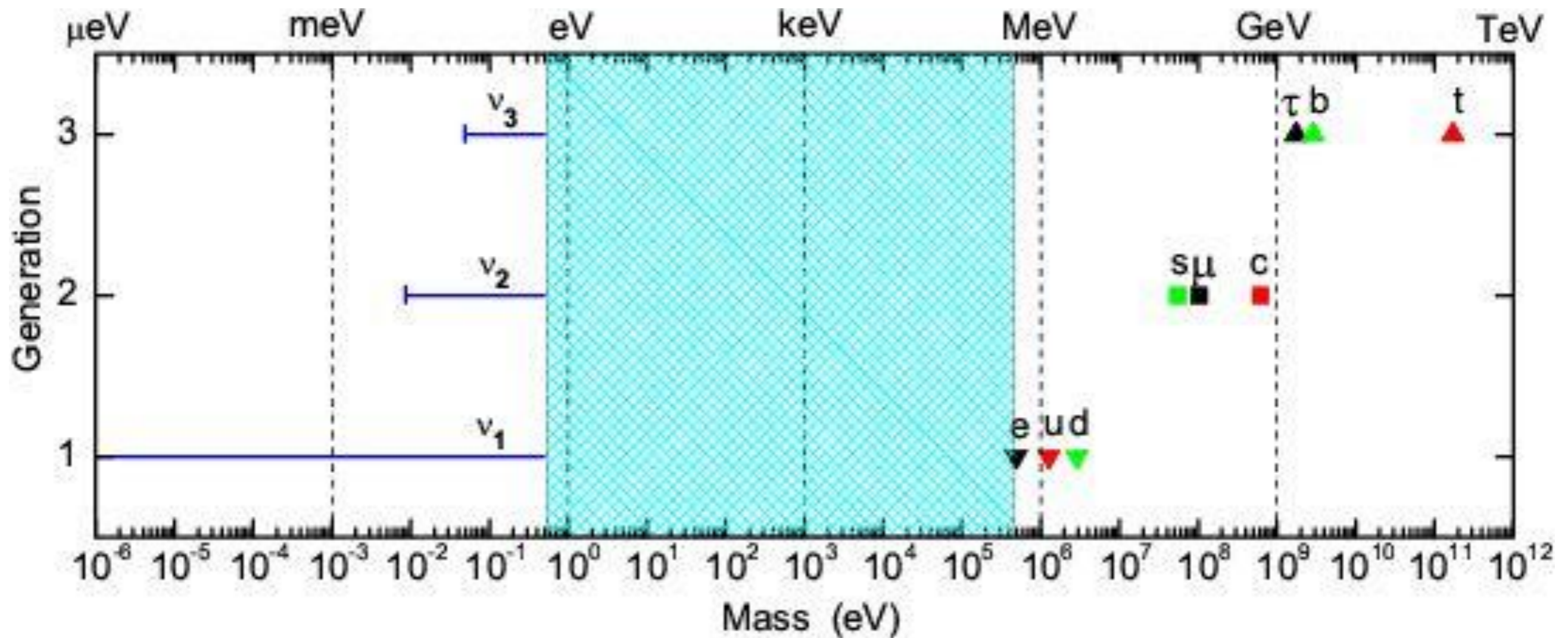
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leptons are just like quarks without colour... but:

- Why are the  $m_i$  so tiny?
- Why is the mixing matrix so different from the CKM matrix?
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# Neutrino Mixing Matrix

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix},$$

“PMNS matrix”  
Pontecorvo-Maki-Nakagawa-Sakata

## Common parameterisation of the matrix $U$ :

$$U_\nu = V^{(23)} U_\delta V^{(13)} U_{-\delta} V^{(12)} \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$$

$$U_{\pm\delta} = \text{diag}(e^{\mp i\delta/2}, 1, e^{\pm i\delta/2}) \quad V^{(12)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V^{(23)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \quad V^{(13)} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 2.6$ )	
	bf $\pm 1\sigma$	$3\sigma$ range	bf $\pm 1\sigma$	$3\sigma$ range
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	0.269 $\rightarrow$ 0.343	$0.304^{+0.012}_{-0.012}$	0.269 $\rightarrow$ 0.343
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	31.27 $\rightarrow$ 35.86	$33.45^{+0.77}_{-0.74}$	31.27 $\rightarrow$ 35.87
$\sin^2 \theta_{23}$	$0.573^{+0.018}_{-0.023}$	0.405 $\rightarrow$ 0.620	$0.578^{+0.017}_{-0.021}$	0.410 $\rightarrow$ 0.623
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	39.5 $\rightarrow$ 52.0	$49.5^{+1.0}_{-1.2}$	39.8 $\rightarrow$ 52.1
$\sin^2 \theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	0.02034 $\rightarrow$ 0.02430	$0.02238^{+0.00064}_{-0.00062}$	0.02053 $\rightarrow$ 0.02434
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	8.20 $\rightarrow$ 8.97	$8.60^{+0.12}_{-0.12}$	8.24 $\rightarrow$ 8.98
$\delta_{CP}/^\circ$	$194^{+52}_{-25}$	105 $\rightarrow$ 405	$287^{+27}_{-32}$	192 $\rightarrow$ 361
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	6.82 $\rightarrow$ 8.04	$7.42^{+0.21}_{-0.20}$	6.82 $\rightarrow$ 8.04
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.515^{+0.028}_{-0.028}$	+2.431 $\rightarrow$ +2.599	$-2.498^{+0.028}_{-0.029}$	-2.584 $\rightarrow$ -2.413

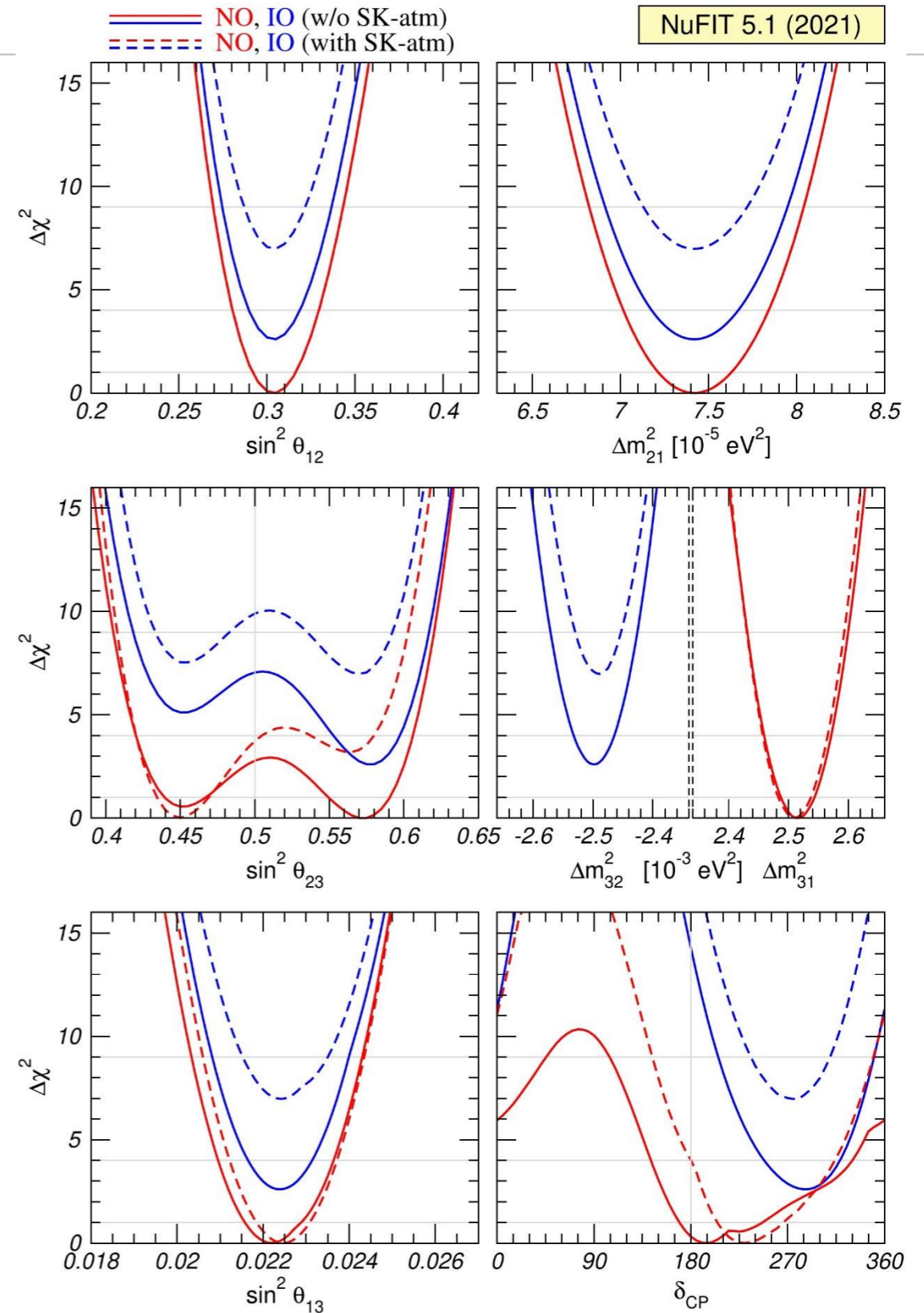
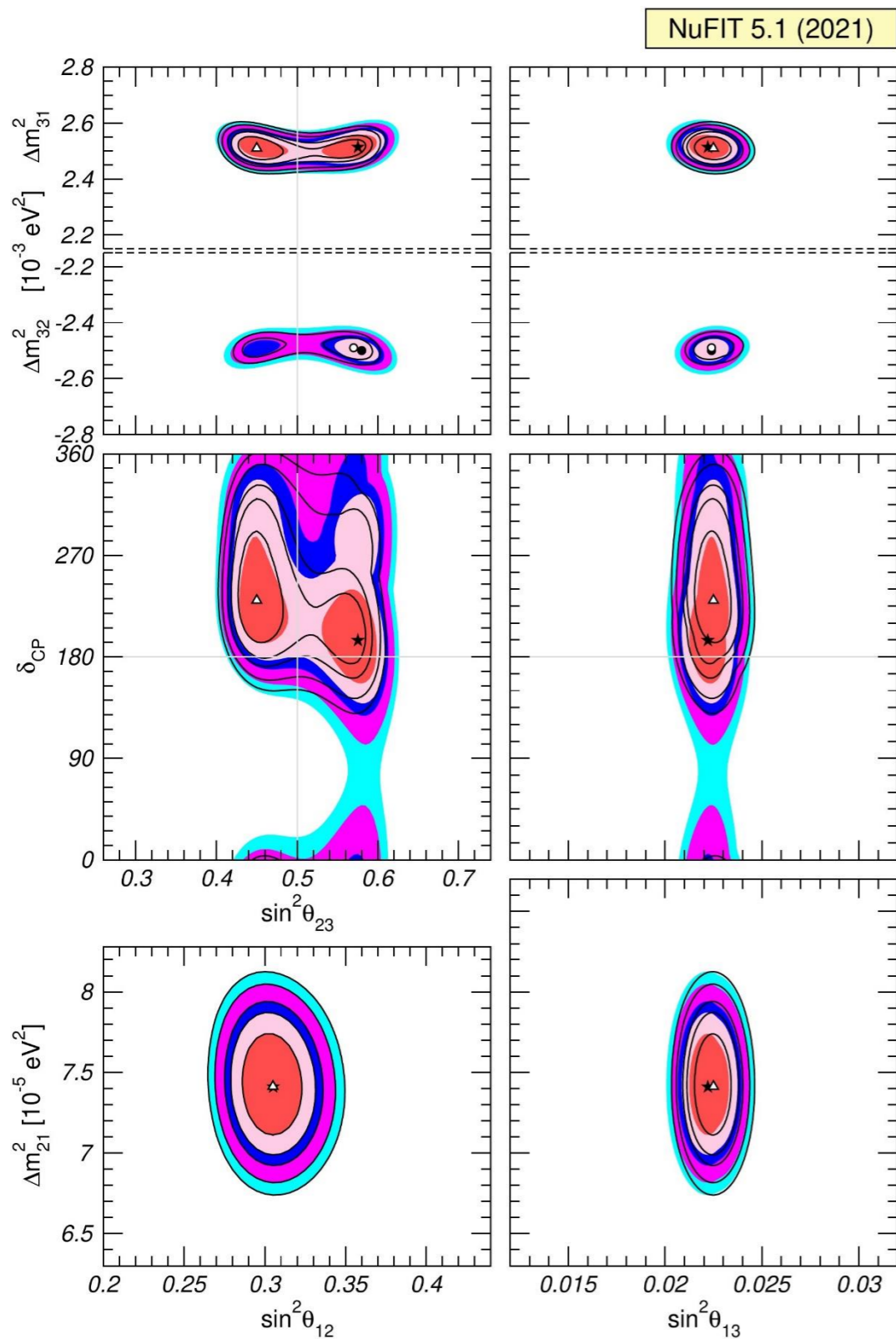
$J_\nu^*$

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# Current Best Fit (NuFit 2021)



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- Majorana fermion  
NEW PHYSICS!
- can be generated in gauge invariant way by higher dim operators  
NEW PHYSICS!  
should be generated by integrating out some heavier states  
with masses  $\sim \Lambda \gg E_\nu$



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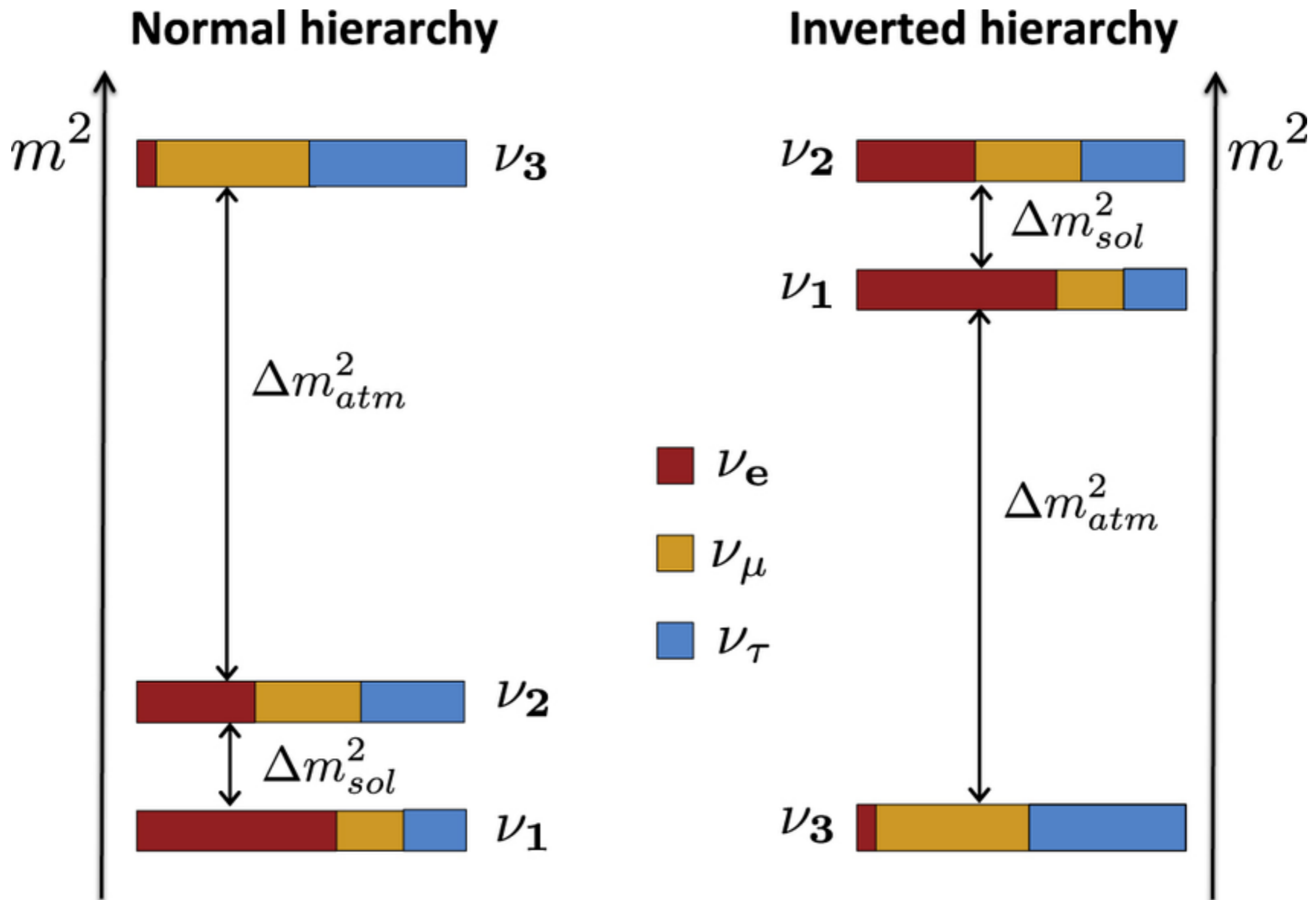
# What we want to know

---

- **the mass hierarchy**  
“normal” or “inverted”?
  - **the absolute mass scale**  
is there a massless elementary fermion?
  - **CP-violation**  
... and connection to baryogenesis?
  - **Dirac or Majorana?**  
Is B-L conserved in nature? Connection to baryogenesis?
- 
- **Are there extra (sterile) neutrinos?**
  - **mechanism of mass generation**  
Hint towards more fundamental theory (e.g. GUT)?  
Scale  $\Lambda$  in the context of the hierarchy problem?  
Connection to Baryogenesis or Dark Matter?

# The Neutrino Mass Hierarchy

# “Normal” and “Inverted” Ordering





---

# Effect of Ordering on Oscillations

---

Electron neutrino appearance probability

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 (A-1)\Delta}{(A-1)^2}$$

To leading order only depends on

- two mixing angles
- the square of the larger mass splitting

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}$$

$$A = +G_f N_e \frac{L}{\sqrt{2}\Delta}$$

# Effect of Ordering on Oscillations

Electron neutrino appearance probability

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 (A-1)\Delta}{(A-1)^2} \\ + 2\alpha \sin \theta_{13} \cos \delta_{CP} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{(A-1)} \cos \Delta \\ - 2\alpha \sin \theta_{13} \sin \delta_{CP} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{(A-1)} \sin \Delta$$

But subleading corrections depend on sign of splittings!

$$\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$$

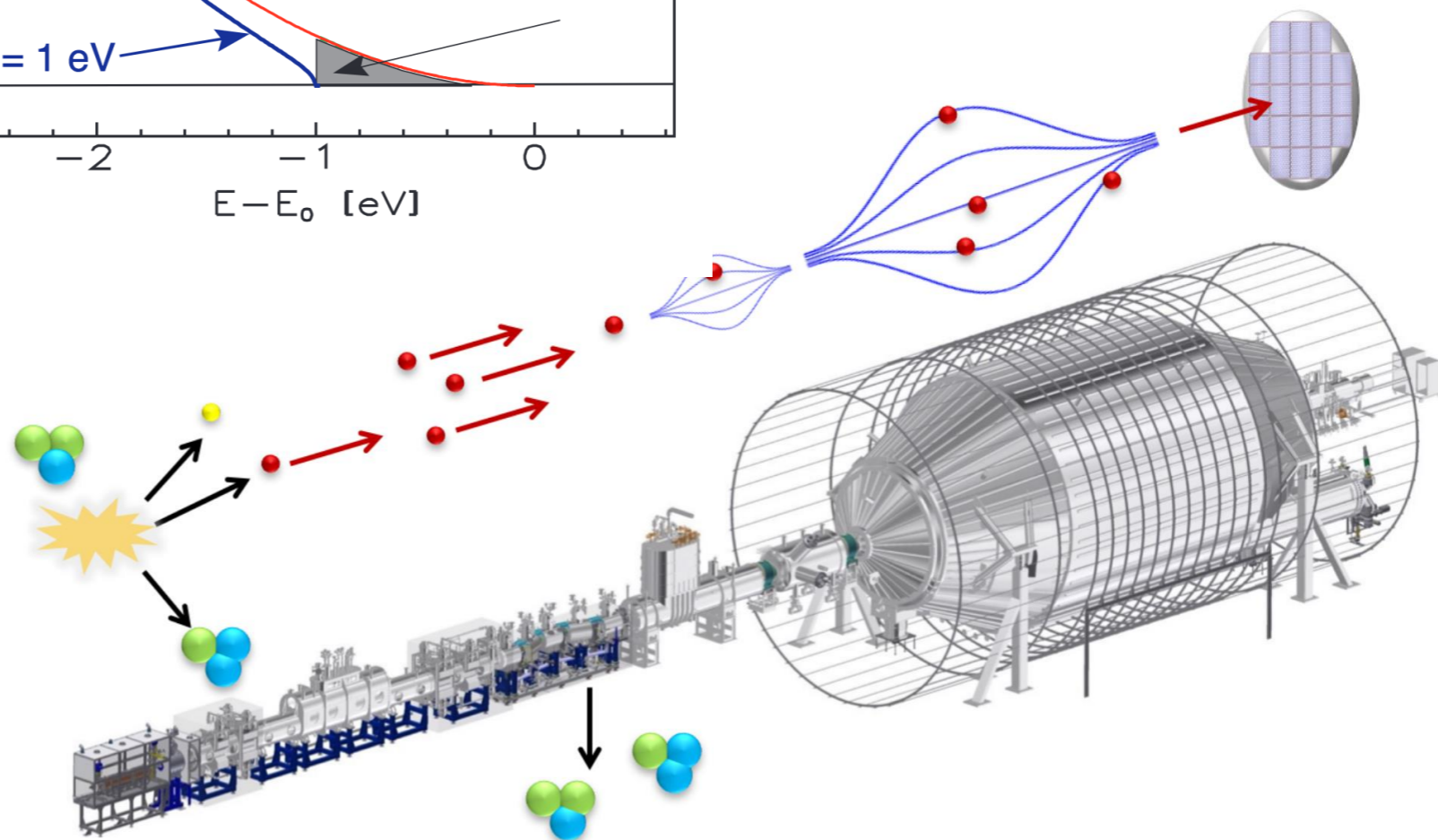
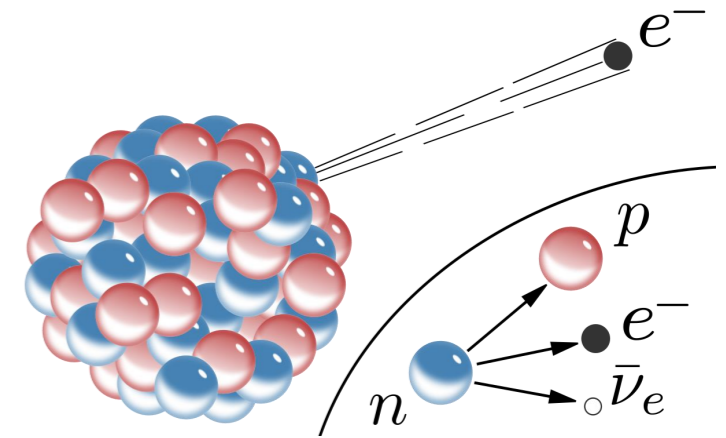
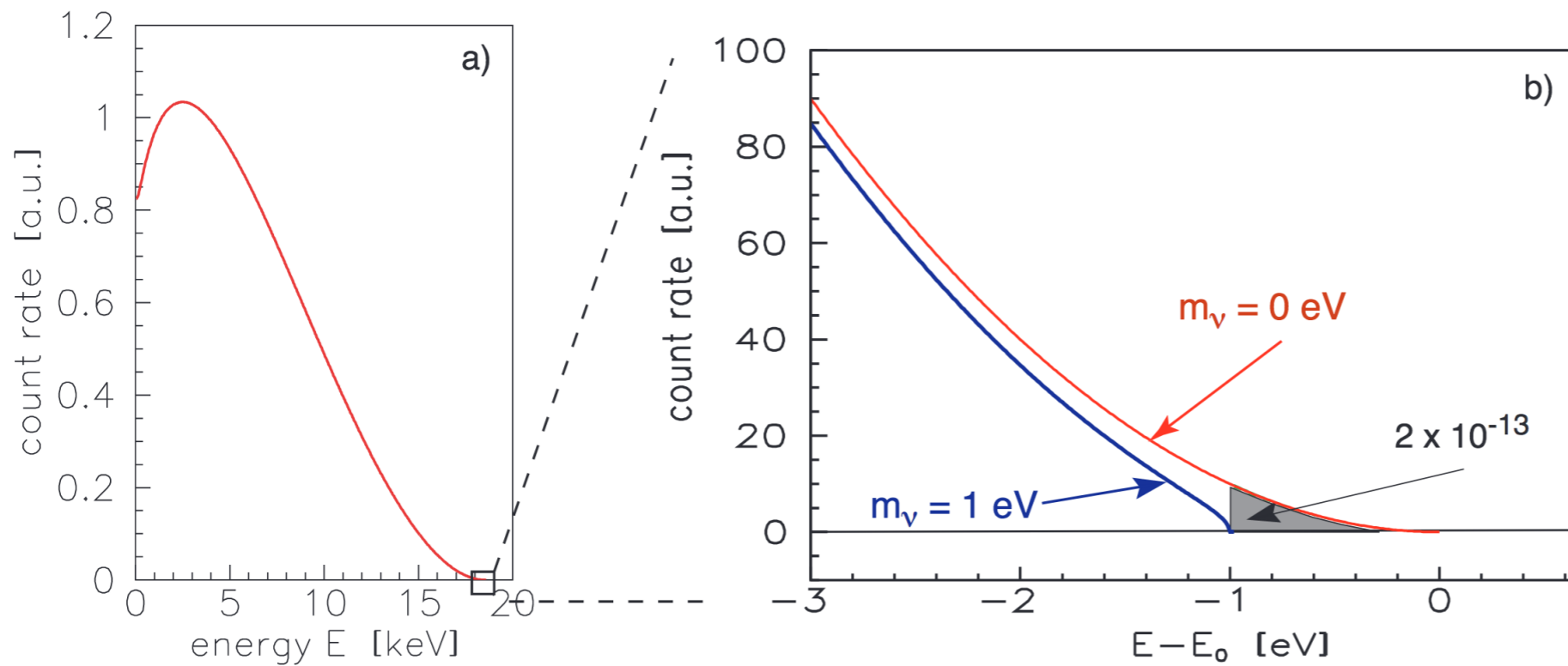
$$\Delta = \frac{\Delta m_{31}^2 L}{4E}$$

$$A = +G_f N_e \frac{L}{\sqrt{2}\Delta}$$

lightest neutrino mass

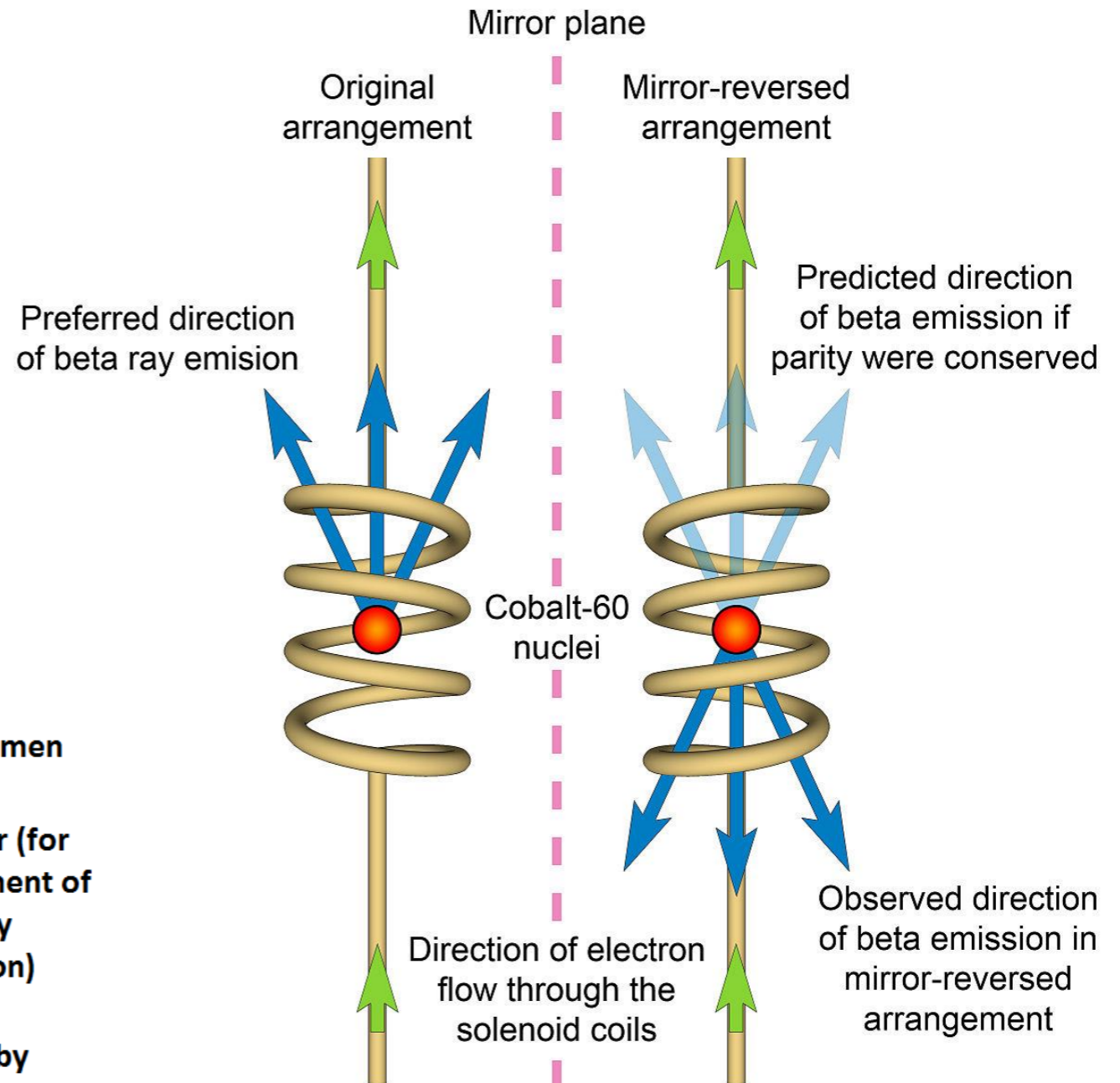
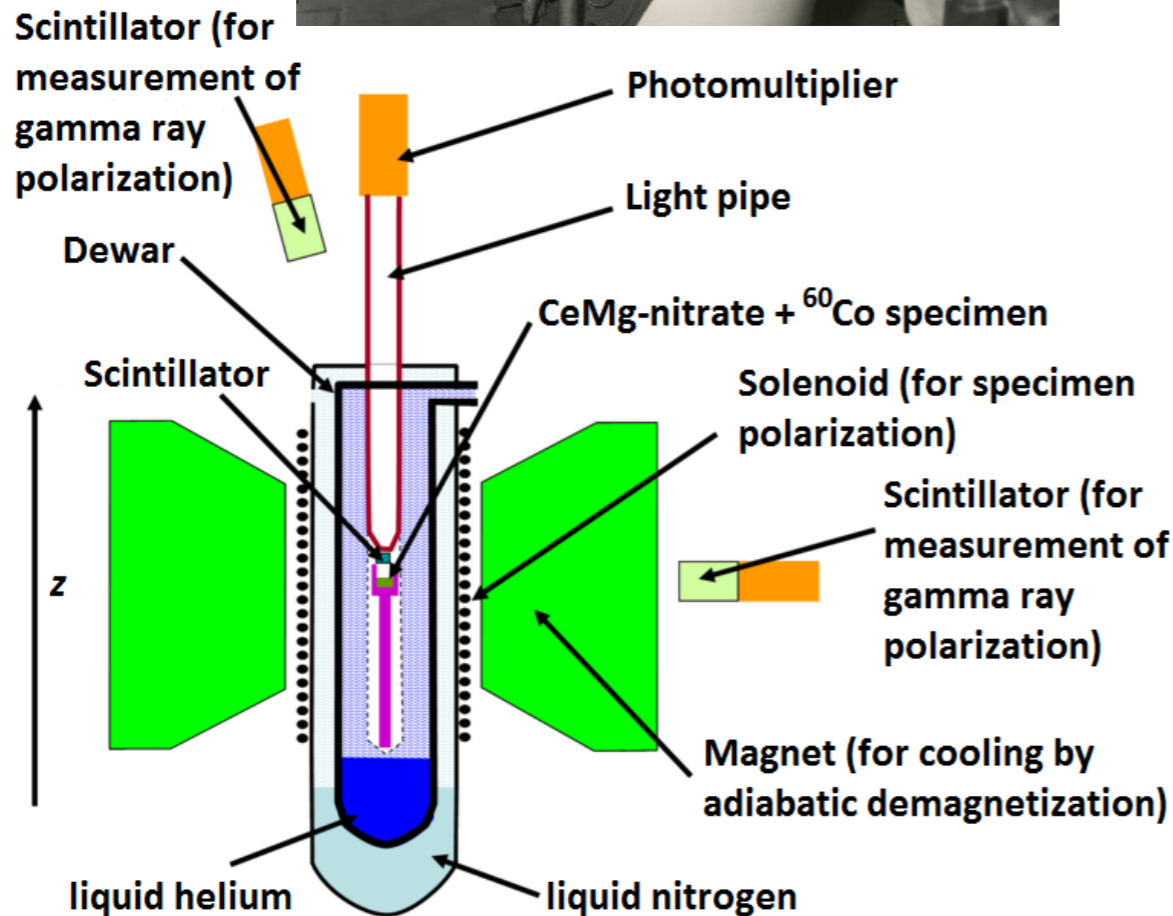


# Direct Measurement of the Neutrino Mass



# Parity Violation

# The Wu Experiment



# The hunt for CP violation



# Effect of CP-Violation on Oscillations

Electron neutrino appearance probability

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 (A-1)\Delta}{(A-1)^2}$$

Is sensitive to  
Dirac phase!

dependence on  
CPV suppressed  
by ratio of mass  
splittings...

$$+ 2\alpha \sin \theta_{13} \cos \delta_{CP} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{(A-1)} \cos \Delta$$

$$- 2\alpha \sin \theta_{13} \sin \delta_{CP} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{(A-1)} \sin \Delta$$

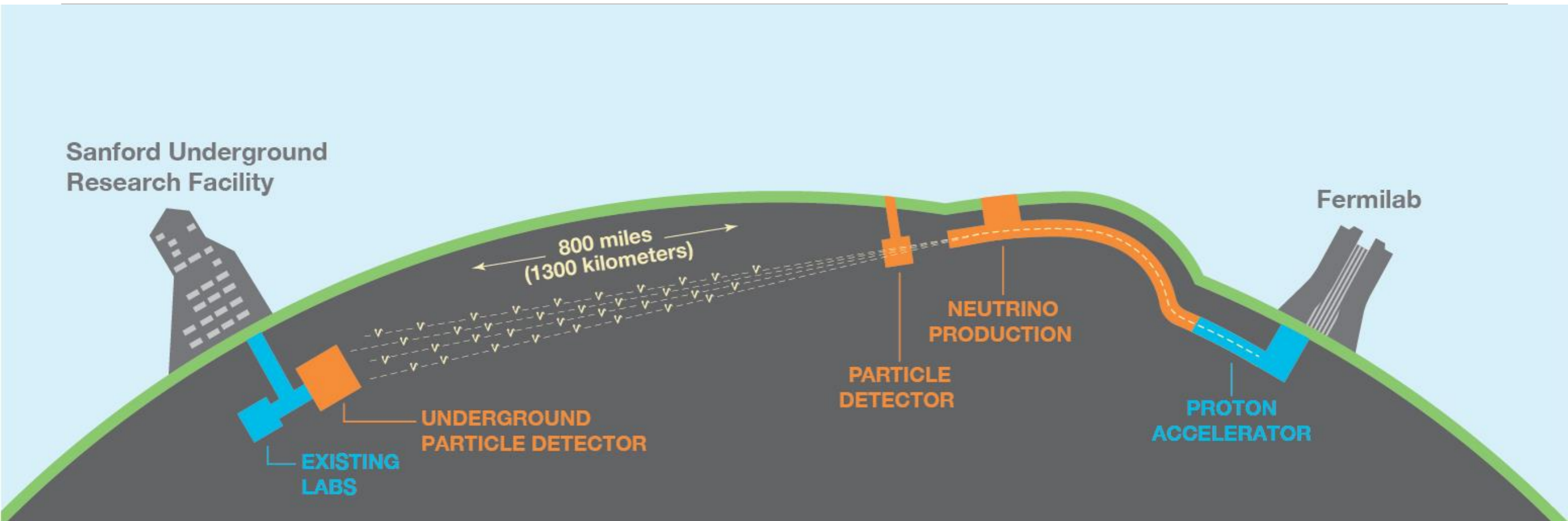
...luckily  $\theta_{13}$  is not too small!

$$\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}$$

$$A = +G_f N_e \frac{L}{\sqrt{2}\Delta}$$

# DUNE



## Liquid Argon Time-Projection Chamber

[https://www.youtube.com/watch?v=R5G1\\_hW0ZUA#action=share](https://www.youtube.com/watch?v=R5G1_hW0ZUA#action=share)

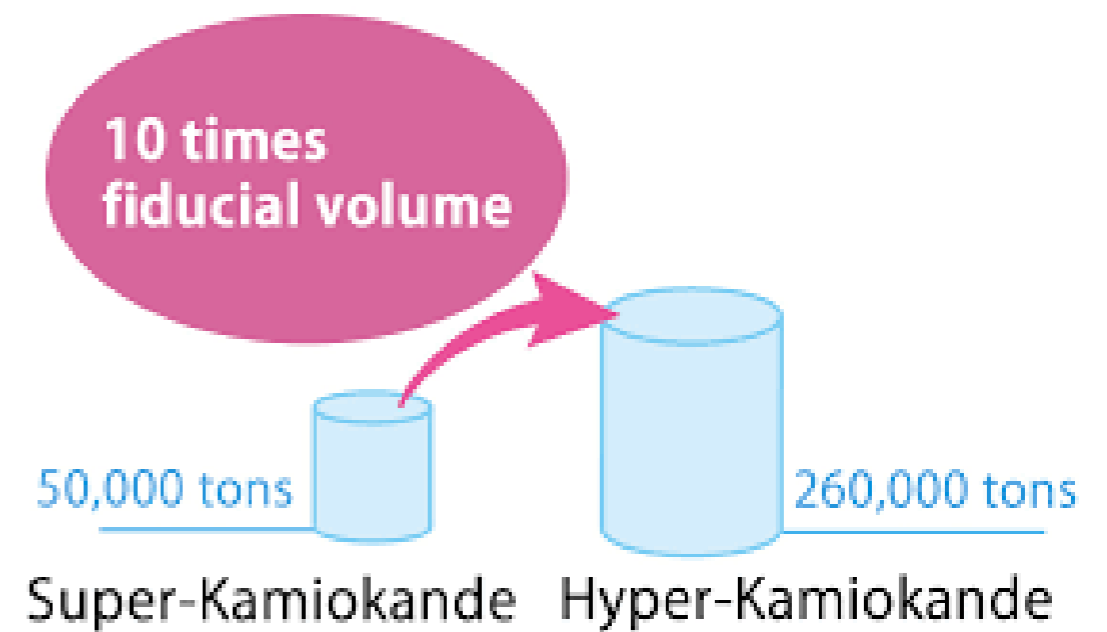
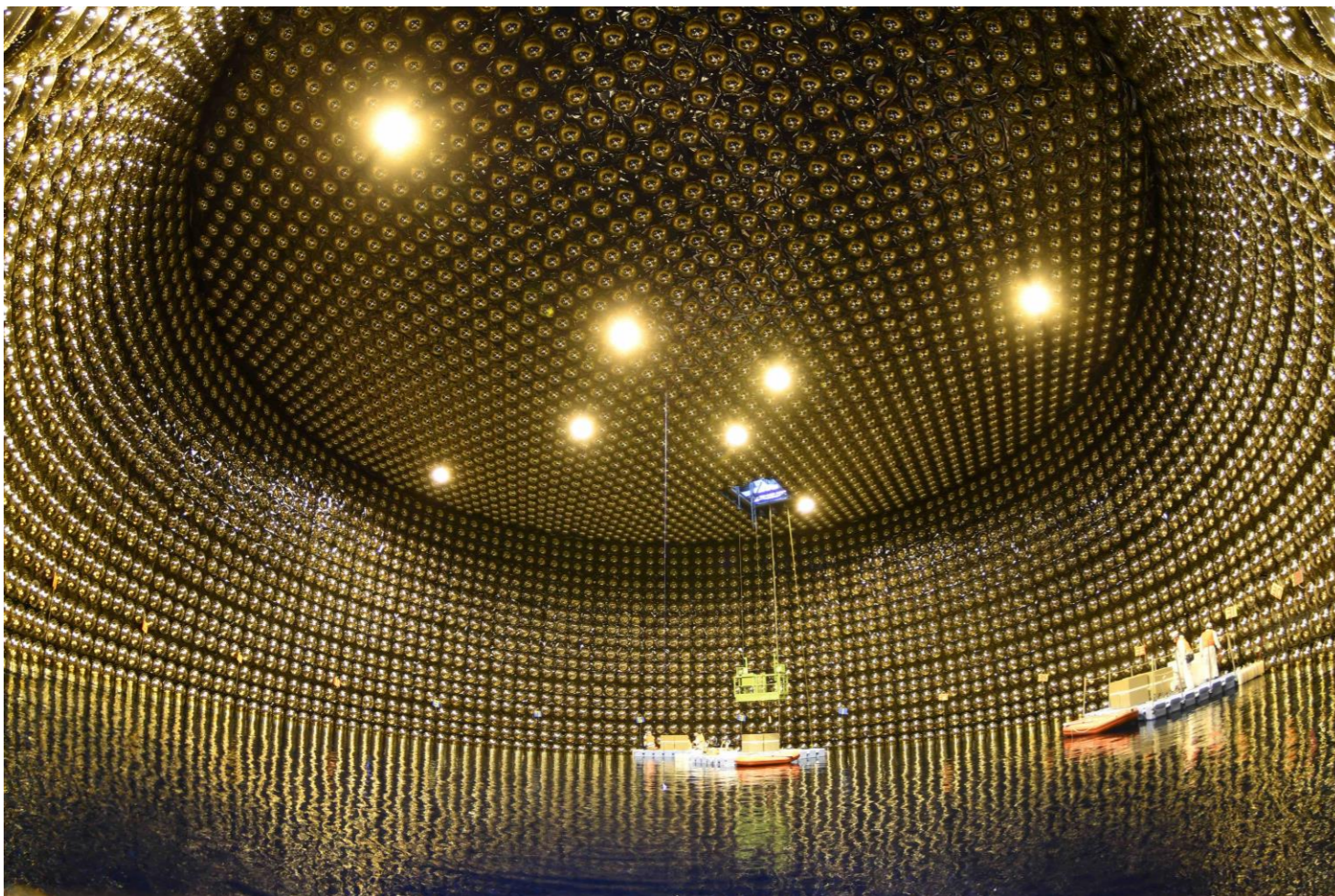
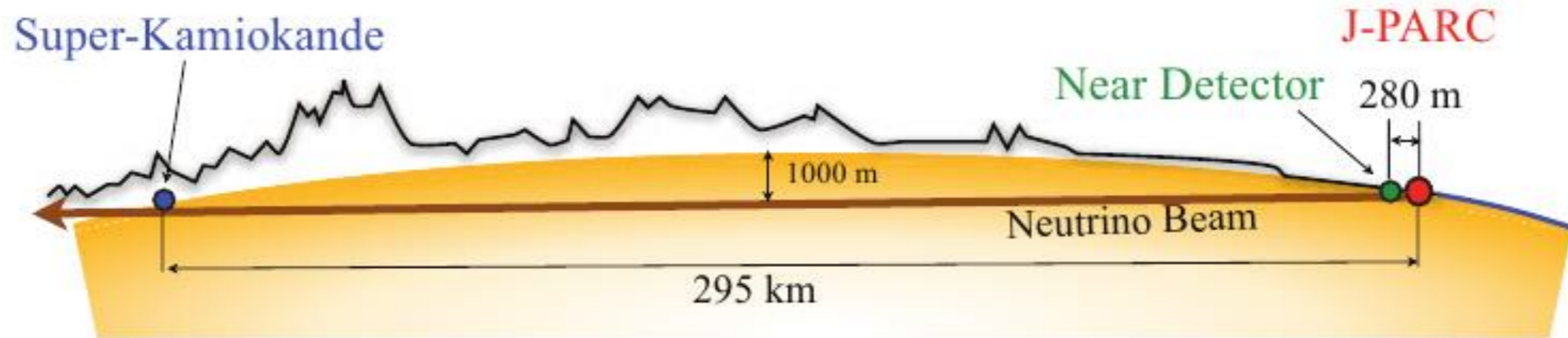
catches beam for Fermilab

[https://www.youtube.com/watch?v=U\\_xWDWKq1CM](https://www.youtube.com/watch?v=U_xWDWKq1CM)

is expected to discover CP violation in electron neutrino appearance

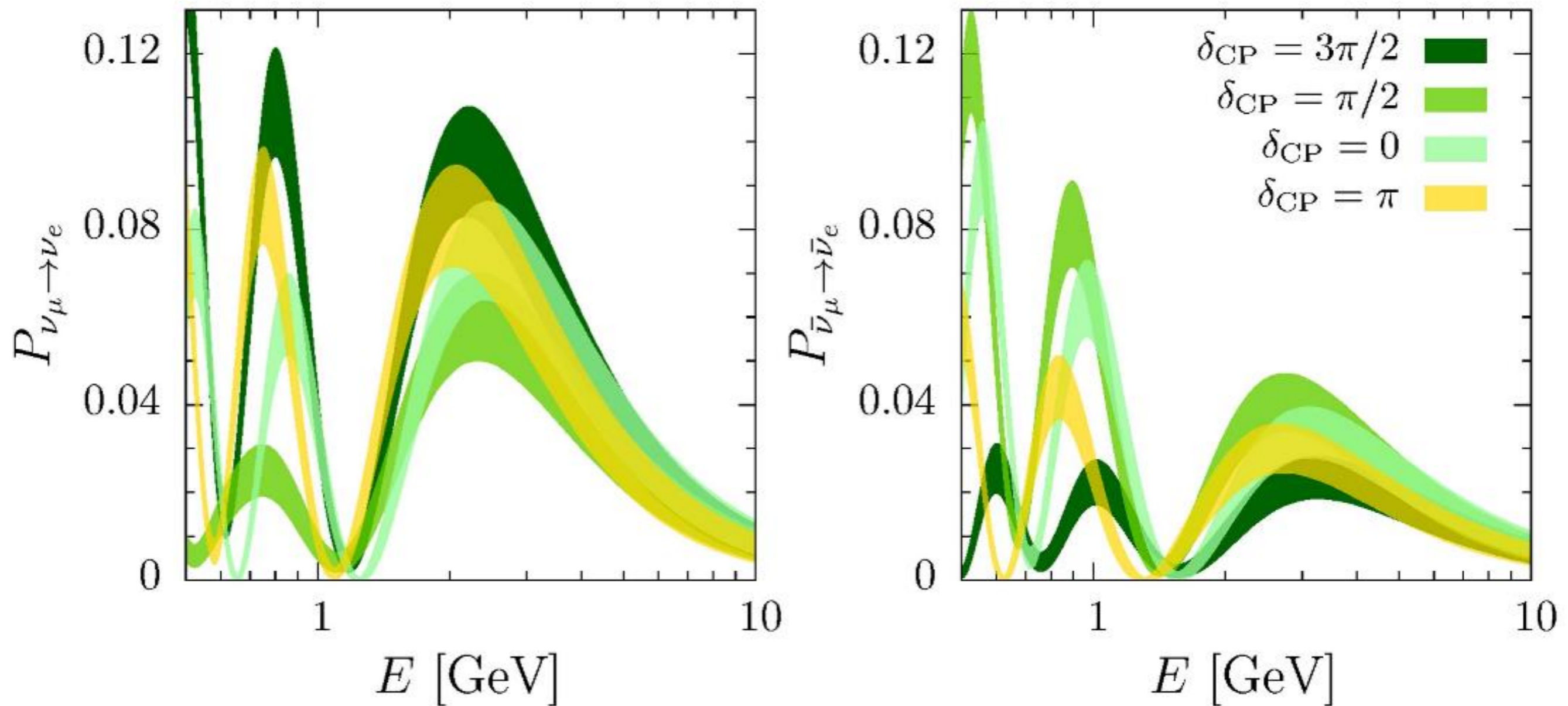


# T2HyperK





# Electron Neutrino Appearance

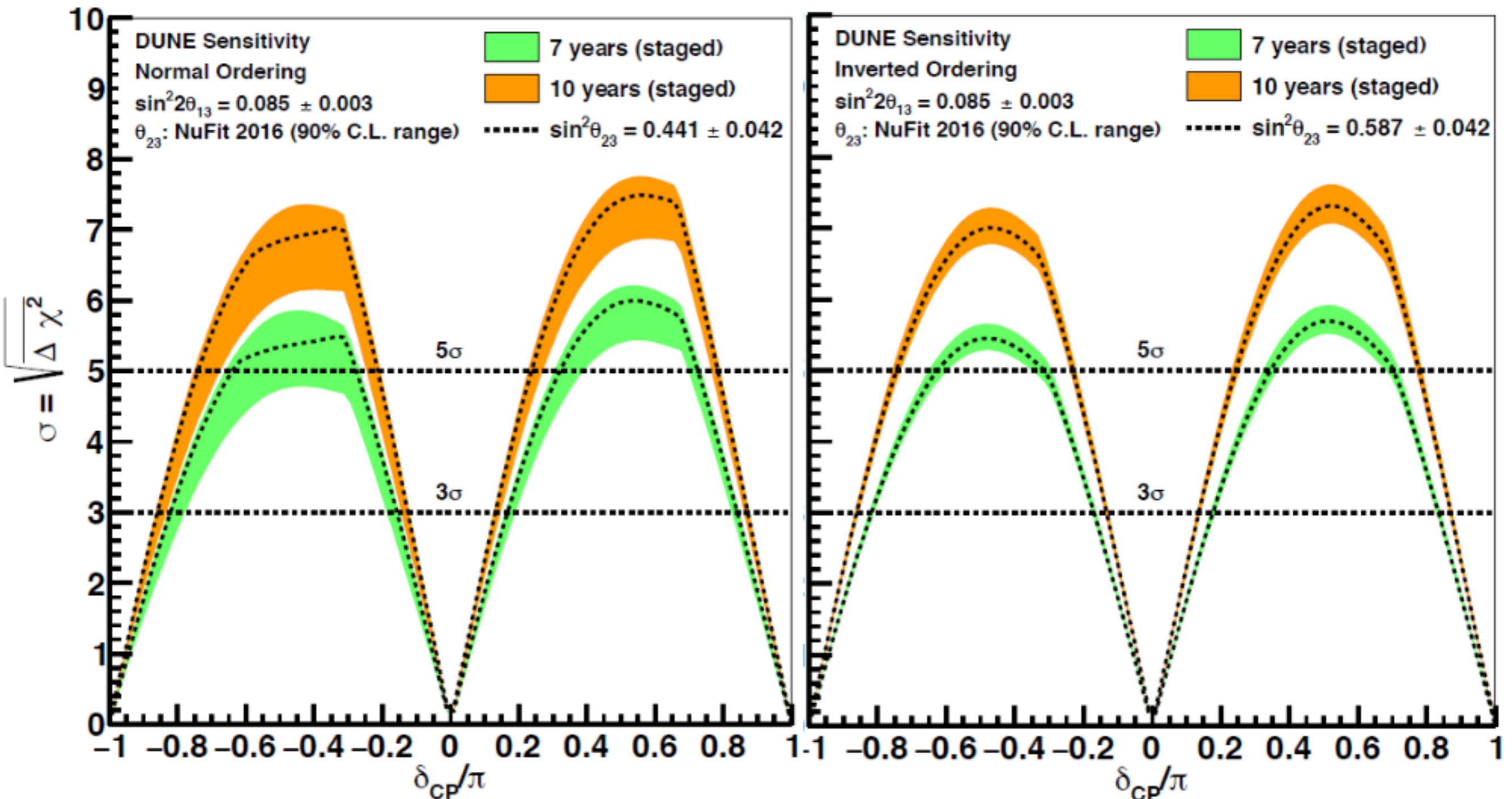




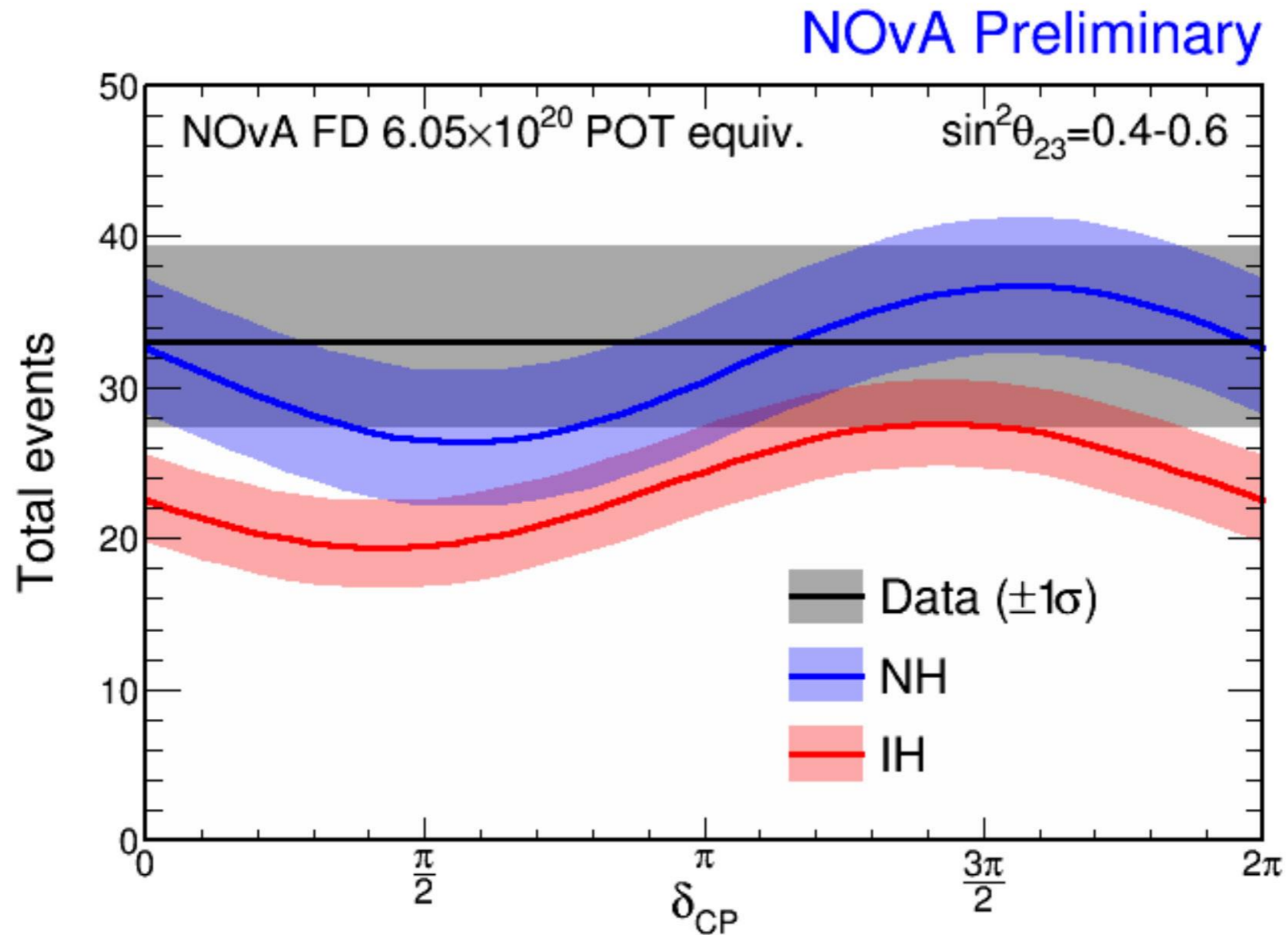
# Expected Sensitivity: Example DUNE

**CP (NH)**

**CP (IH)**



# Example: NovA



Dirac or Majorana?



# Neutrinoless Double $\beta$ Decay

Rate depends on nuclear matrix element

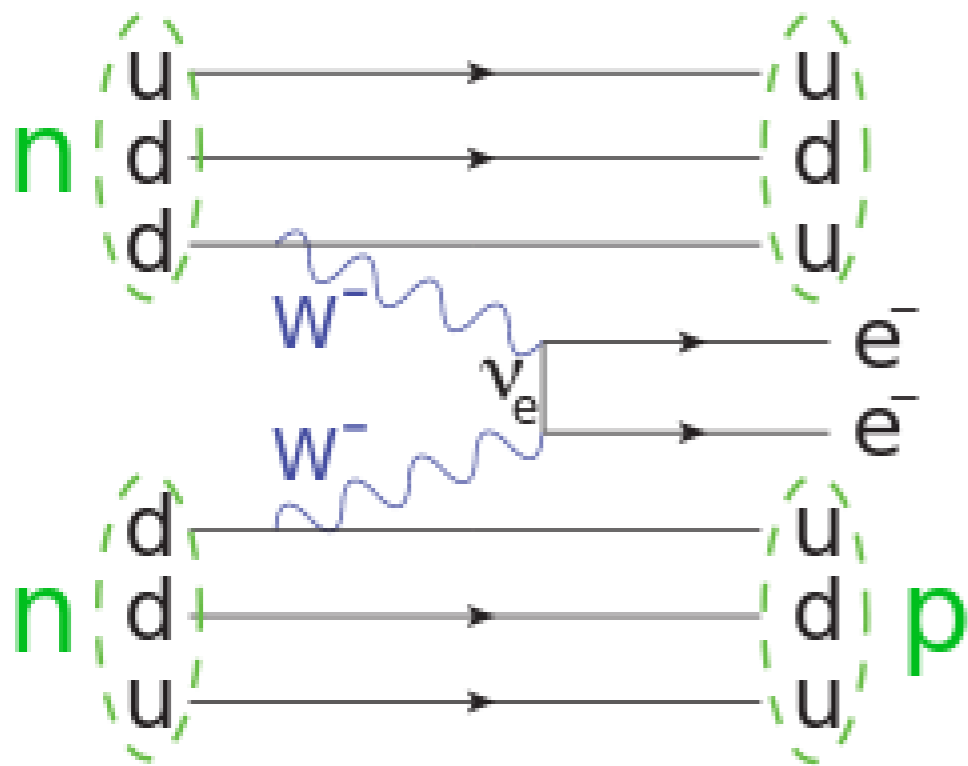
$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \left| \sum_N (U_{eN}^2 m_N) m_p M'^{0\nu}(m_N, g_A^{\text{eff}}) \right|^2$$

Often assumed: factorisation!

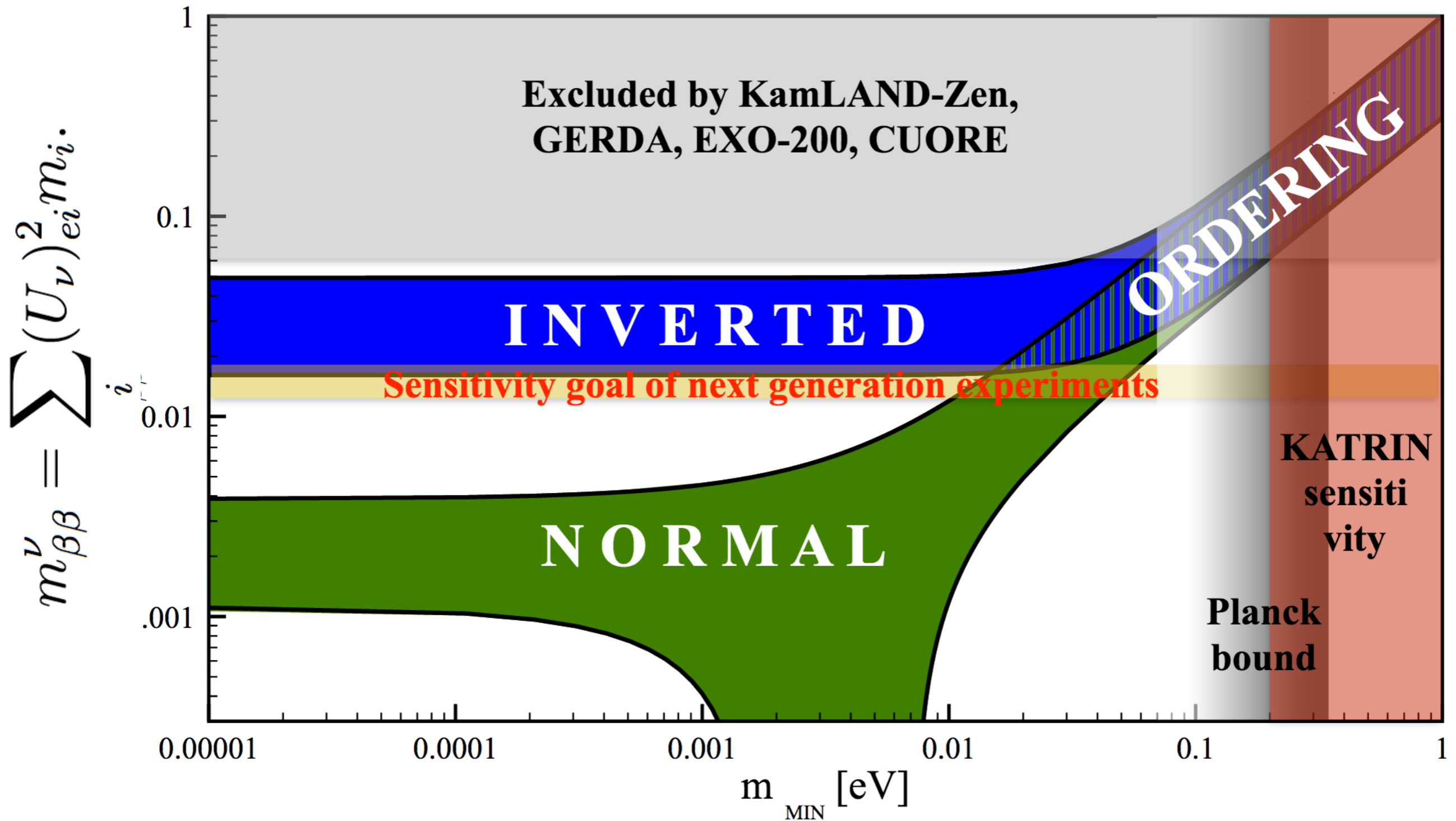
$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|$$

All neutrino physics inside one number

$$m_{\beta\beta}^\nu = \sum_i (U_\nu)_{ei}^2 m_i$$



# Neutrinoless Double $\beta$ Decay



# Sterile neutrinos



# Sterile neutrino - mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

**Sterile neutrino is gauge singlet, feels no electromagnetic or nuclear forces!**

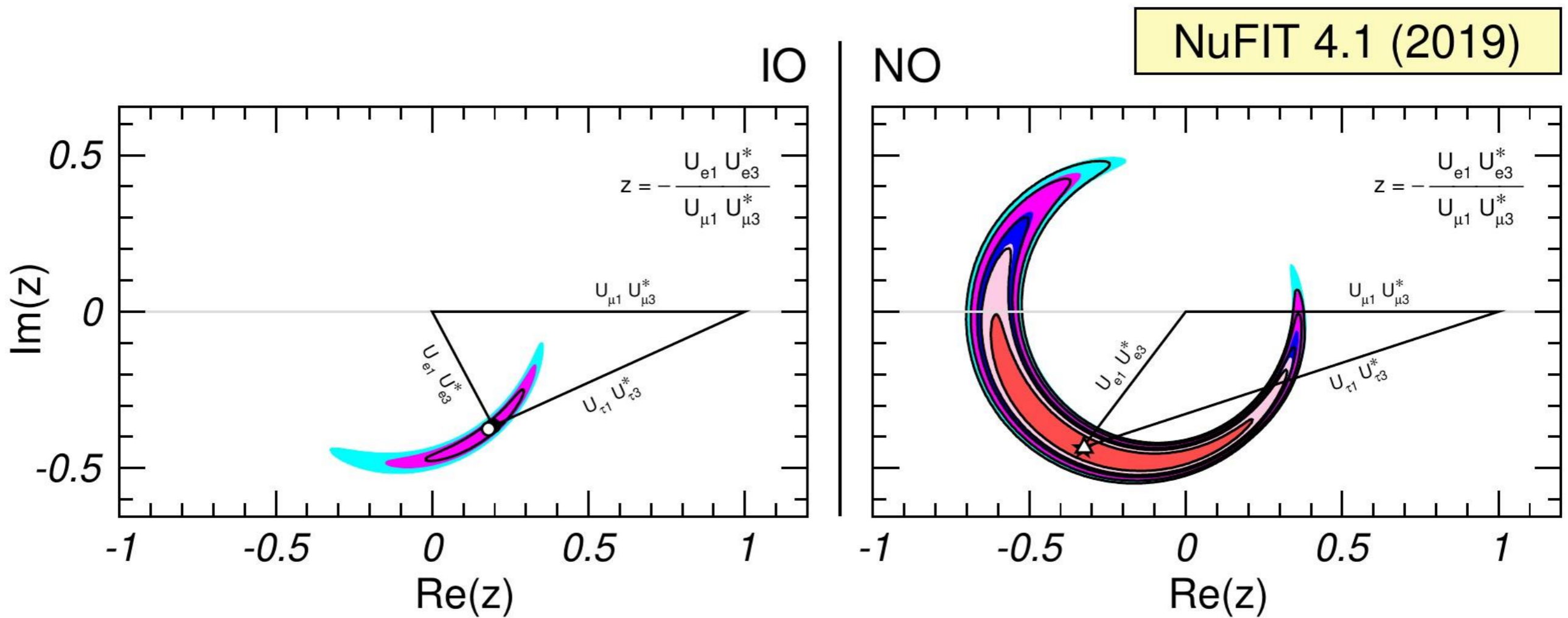
**But mixing with ordinary neutrinos affects neutrino oscillation data:**

- if the new states are light enough, neutrinos can oscillate into them

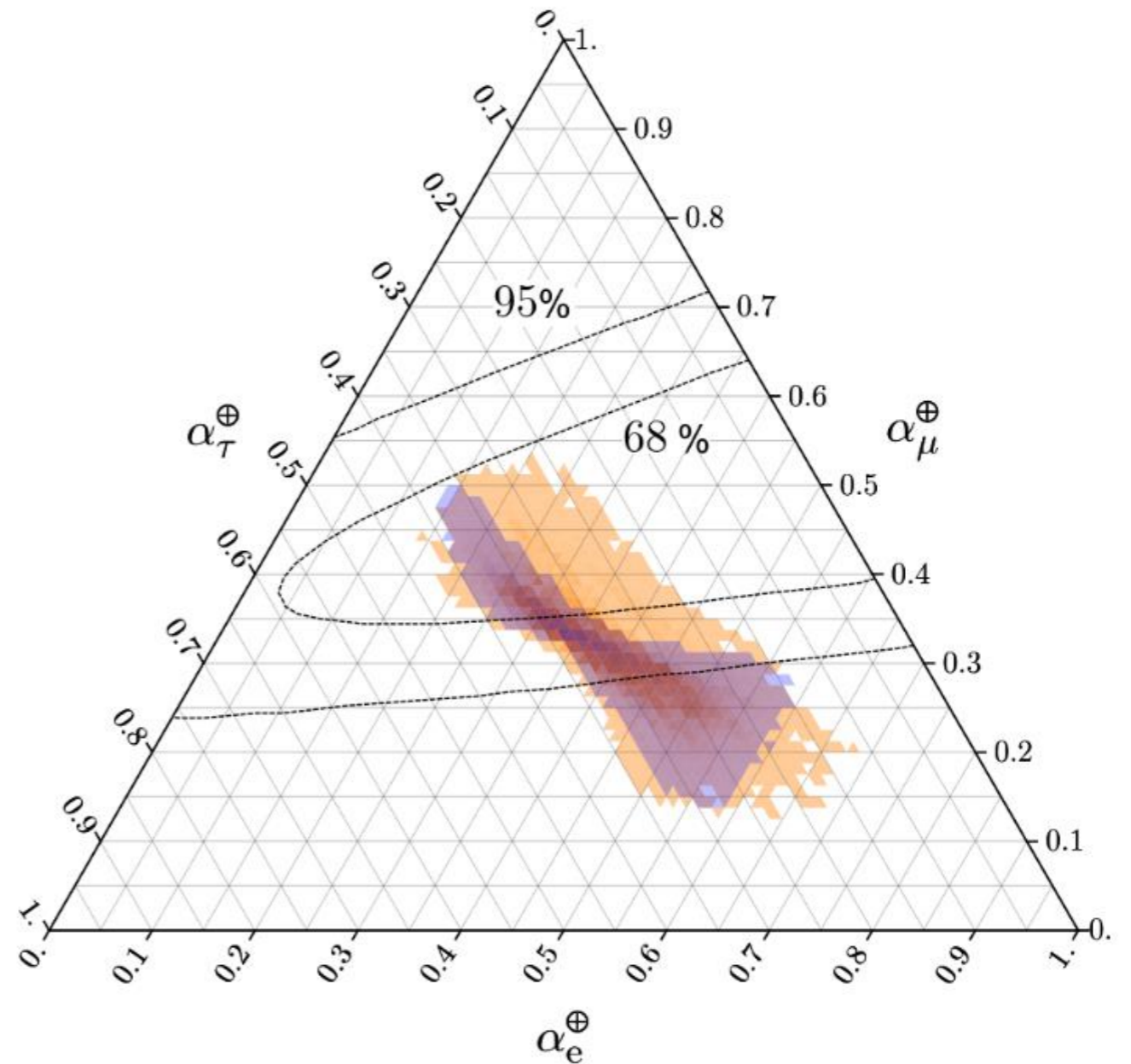
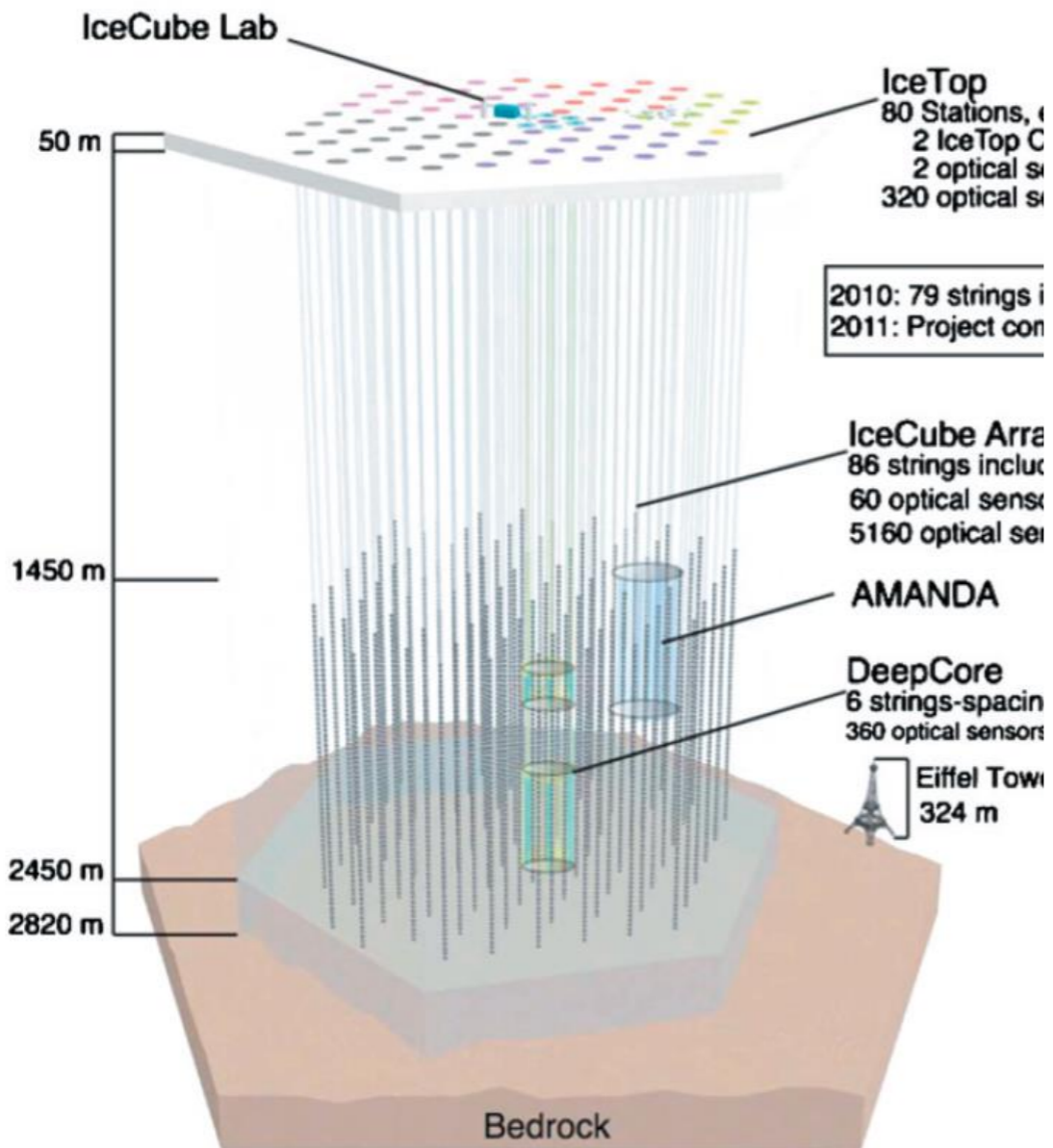
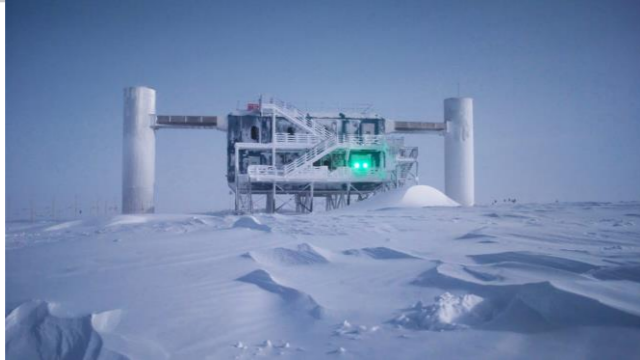
- if they are too heavy, they induce a non-unitarity in the observable 3x3 submatrix of the full neutrino mixing matrix

$$\nu_{L\alpha} = (V_\nu)_{\alpha i} \nu_i. \quad V_\nu = (1 + \eta) U_\nu.$$

# Unitarity Triangle

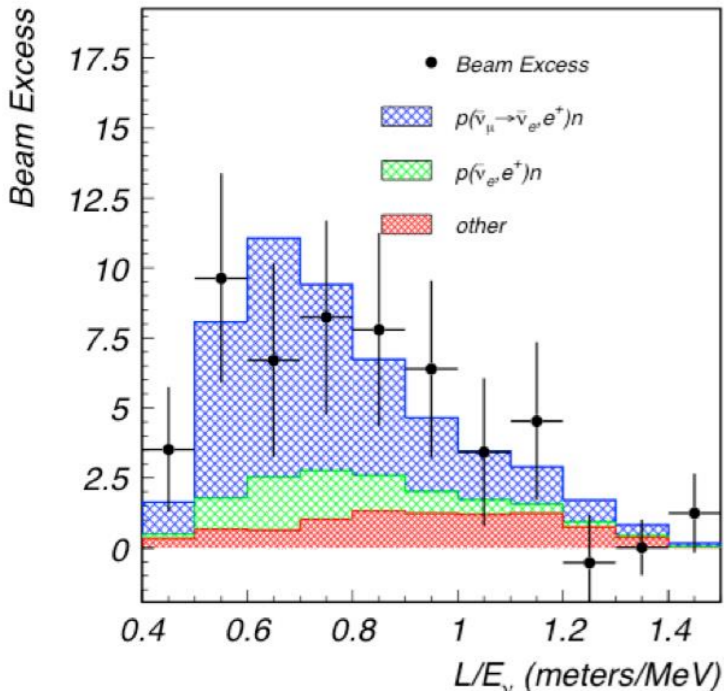


# Unitarity test with IceCube



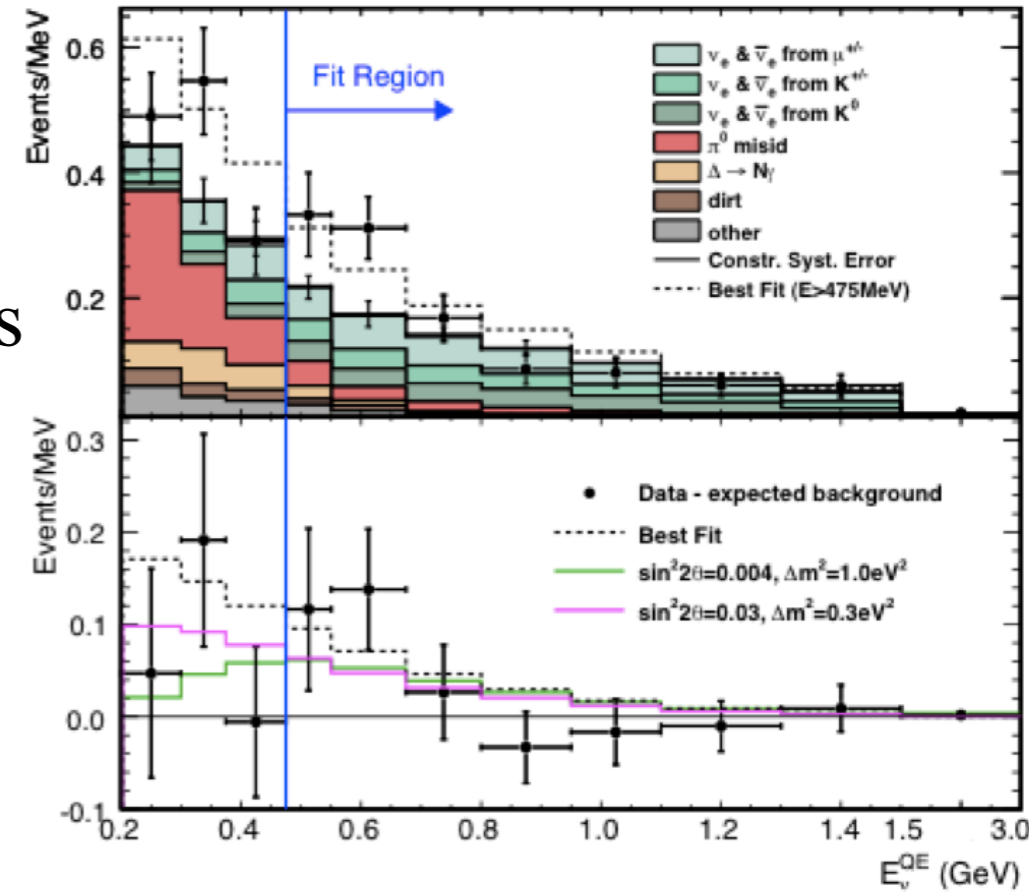


# Have we seen it?



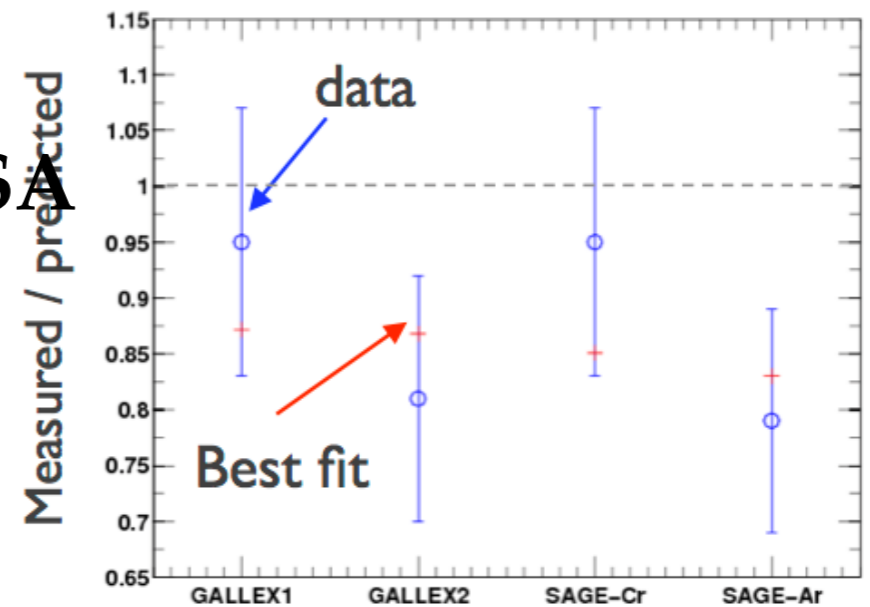
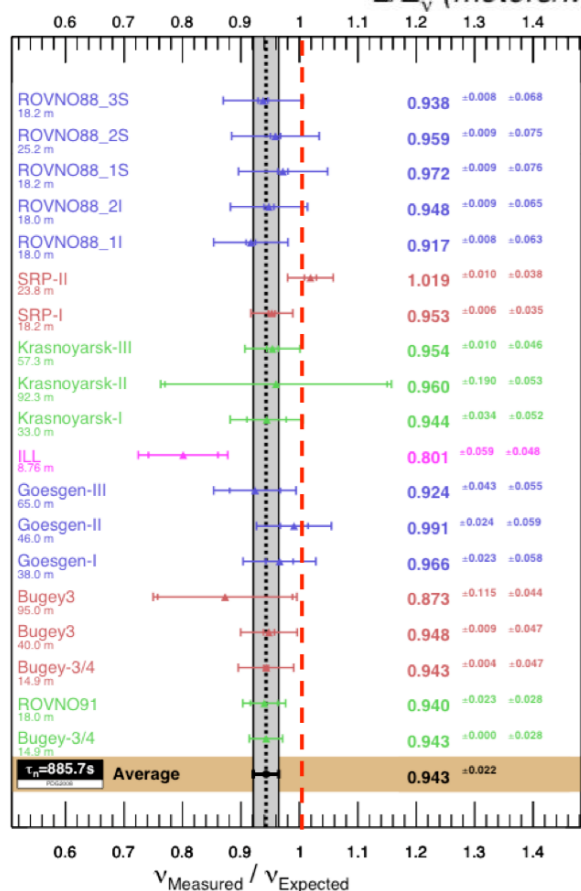
**LSND:**  
excess of  
electron  
antineutrinos

**MiniBoone:**  
extra electron  
(anti)neutrinos  
at low energy



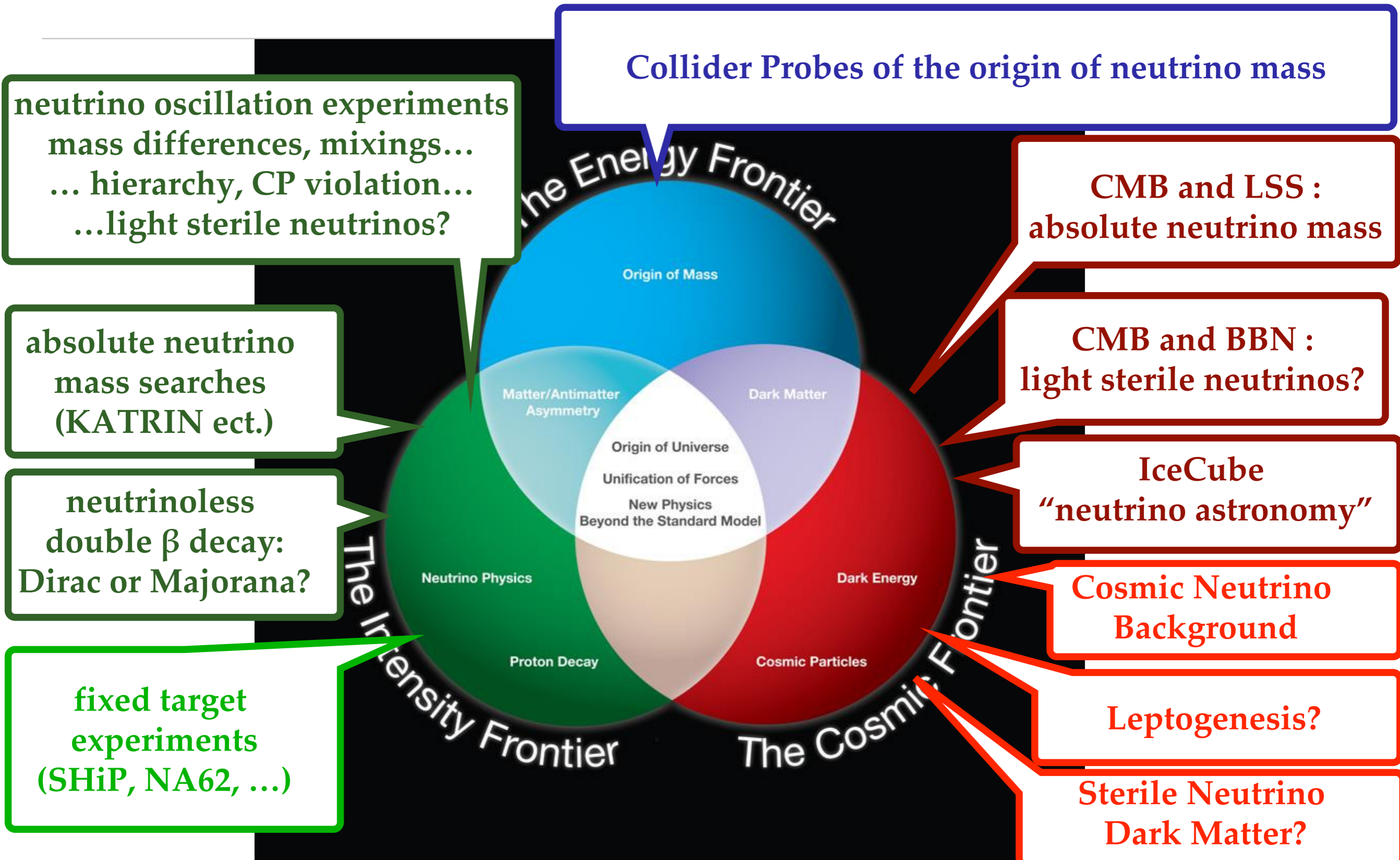
**Reactor  
anomaly:**  
too few  
neutrinos from  
reactors?

**GALLEX/SA  
GE:**  
missing  
electron  
neutrinos





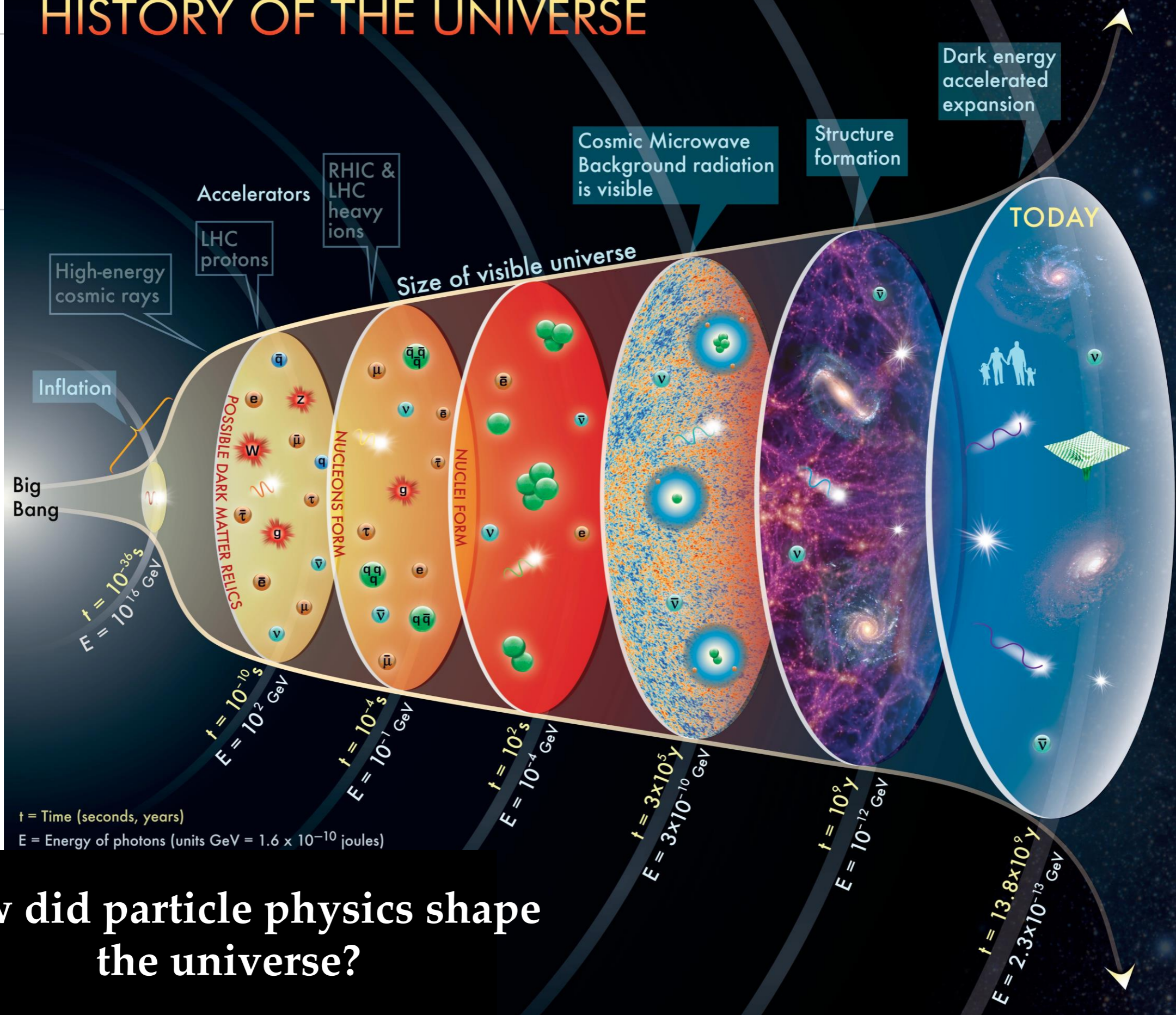
# A Multi-Frontier Problem



# Neutrinos in cosmology



# HISTORY OF THE UNIVERSE

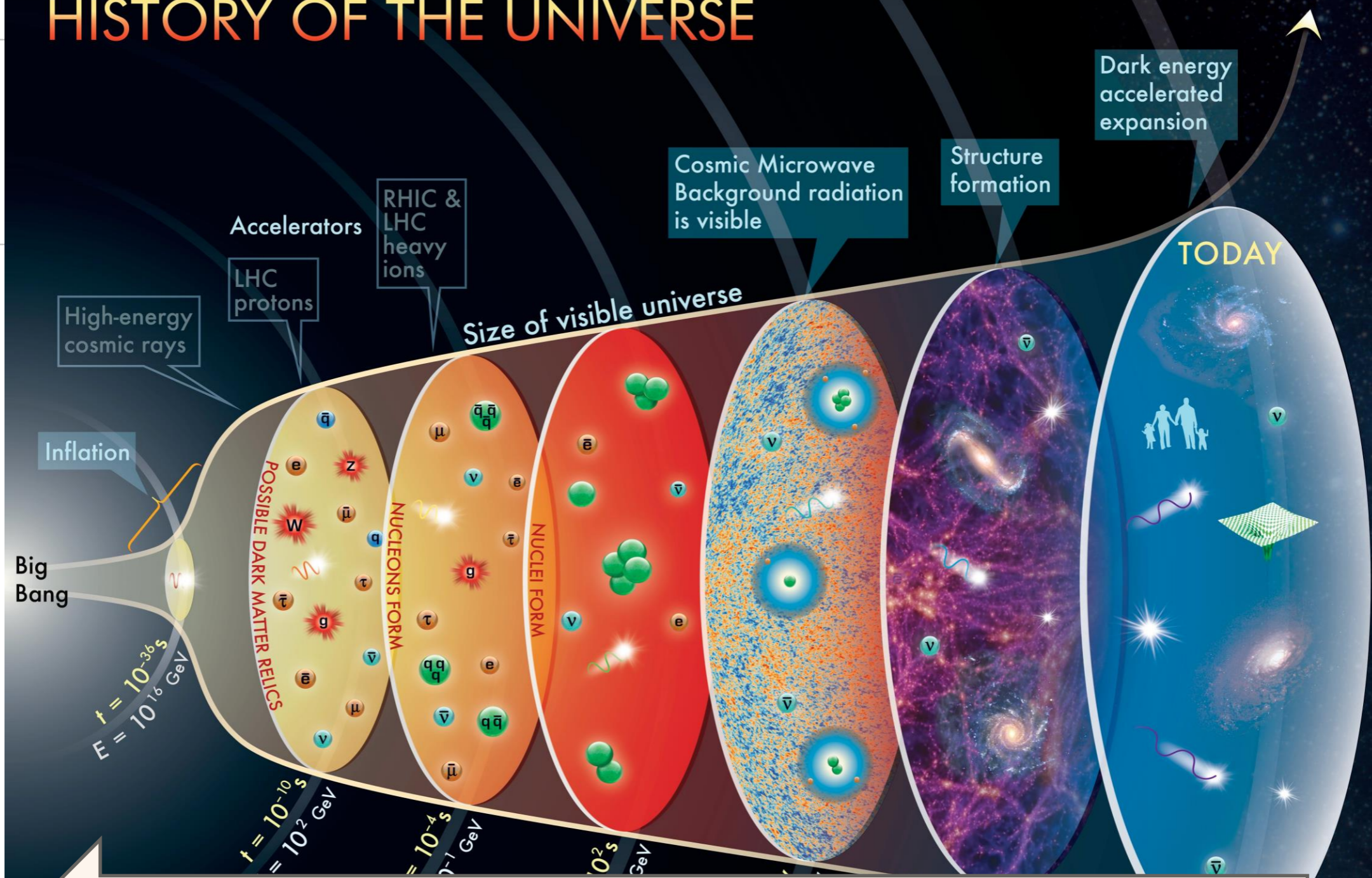


How did particle physics shape the universe?

The concept for the above figure originated in a 1986 paper by Michael Turner.



# HISTORY OF THE UNIVERSE



energy density, temperature

cosmic time

The concept for the above figure originated in a 1986 paper by Michael Turner.

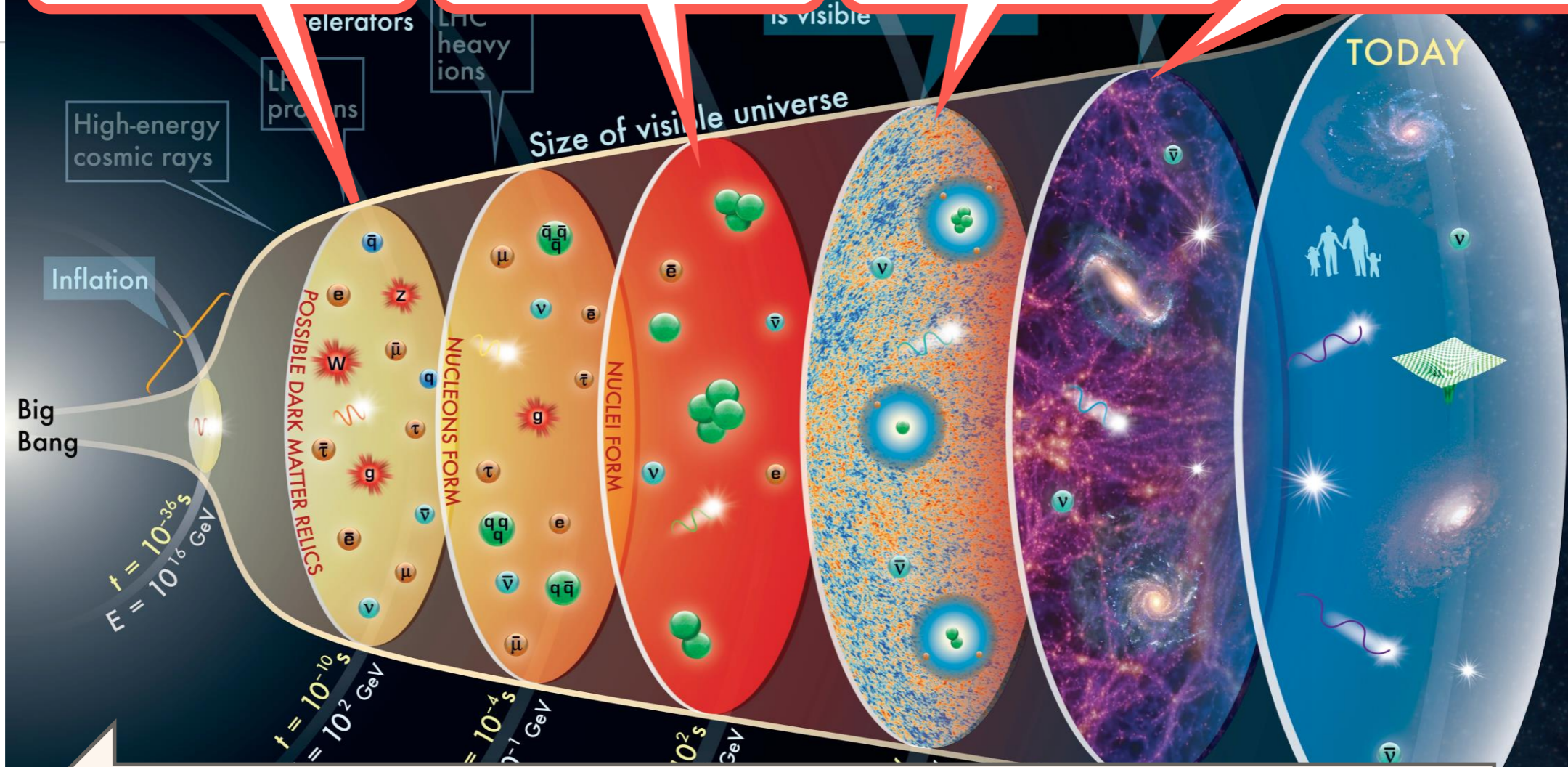


Large Hadron Collider

light element abundances

Cosmic Microwave Background

optical astronomy



energy density, temperature

cosmic time

The concept for the above figure originated in a 1986 paper by Michael Turner.

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# Crucial Events in Cosmic History

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**T ~ 1.4 MeV: neutrino decoupling**

fixes neutrino number density

**T ~ 0.5 MeV: electron-positron annihilation**

“heats” CMB relative to CvB, slightly distorts neutrino spectra

**T ~ 0.1 MeV: big bang nucleosynthesis**

sensitive to  $N_{eff}$  and baryon density

**T ~ 0.8 eV: matter-radiation equality**

structures start to grow

**T ~ 0.25 eV: photon decoupling**

CMB spectrum fixed

**T ~ 0.05 eV: neutrinos become non-relativistic**

free streaming ends, neutrinos behave like Dark Matter

---

# Background and Perturbation Effects

---

## Effect on background evolution

Neutrinos affect the background evolution through Friedmanns equation

$$H^2 = \frac{8\pi}{3} G \rho,$$

$$\rho_\gamma + \rho_{\text{neutrinos}} + [\text{new physics effects}] \equiv \rho_\gamma + N_{\text{eff}} \rho_\nu = \frac{\pi^2}{15} T_\gamma^4 \left[ 1 + N_{\text{eff}} \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \right]$$

This indirectly affects the cosmological perturbations that we observe

## Effect on perturbations

Perturbations are also affected directly because neutrinos stream and cluster differently from other components



---

# Background Effects

---

Neutrinos affect the evolution of the universe in different ways:

1) When they are relativistic they contribute to the radiation pressure

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} N_{\text{eff}} \left( \frac{4}{11} \right)^{4/3}$$

2) When they are non-relativistic they contribute to the matter density

$$\Omega_\nu = \frac{\rho_\nu^0}{\rho_{\text{crit}}^0} = \frac{\sum m_\nu}{93.14 h^2 \text{ eV}}$$

---

# Effects of $N_{\text{eff}}$ and $\Omega_\nu$

---

Physical quantities affected by the presence of neutrinos

- **moment of matter-radiation equality**
- **baryonic to Dark Matter density ratio**
- **free streaming**

This affects observables in different ways:

- **BBN** (expansion rate of the universe, neutrinos in nuclear reactions)
- **CMB** (expansion rate of the universe, Silk damping, anisotropic stress, gravitation between neutrino and photon perturbations, weak lensing of the CMB...)
- **Matter spectrum** (matter-radiation equality, expansion rate, enhancement of “collapsing” component increases growth of structure that are larger than neutrino free streaming length...)

---

# Effects of Neutrino Masses

---

- **CMB**

- 1) Matter density at late times affects expansion rate and ISW effect.
- 2) Change in equation of state when becoming non-relativistic
- 3) Decrease in matter power spectrum reduce weak lensing
- 4) Low momentum neutrinos that become relativistic earlier, affects photons through gravity

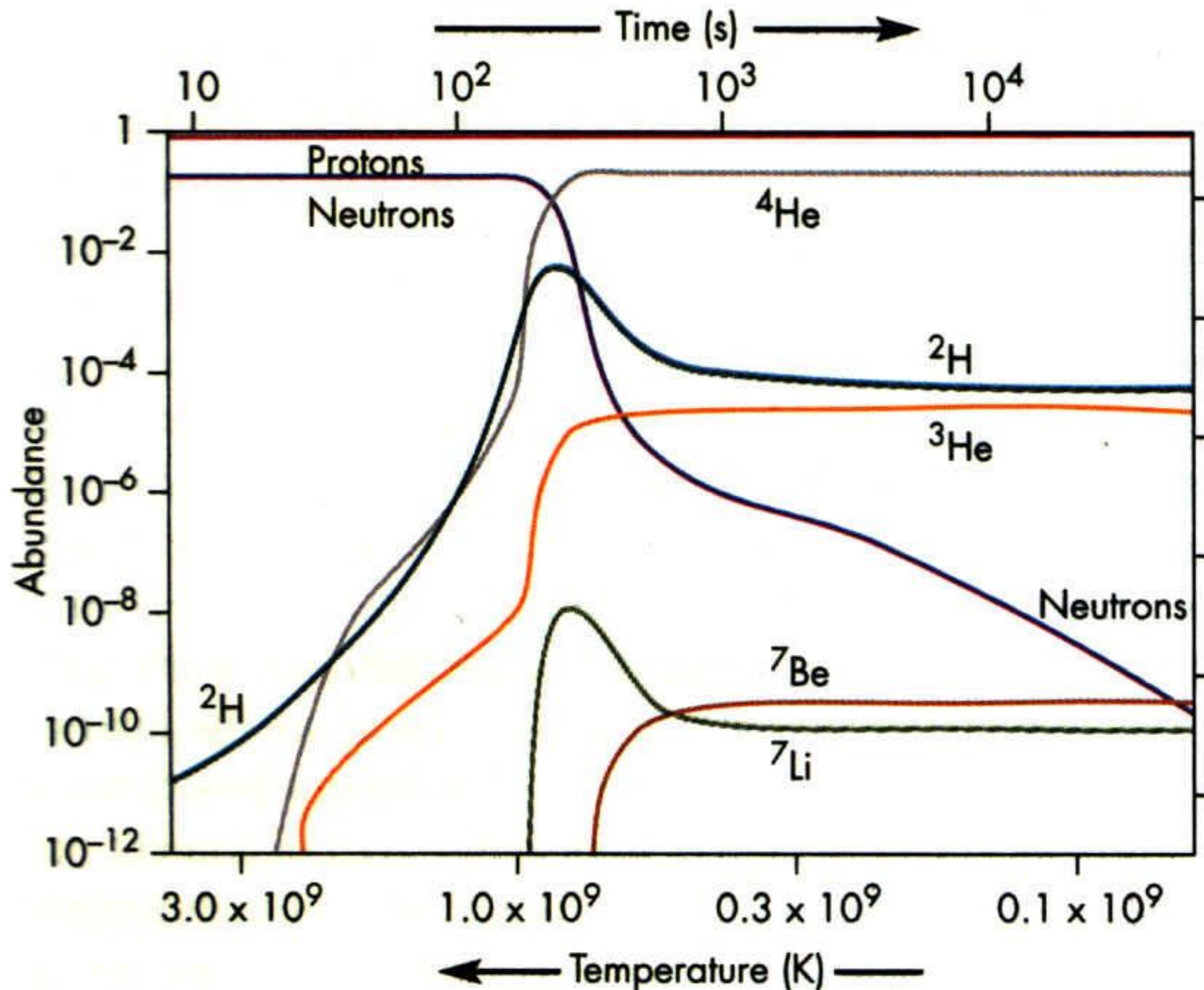
- **Matter spectrum**

- 1) neutrinos only contribute to growth of structures larger than their free streaming
- 2) they contribute to average matter density, but not to fluctuations on small scales

**Cosmology currently provides best upper bound on the sum of neutrino masses  $< 0.12$  eV**



# Big Bang Nucleosynthesis

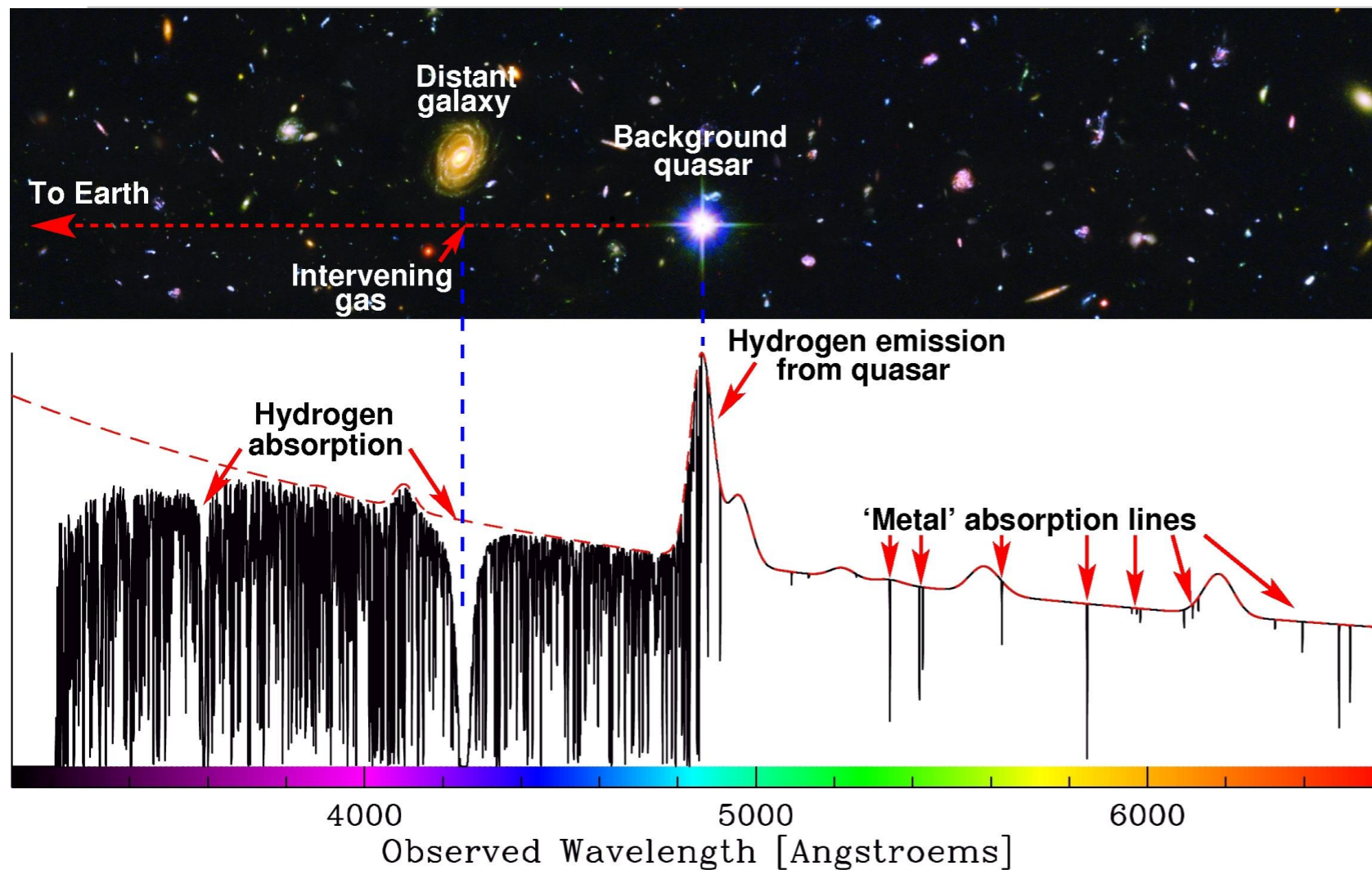


light elements are produced in a chain of nuclear reactions.

sensitive to  $N_{\text{eff}}$  through expansion rate and neutrino density through Boltzmann equation

only unknown parameter is the baryon-to-photon ratio.

# Big Bang Nucleosynthesis



- ❖ light element abundances in intergalactic medium can be measured in quasar spectra
- ❖ Deuterium is sensitive to baryon asymmetry and not produced in stars
- ❖ Helium then fixes  $N_{eff}$

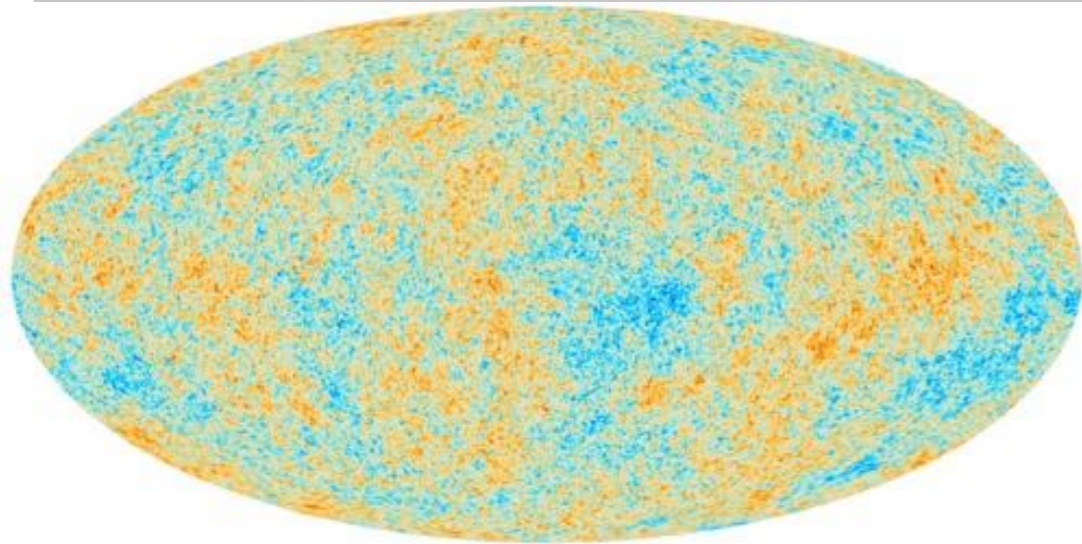
$$5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$$

PDG 2016

$$2.3 < N_{eff} < 3.4$$



# CMB

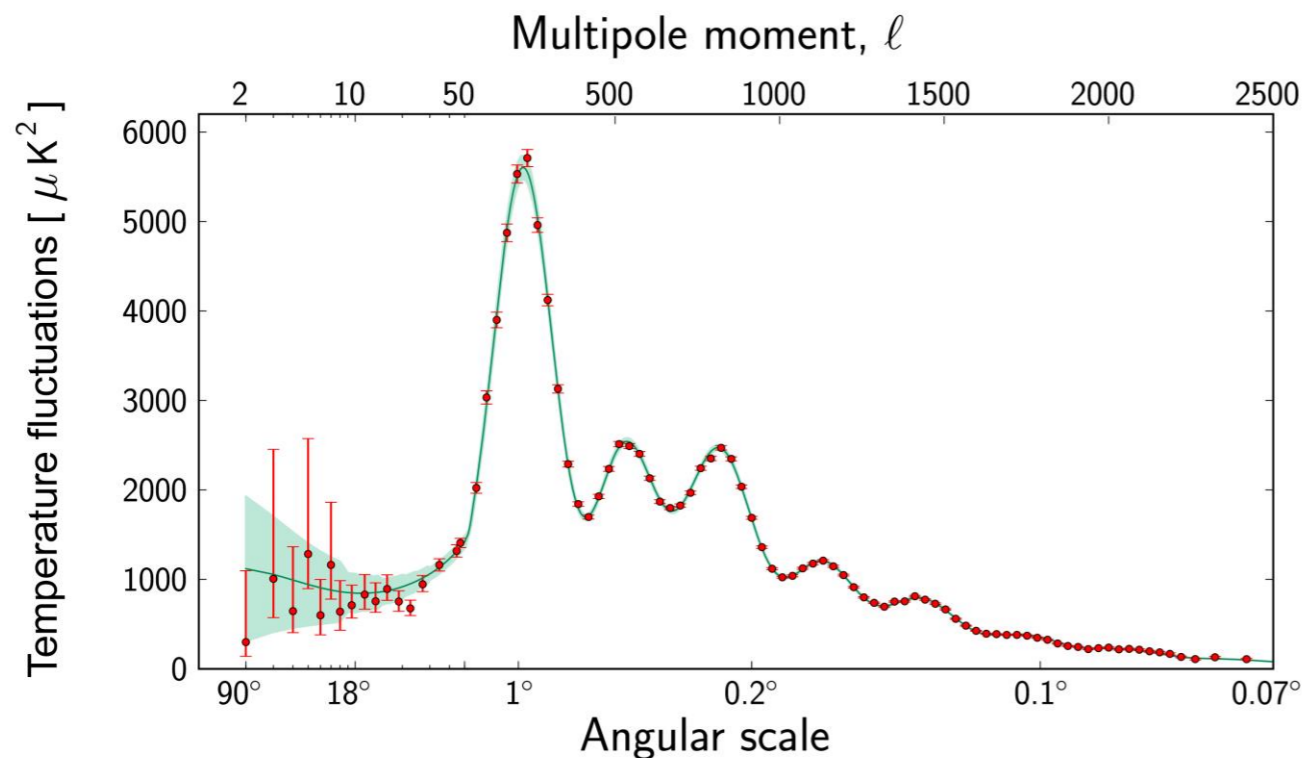


## Before photon decoupling

- equation of state
- matter to radiation equality
- free streaming / Silk damping
- ...

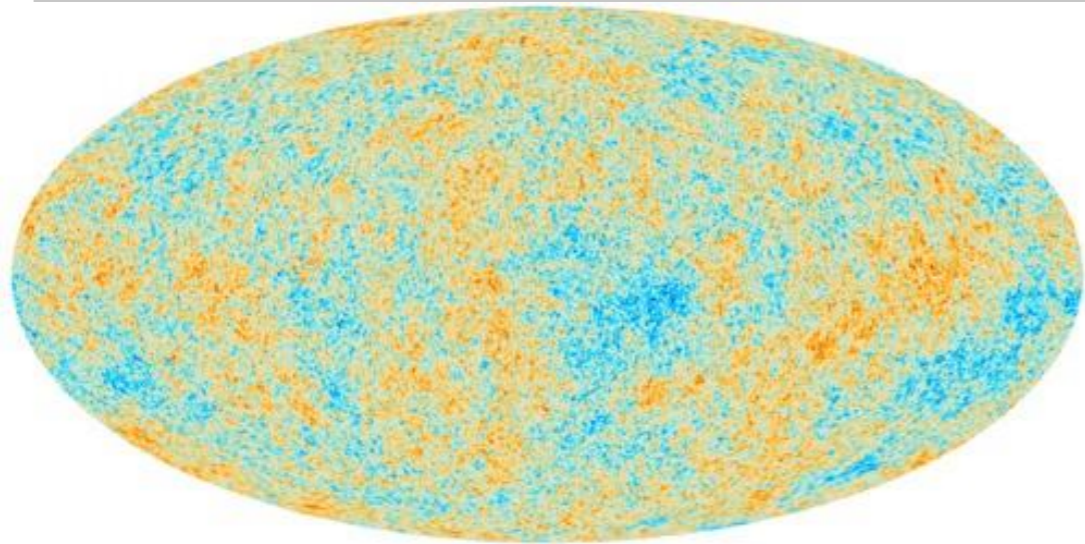
## After photon decoupling

- expansion history (angular size of perturbations)
- weak lensing (via matter power spectrum)
- gravitational potential that photons feel (also via effect on matter power spectrum)

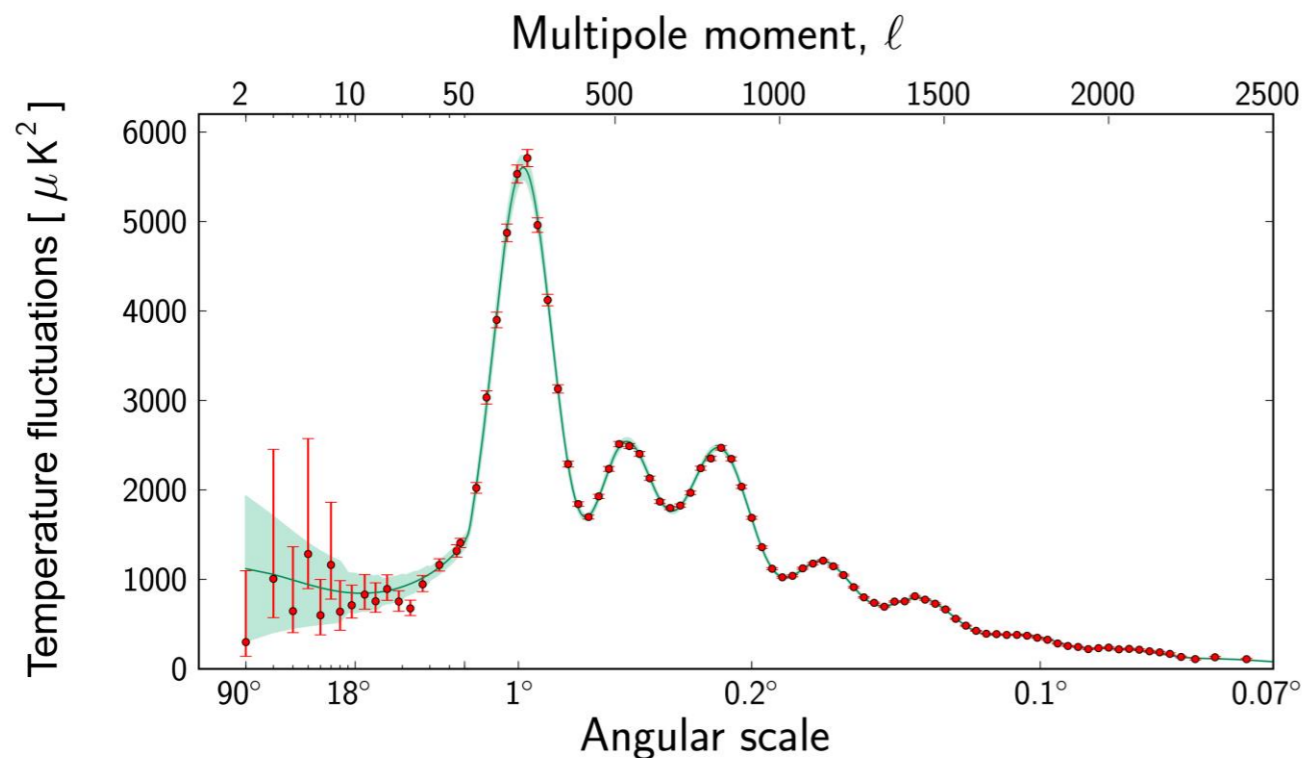




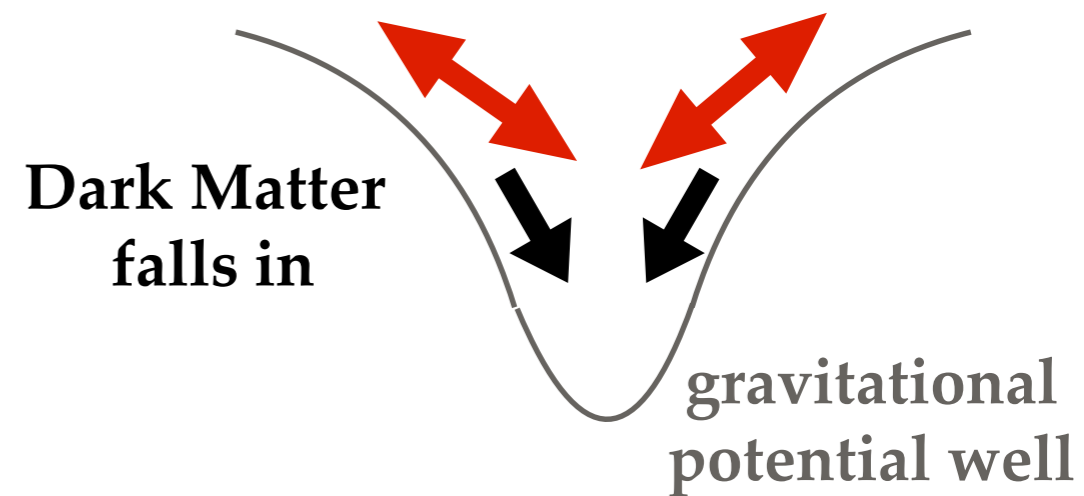
# Baryon Acoustic Oscillations



Radiation, baryons and Dark Matter affect the acoustic oscillations in the primordial plasma in different ways...



ordinary matter/radiation oscillates due to radiation pressure



neutrinos stream freely until they become non-relativistic

---

# What to fix?

---

**How neutrinos affect the CMB strongly depends on what quantities one keeps fixed while varying the neutrino parameters.**

## **Example: changing $N_{\text{eff}}$**

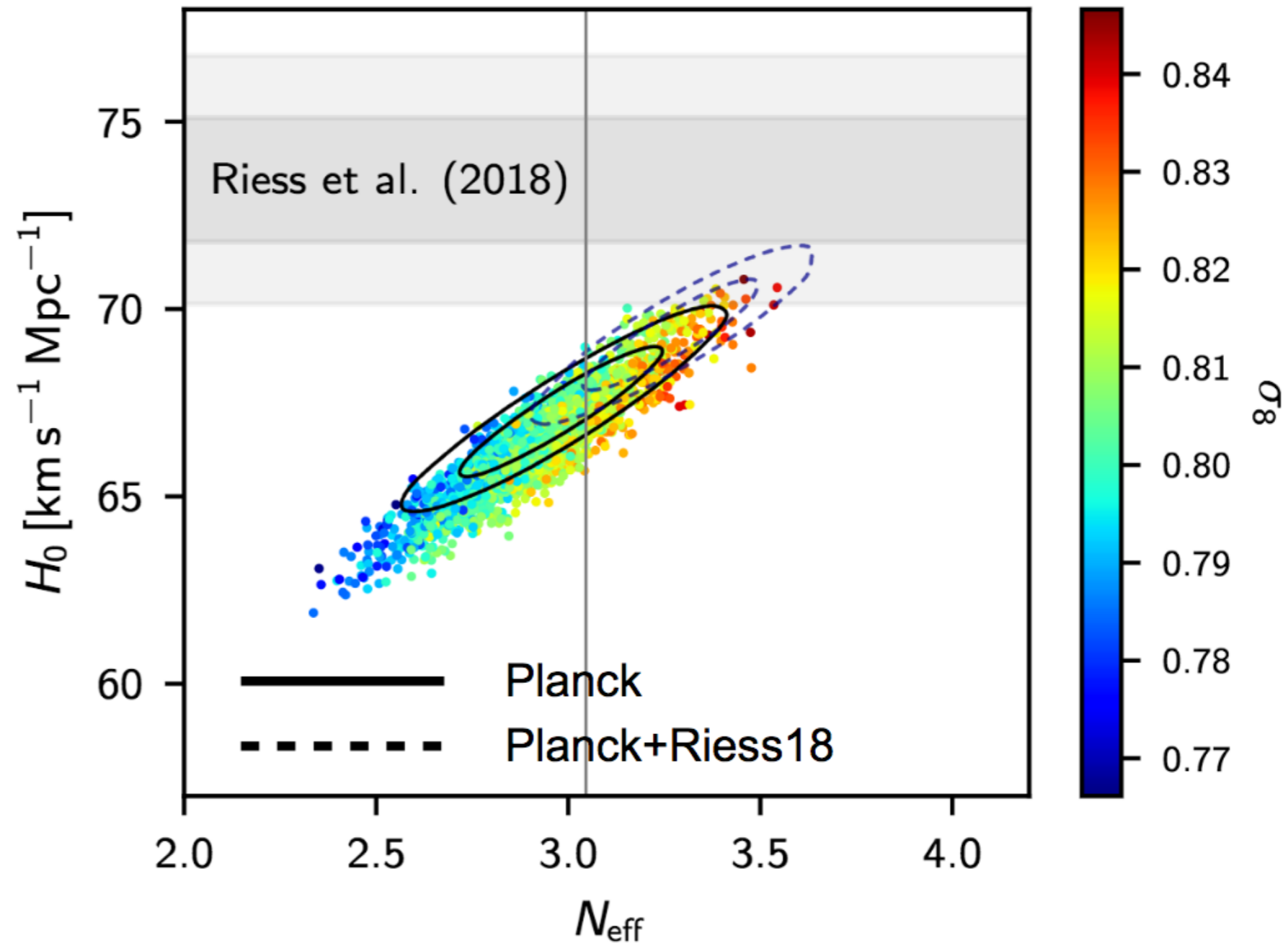
- *keep all other densities fixed*: Changing  $N_{\text{eff}}$  changes matter radiation equality, this affects the position and amplitude of CMB peaks
- *keep ratios between matter, radiation and Dark Energy fixed*: Changing  $N_{\text{eff}}$  changes the scale of “Silk damping”

## **Example: changing the neutrino masses**

- *keep matter density today fixed*: Changing  $m_\nu$  changes matter radiation equality
- *keep matter-radiation equality fixed*: Changing  $m_\nu$  modifies today's total matter density

# The $H_0$ Tension

- Local measurement of the Hubble rate disagrees with cosmological measurement
- This is degenerate with  $N_{\text{eff}}$

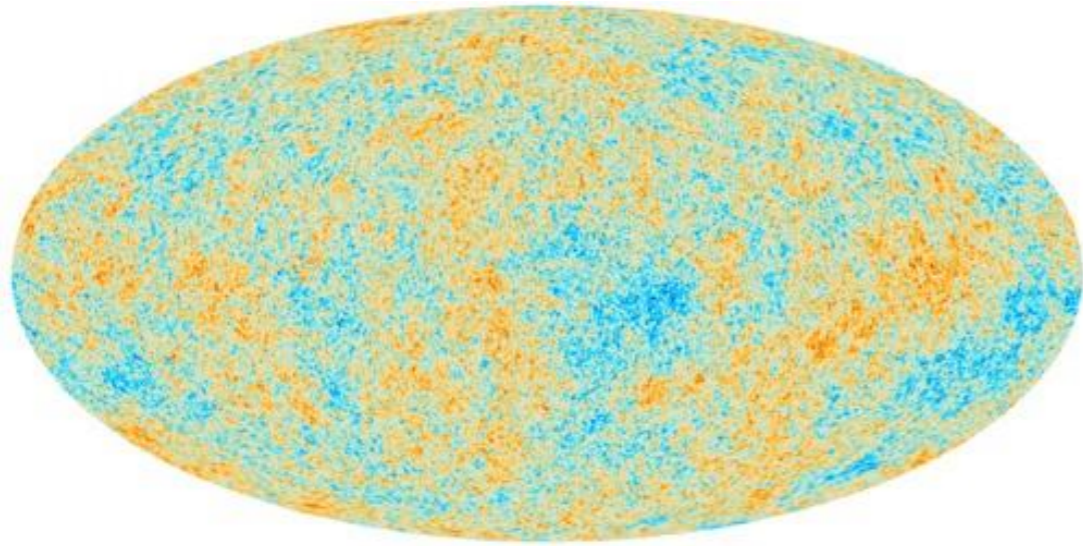




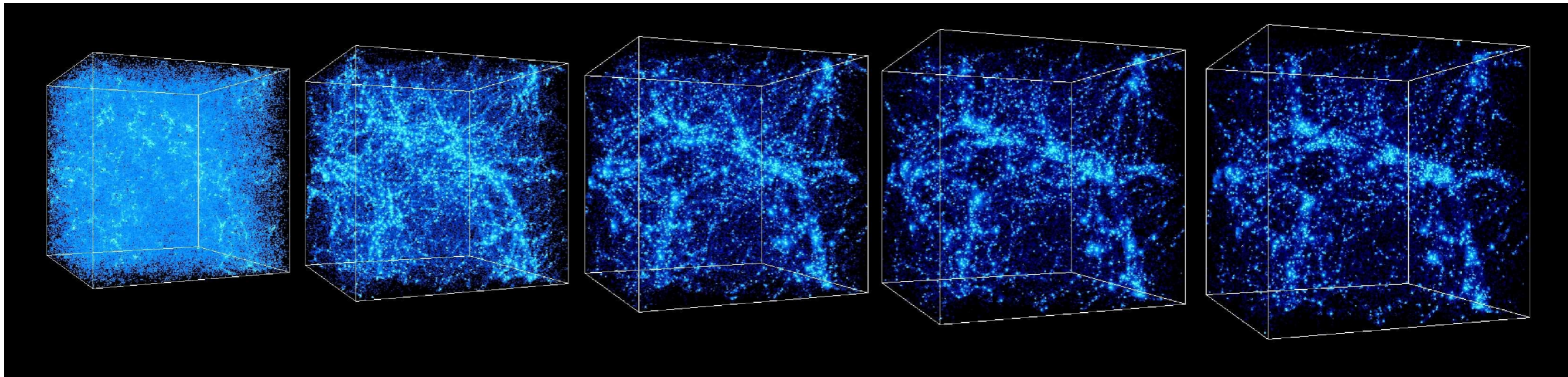
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# Structure Formation

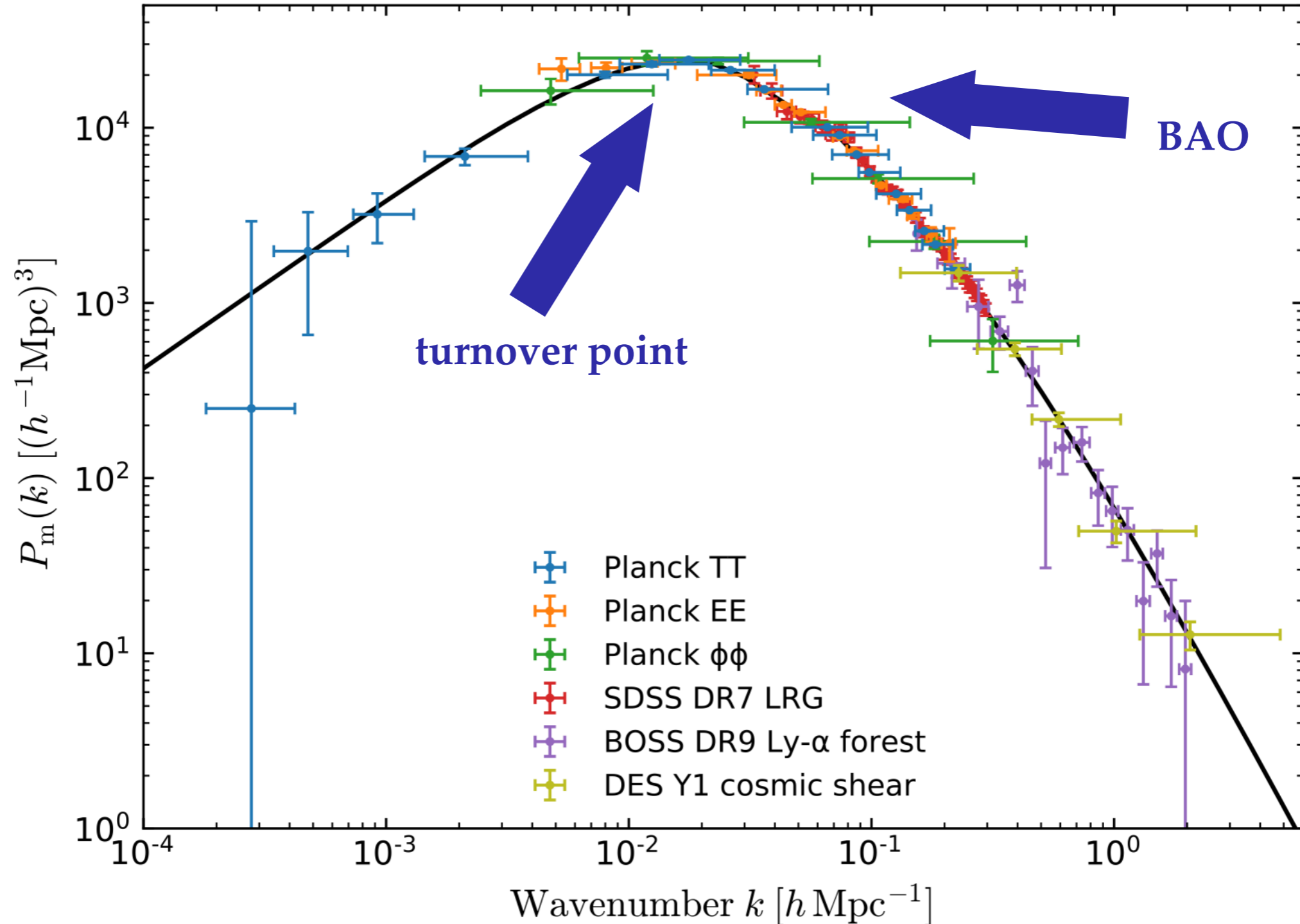
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The early universe was very homogeneous and isotropic...  
... structure formed by gravitational collapse of small inhomogeneities  $\sim 1/100.000$



# Matter Power Spectrum



---

# Effect of $N_{\text{eff}}$ on the Turnover Point

---

## The growth of perturbations

- cosmic perturbations are “frozen” when they are larger than the horizon
- in the radiation dominated era, sub-horizon modes grow slower than in the matter dominated era

## How $N_{\text{eff}}$ comes into play

- initial power spectrum is almost scale invariant
- modes that entered the horizon during radiation domination grew only slowly
- modes that entered after matter-radiation equality grew quicker
- $N_{\text{eff}}$  modifies the moment of matter-radiation equality and shifts the turnover point

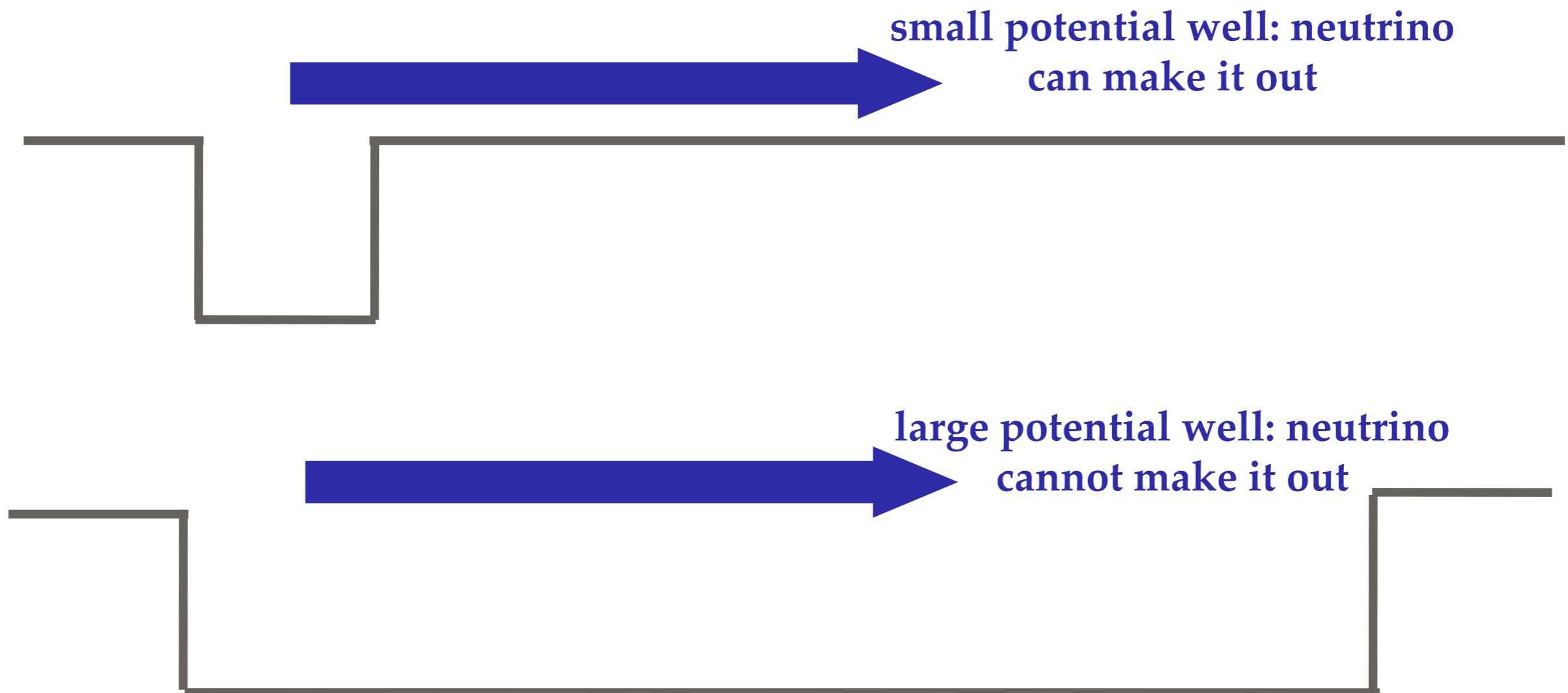


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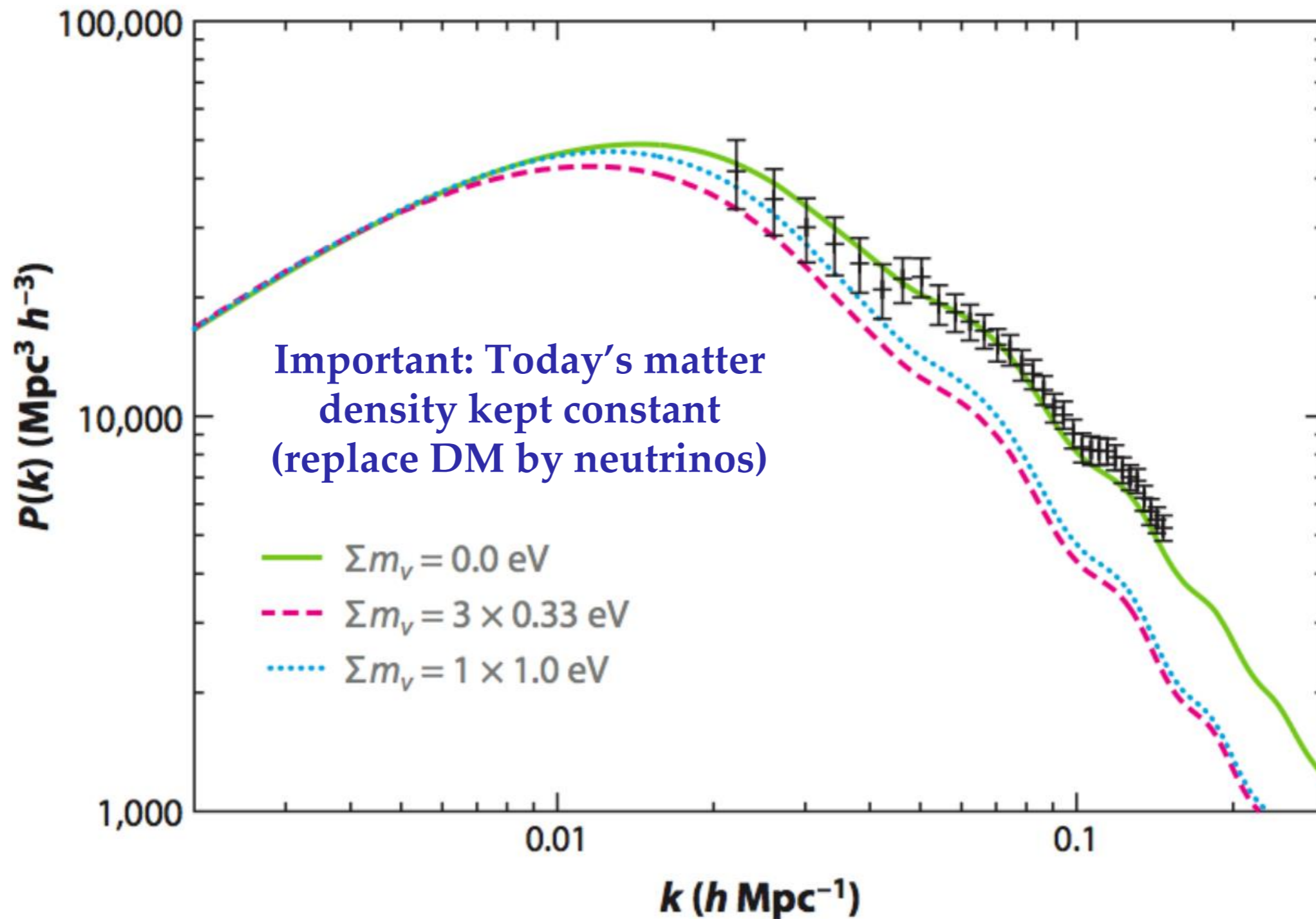
# Effects of Massive Neutrinos on Clustering

---

**Neutrinos only contribute to the growth of structures that are larger than their free streaming length**



# Effects of Massive Neutrinos on Clustering



---

# Indirect Effects on the CMB

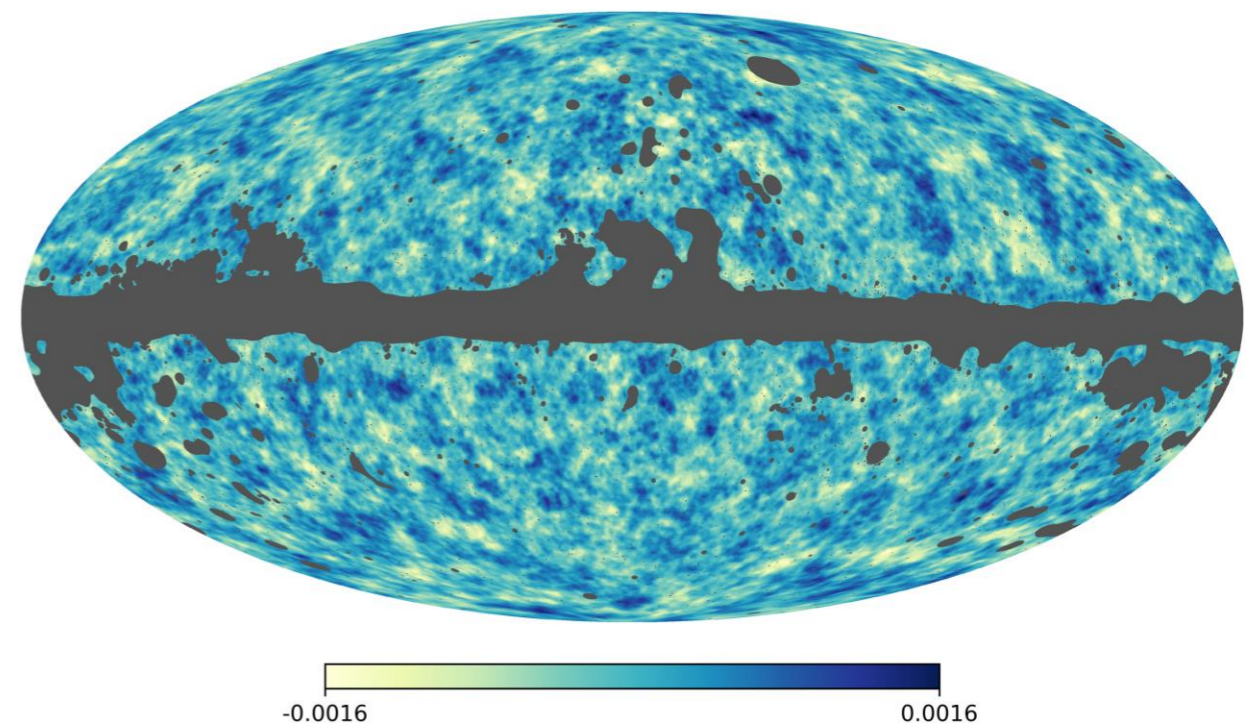
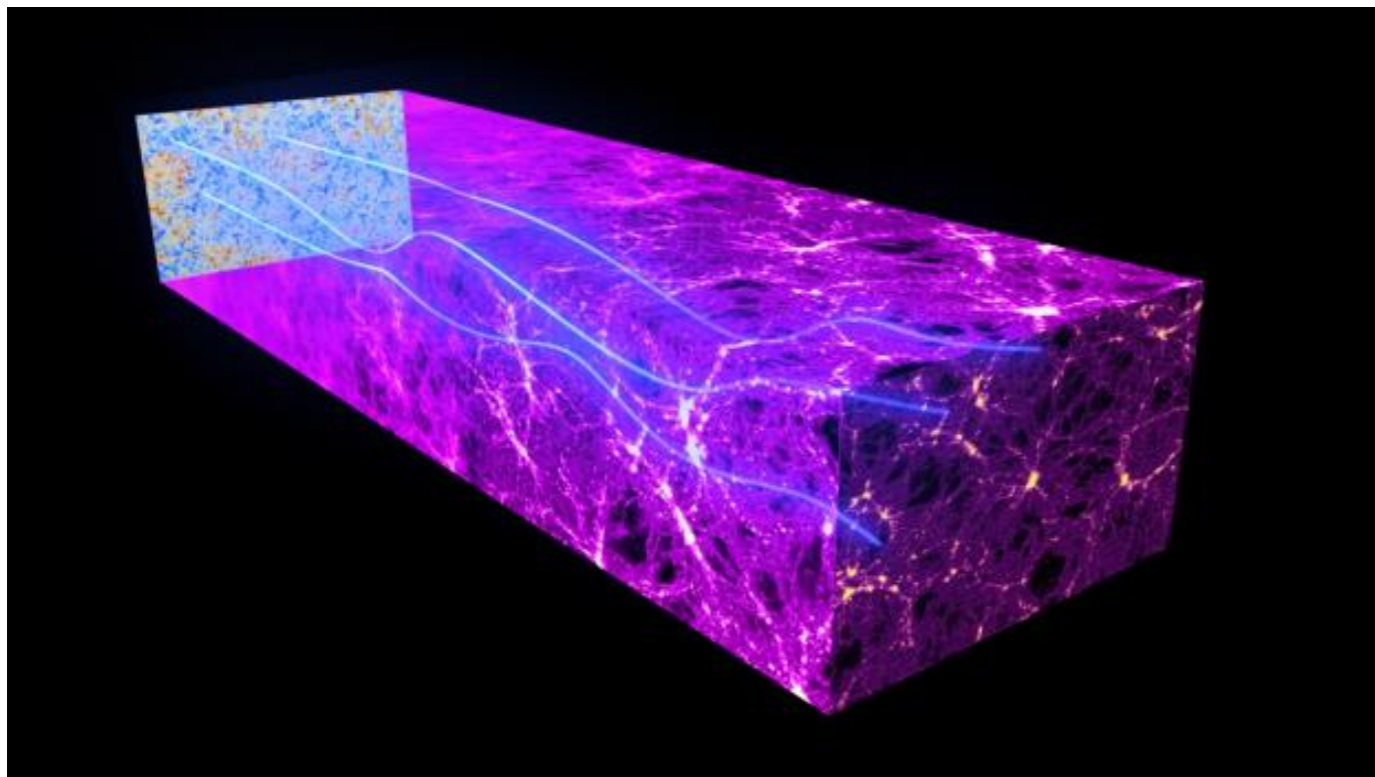
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## Integrated Sachs-Wolfe Effect (ISW)

CMB photons fall in and climb out of gravitational wells on their way to us... but the wells change why they do so

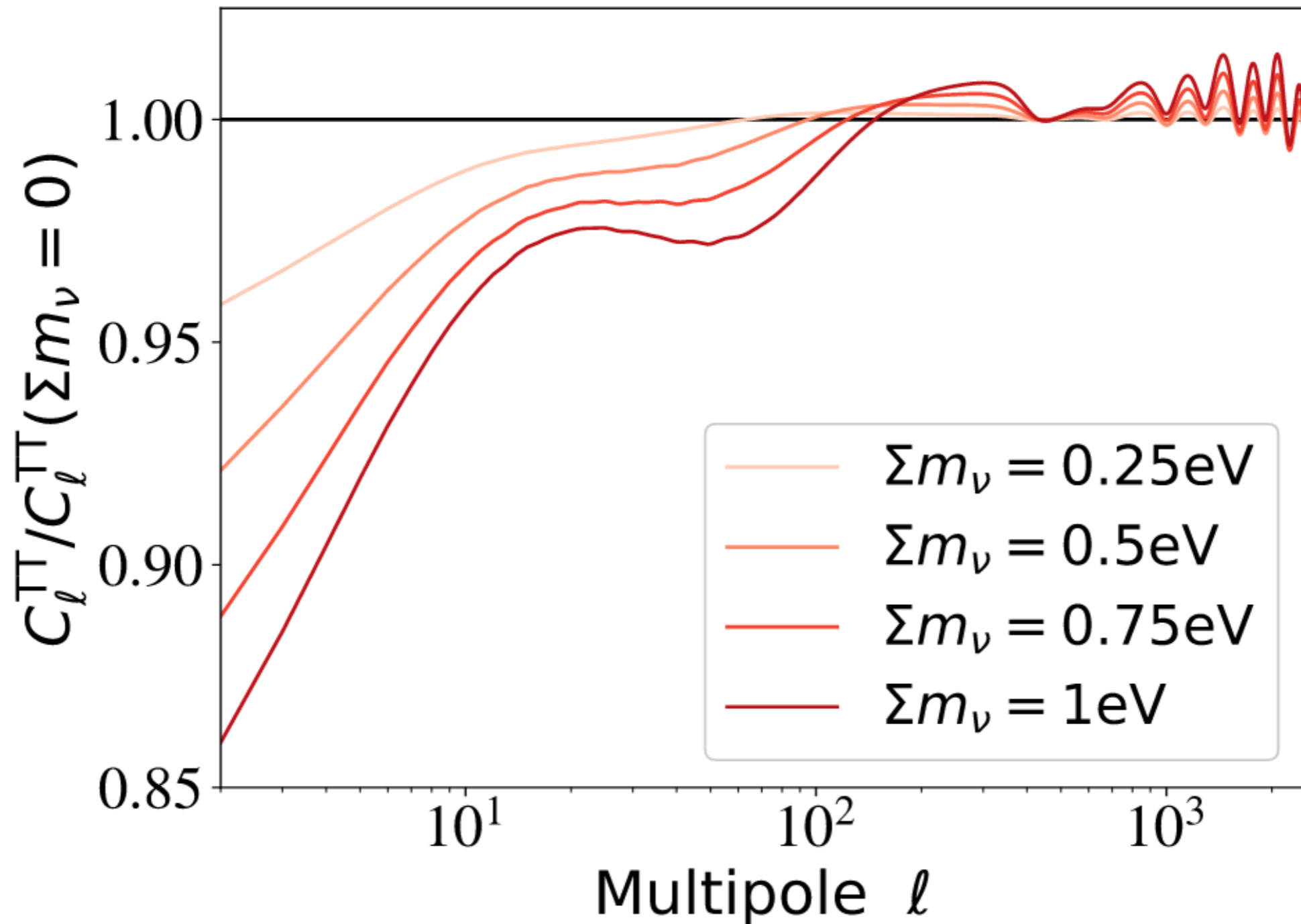
## Weak Lensing

CMB photons get lensed on their way so us

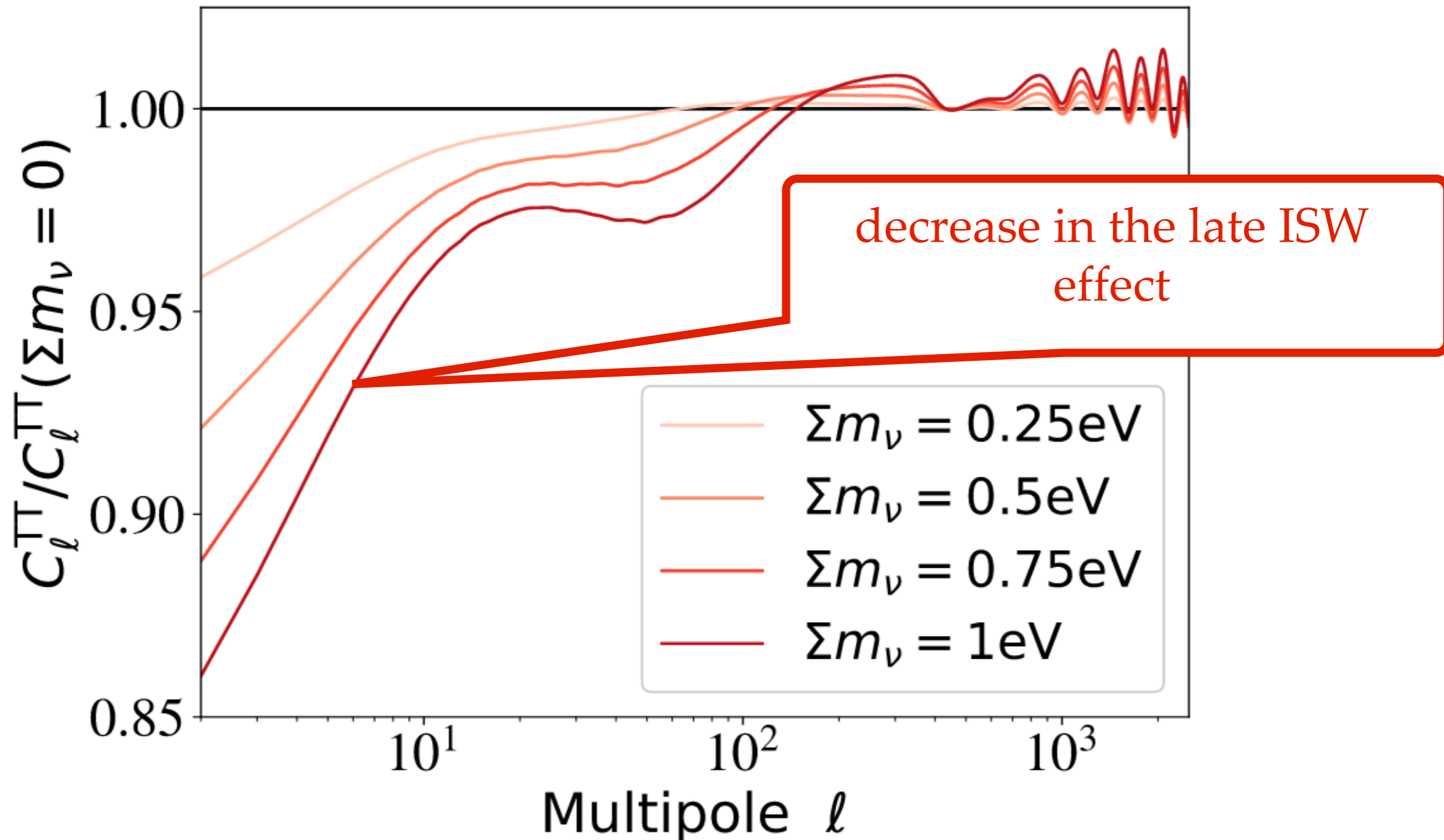




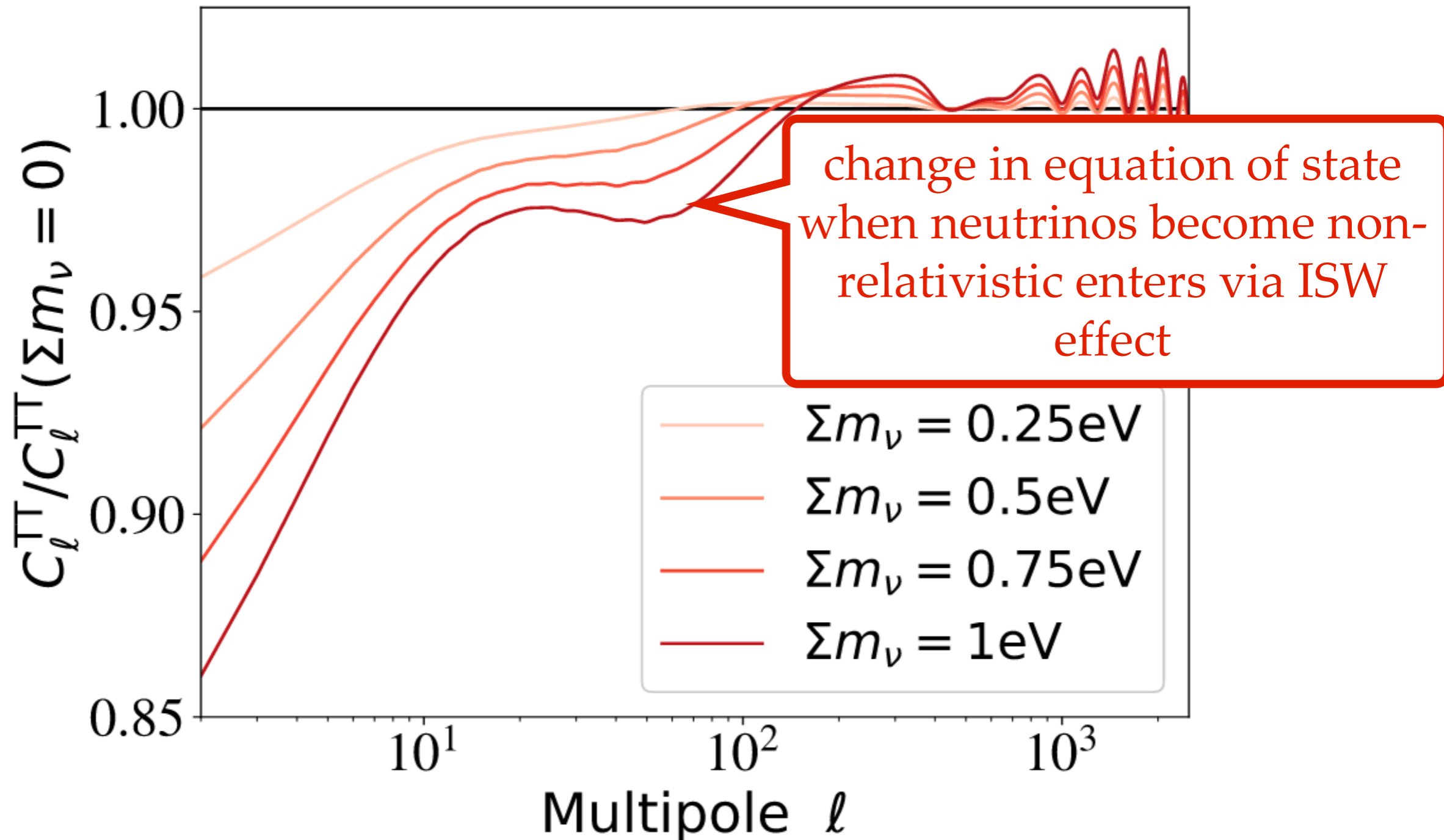
# Changing $m_\nu$ with fixed $d_A$



# Changing $m_\nu$ with fixed $d_A$

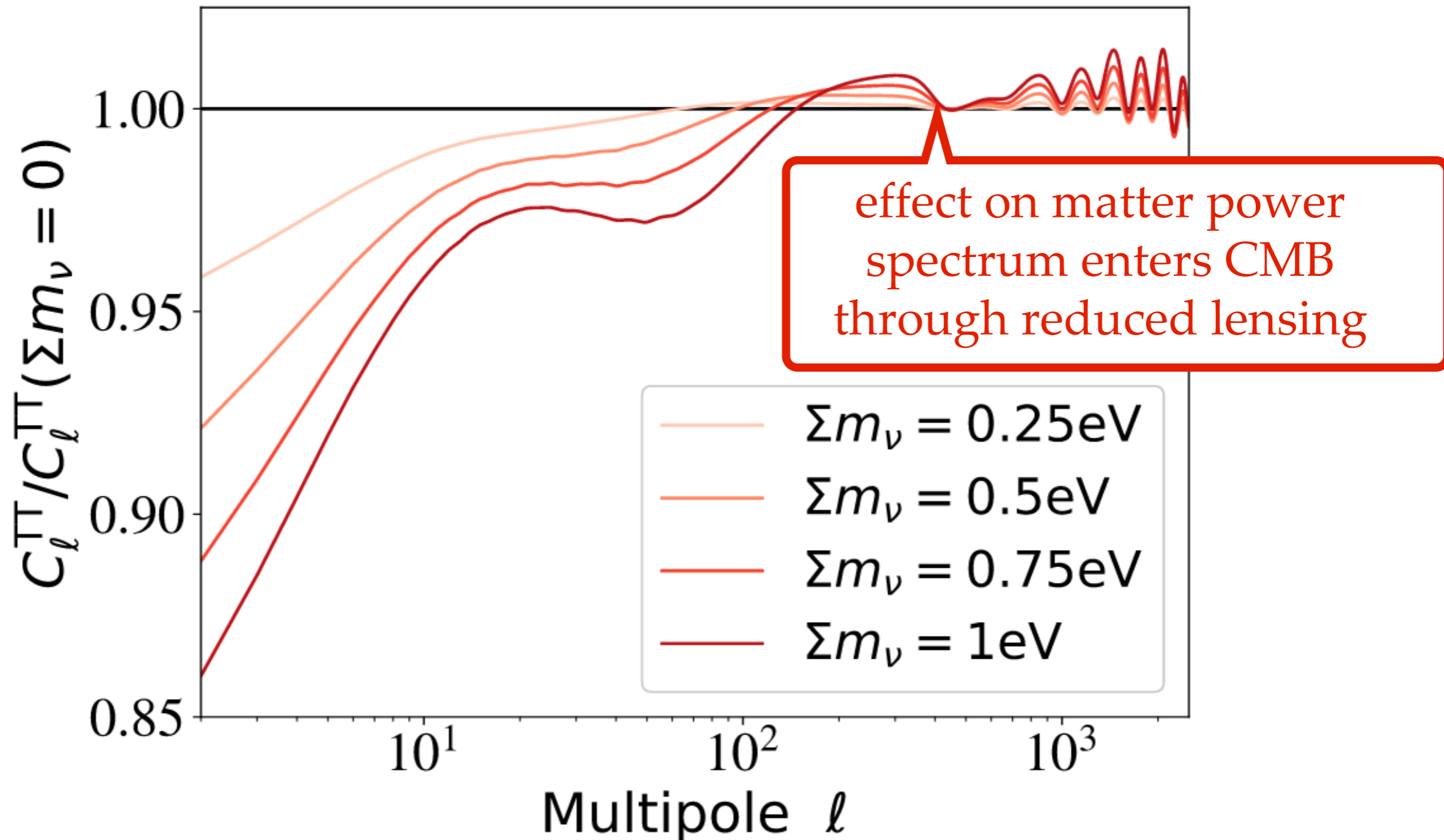


# Changing $m_\nu$ with fixed $d_A$

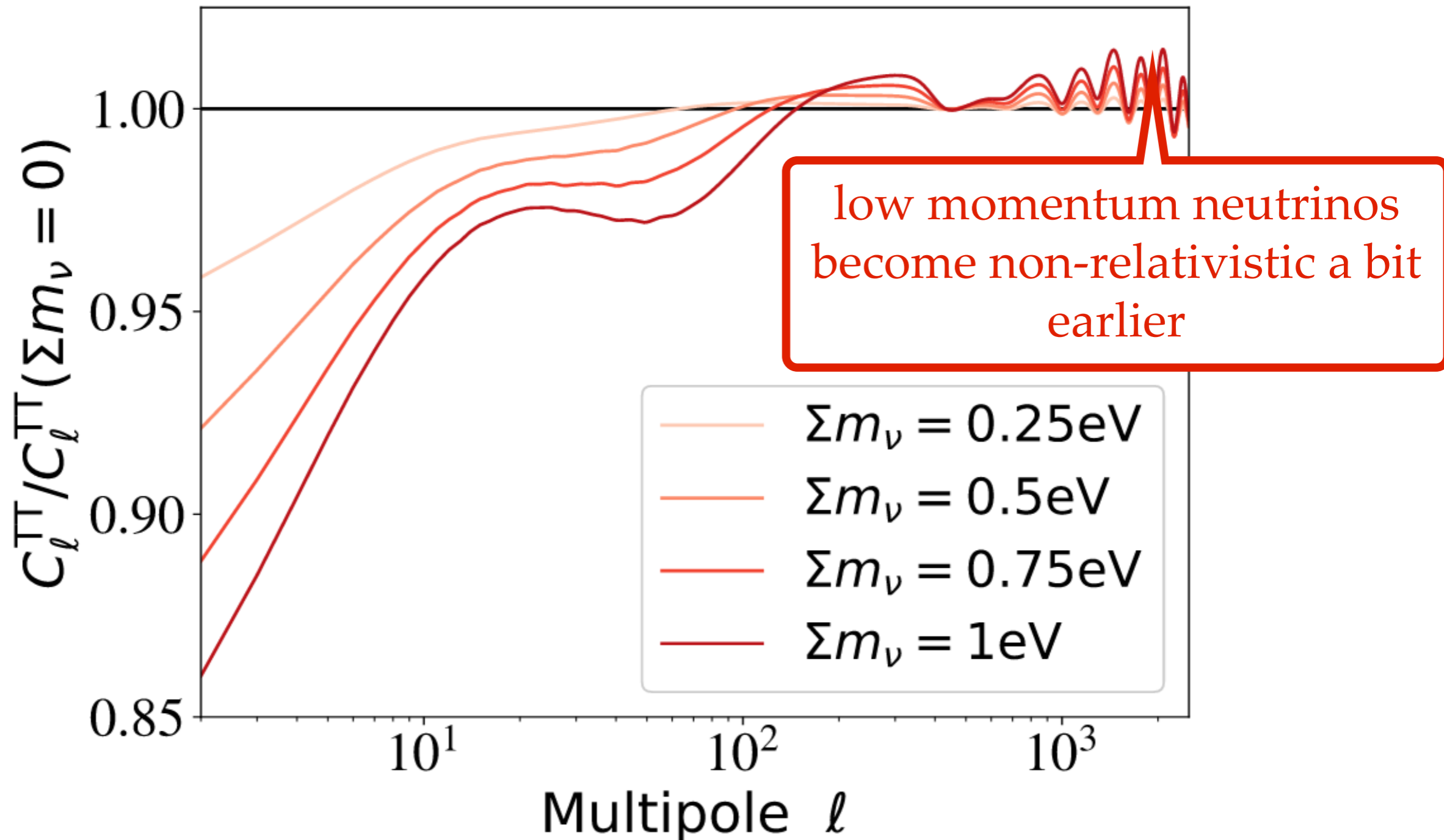




# Changing $m_\nu$ with fixed $d_A$



# Changing $m_\nu$ with fixed $d_A$

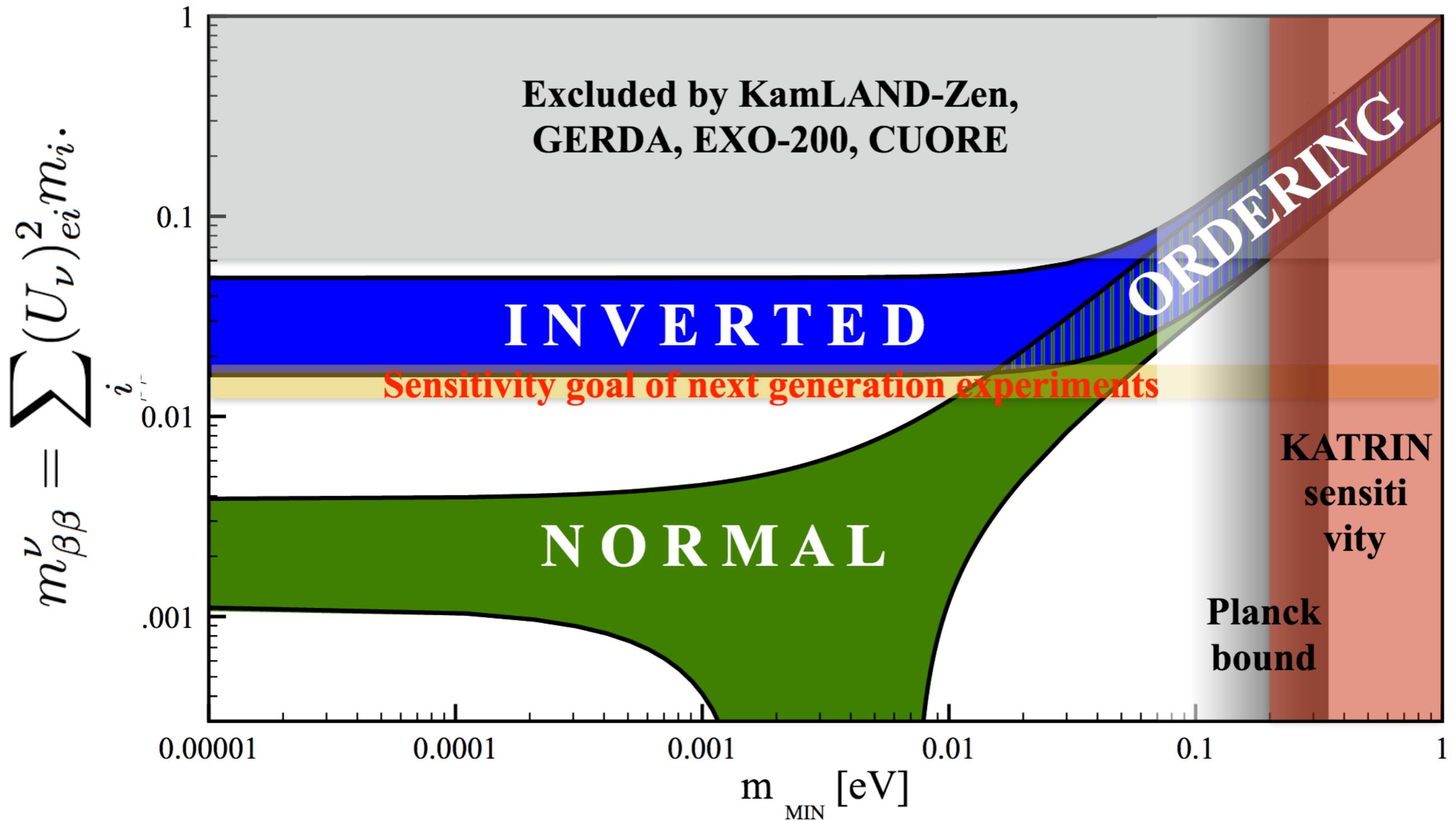


	Model	68%CL
<b>CMB alone</b>		
P115[TT+lowP]	$\Lambda\text{CDM}+N_{\text{eff}}$	$3.13 \pm 0.32$
P115[TT+lowP]	$\Lambda\text{CDM}+N_{\text{eff}}+\sum m_\nu$	$3.08 \pm 0.31$
<b>CMB + probes of background evolution</b>		
P115[TT+lowP] + BAO	$\Lambda\text{CDM}+N_{\text{eff}}$	$3.15 \pm 0.23$
P115[TT+lowP] + BAO	$\Lambda\text{CDM}+N_{\text{eff}}+\sum m_\nu$	$3.18^{+0.24}_{-0.27}$
<b>CMB + probes of background evolution + LSS</b>		
P115[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+N_{\text{eff}}$	$3.08^{+0.22}_{-0.24}$
” + BAO + JLA + HST	$\Lambda\text{CDM}+N_{\text{eff}}$	$3.41 \pm 0.22$
” + BAO	$\Lambda\text{CDM}+N_{\text{eff}}+\sum m_\nu$	$3.2 \pm 0.5$
P115[TT,TE,EE+lowP+lensing]	$\Lambda\text{CDM}+N_{\text{eff}}+5\text{-params.}$	$2.93^{+0.51}_{-0.48}$

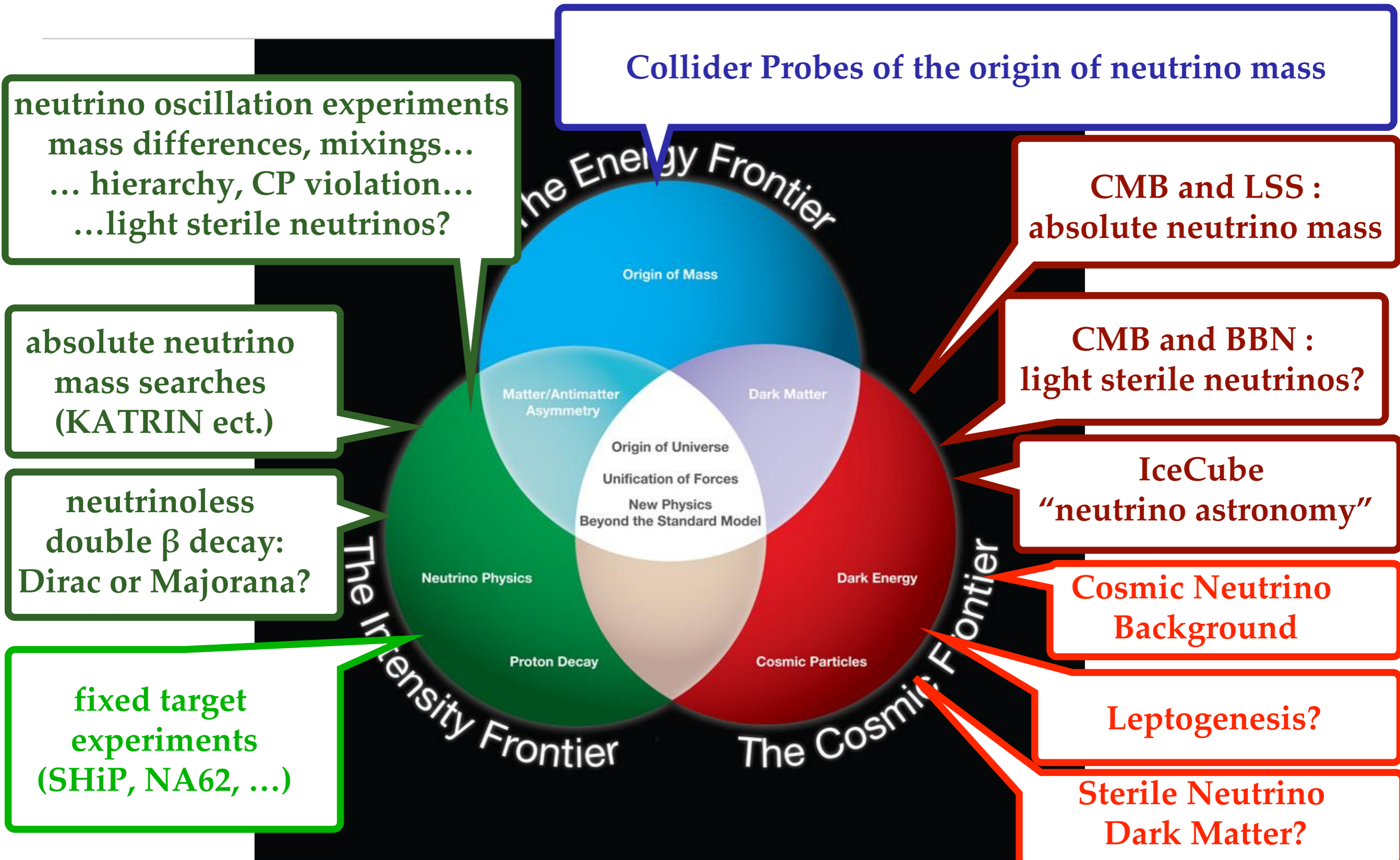
	Model	95% CL (eV)
<b>CMB alone</b>		
P115[TT+lowP]	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.72$
P115[TT+lowP]	$\Lambda\text{CDM}+\sum m_\nu+N_{\text{eff}}$	$< 0.73$
P116[TT+SimLow]	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.59$
<b>CMB + probes of background evolution</b>		
P115[TT+lowP] + BAO	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.21$
P115[TT+lowP] + JLA	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.33$
P115[TT+lowP] + BAO	$\Lambda\text{CDM}+\sum m_\nu+N_{\text{eff}}$	$< 0.27$
<b>CMB + probes of background evolution + LSS</b>		
P115[TT+lowP+lensing]	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.68$
P115[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.25$
P115[TT+lowP] + $P(k)_{\text{DR12}}$	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.30$
P115[TT,TE,EE+lowP] + BAO+ $P(k)_{\text{WZ}}$	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.14$
P115[TT,TE,EE+lowP] + BAO+ $P(k)_{\text{DR7}}$	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.13$
P115[TT+lowP+lensing] + $\text{Ly}\alpha$	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.12$
P116[TT+SimLow+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu$	$< 0.17$
P115[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu+\Omega_k$	$< 0.37$
P115[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu+w$	$< 0.37$
P115[TT+lowP+lensing] + BAO	$\Lambda\text{CDM}+\sum m_\nu+N_{\text{eff}}$	$< 0.32$
P115[TT,TE,EE+lowP+lensing]	$\Lambda\text{CDM}+\sum m_\nu+5\text{-params.}$	$< 0.66$



# Neutrinoless Double $\beta$ Decay



# A Multi-Frontier Problem



# Neutrinos and New Physics

# Origin of Neutrino Mass

- neutrinos undergo flavour oscillations
- those can be explained by a Dirac or Majorana mass term

$$\bar{\nu}_L m_D \nu_R + h.c.$$

$$\bar{\nu}_L m_M \nu_L^c + h.c.$$

- ...which can be diagonalised as

$$\text{diag}(m_1, m_2, m_3) = U_\nu^\dagger m_D \tilde{U}_\nu \quad \text{diag}(m_1, m_2, m_3) = U_\nu^\dagger m_M U_\nu^*$$

leptons are just like quarks without colour... but:

- Why are the  $m_i$  so tiny?
- Why is the mixing matrix so different from the CKM matrix?
- What forbids the Majorana mass for the RH neutrino?



# Origin of Neutrino Mass

- neutrinos undergo flavour oscillations
- those can be explained by a Dirac or Majorana mass term

$$\bar{\nu}_L m_D \nu_R + h.c.$$

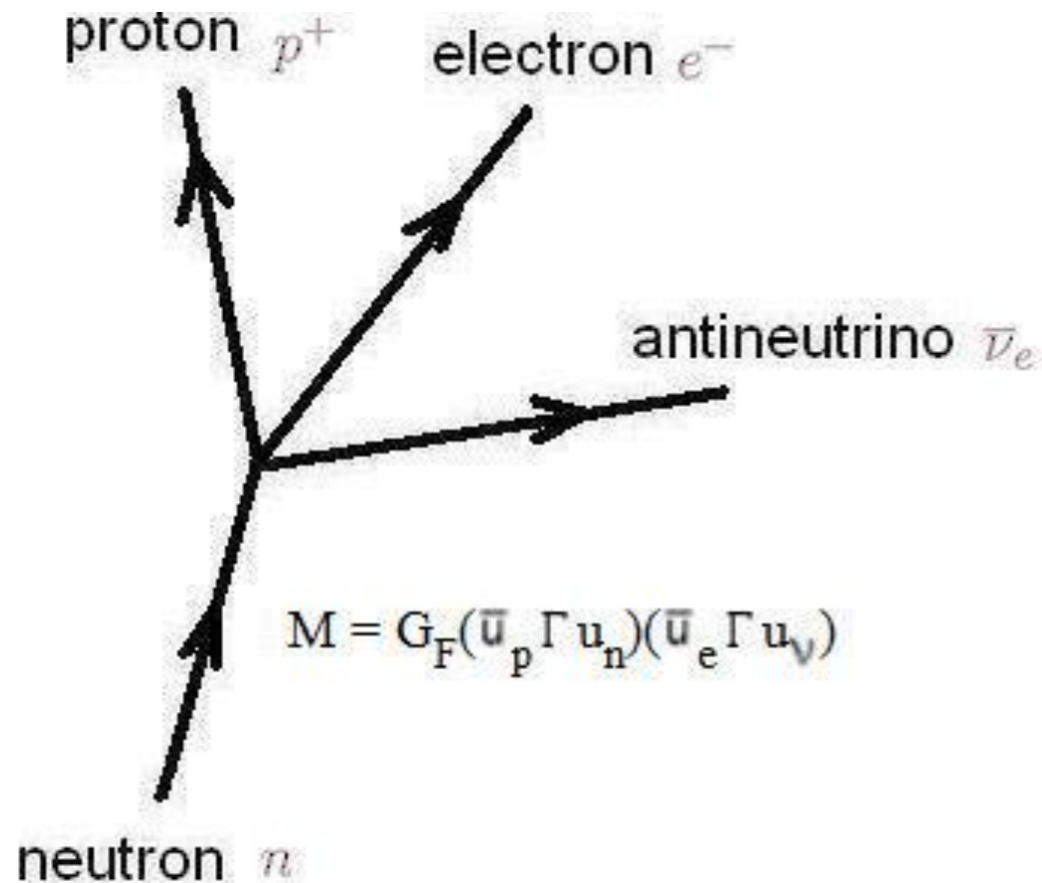
$$\bar{\nu}_L m_M \nu_L^c + h.c.$$

- ...which can be diagonalised as

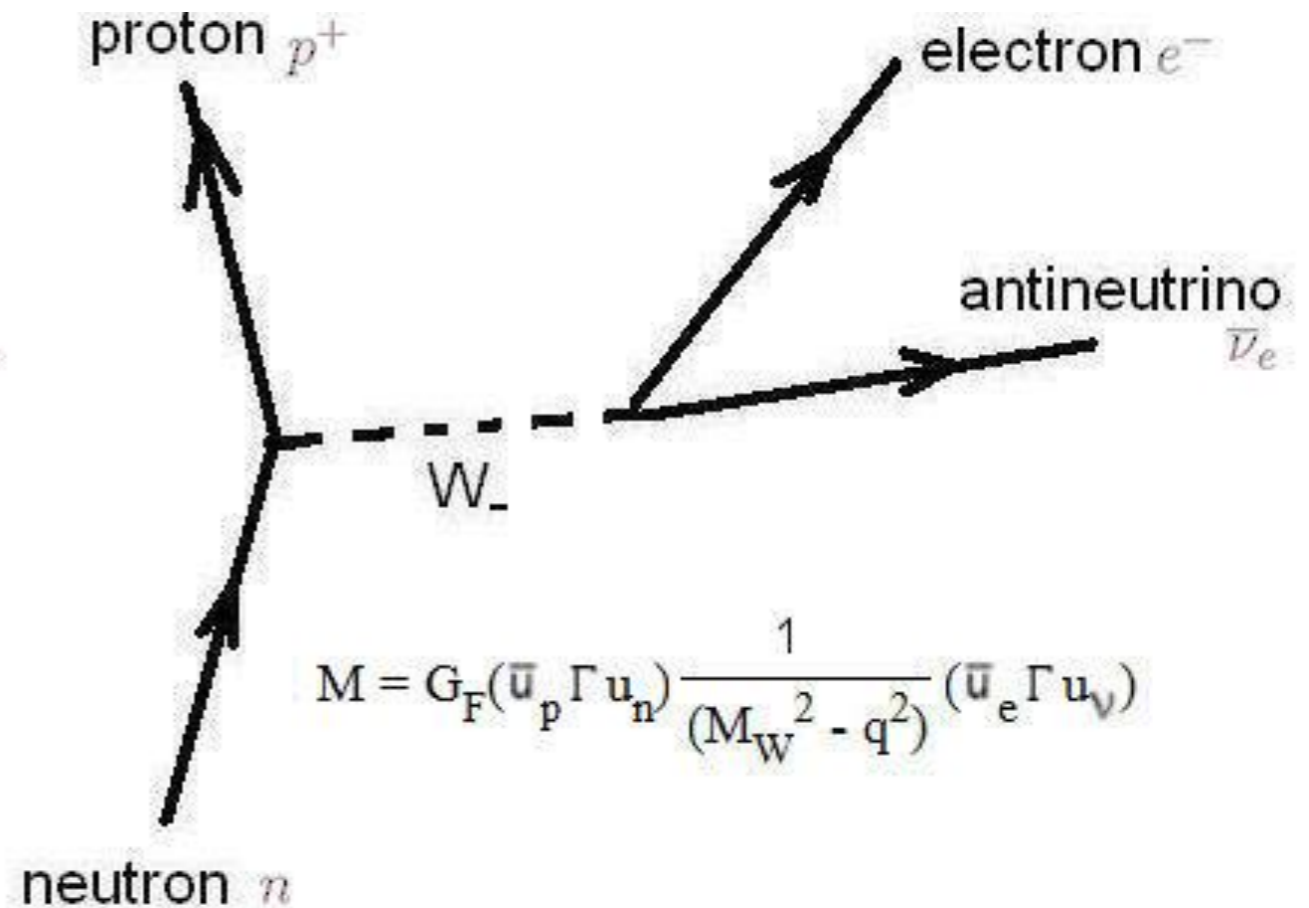
$$\text{diag}(m_1, m_2, m_3) = U_\nu^\dagger m_D \tilde{U}_\nu \quad \text{diag}(m_1, m_2, m_3) = U_\nu^\dagger m_M U_\nu^*$$

- Majorana fermion  
NEW PHYSICS!
- can be generated in gauge invariant way by higher dim operators  
NEW PHYSICS!  
should be generated by integrating out some heavier states  
with masses  $\sim \Lambda \gg E_\nu$

# “Integrating out” Heavy Particles



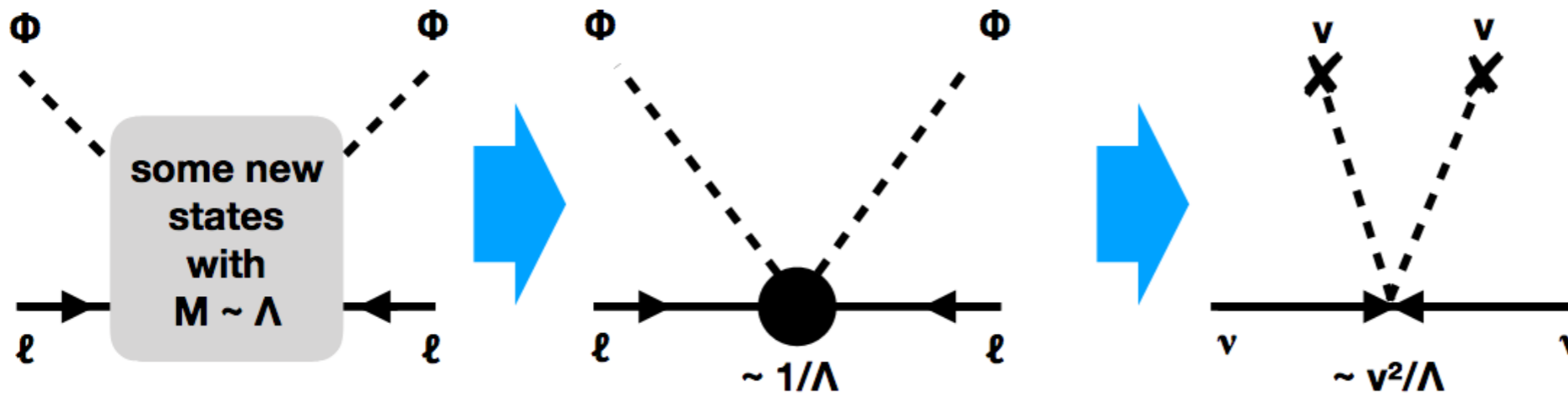
a. Fermi's 4-point Interaction, 1934



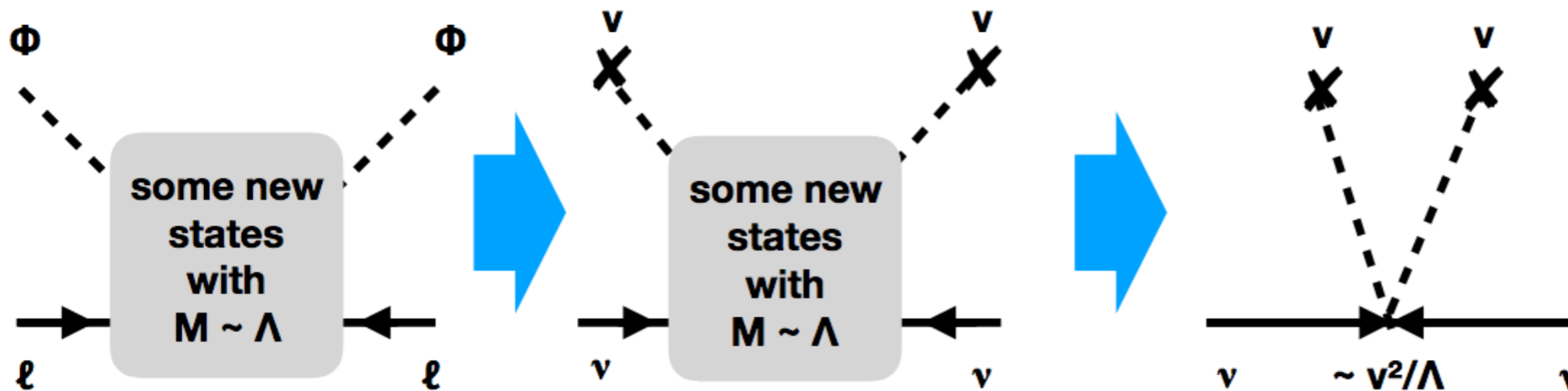
b. Weak Interaction mediated by boson, 1938

The weak gauge bosons are too heavy to be produced in the decay...  
...but appear as “virtual particles”, giving rise to an effective vertex

# Neutrino Mass as a Portal to New Physics



High Scale Seesaw:  $\Lambda \gg v$



Low Scale Seesaw:  $\Lambda \lesssim v$

$$\Phi = (0, v) + \text{fluctuations } h$$

$$\begin{aligned} \Phi l l \Phi / \Lambda = & \\ & v \nu_L \nu_L v / \Lambda \\ & + h \nu_L \nu_L h / \Lambda \\ & + \text{mixed} / \Lambda \end{aligned}$$

$$\overline{\nu}_L m_M \nu_L^c + h.c.$$

“integrating out” heavier states with masses  $\sim \Lambda \gg E_v$

# Tree Level Seesaw Mechanisms

$$\overline{\nu}_L m_M \nu_L^c + h.c.$$

- **Type I: fermionic singlet  $N$  (“right handed neutrino”)**

Minkowski 79, Gell-Mann/Ramond/Slansky 79,  
Mohapatra/Senjanovic 79, Yanagida 80

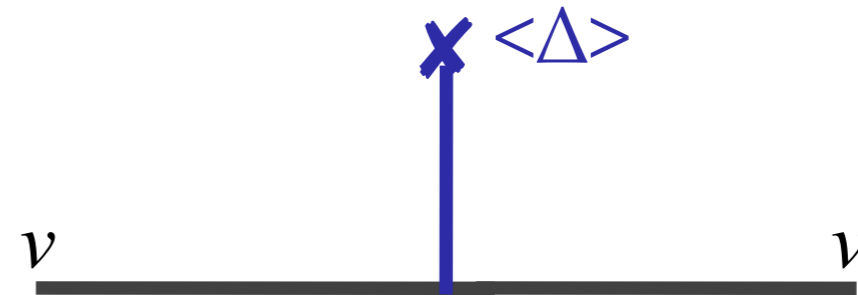
$$-\overline{\ell}_L Y_I N \tilde{\Phi} - \frac{1}{2} \overline{N} M N$$



- **Type II: scalar triplet  $\Delta$**

Schechter/Valle 80, Cheng/Li 1980,  
Lazarides/Shafi/Wetterich 80, Mohapatra/Senjanovic 81

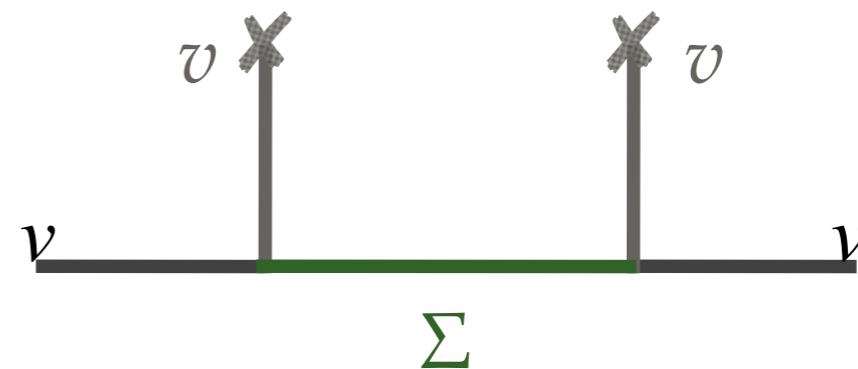
$$-\overline{\ell}_L^c Y_{II} i\sigma_2 \Delta_L \ell$$



- **Type III: fermionic triplet  $\Sigma$**

Foot/Lew/He/Joschi 89

$$-\overline{\ell}_L Y_{III} \Sigma_R^c \tilde{\Phi}$$





# The Standard Model of Particle Physics

"fermions" = matter particles				"bosons" = force carriers		"Higgs boson" = gives mass	
	I	II	III				
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0	0	0	125 GeV
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0	0	0
name →	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b><math>\gamma</math></b> photon	<b>Z<sup>0</sup></b> weak force	<b>H</b> Higgs boson
	Left Right	Left Right	Left Right				
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom				
Quarks	Left Right	Left Right	Left Right				
	0 eV	0 eV	0 eV				
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino				
	Left Right	Left Right	Left Right				
	0.511 MeV	105.7 MeV	1.777 GeV				
	-1	-1	-1				
Leptons	Left Right	Left Right	Left Right				
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau				
	Left Right	Left Right	Left Right				
				Bosons (Forces) spin 1			
				80.4 GeV	<b>W<sup>±</sup></b> weak force		
				±1			
						spin 0	

The "periodic table" of elementary particles - who is missing?

# The Standard Model of Particle Physics

**"fermions" = matter particles**

	I	II	III	"bosons"	"Higgs boson"
mass →	2.4 MeV	1.27 GeV			
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$		
name →	Left <b>u</b> Right up	Left <b>c</b> Right charm	Left top Right		
Quarks	4.8 MeV $-\frac{1}{3}$ Left <b>d</b> Right down	104 MeV $-\frac{1}{3}$ Left <b>s</b> Right strange	4.2 GeV $-\frac{1}{3}$ Left <b>b</b> Right bottom	gluon	
	0 eV 0 Left <b><math>\nu_e</math></b> electron neutrino	0 eV 0 Left <b><math>\nu_\mu</math></b> muon neutrino	0 eV 0 Left <b><math>\nu_\tau</math></b> tau neutrino	$\gamma$ photon	
	0.511 MeV -1 Left <b>e</b> Right electron	105.7 MeV -1 Left <b><math>\mu</math></b> Right muon	1.777 GeV -1 Left <b><math>\tau</math></b> Right tau	91.2 GeV 0 Left <b>Z<sup>0</sup></b> weak force	125 GeV 0 Left <b>H</b> Higgs boson
Leptons				80.4 GeV $\pm 1$ Left <b>W<sup>±</sup></b> weak force	spin 0

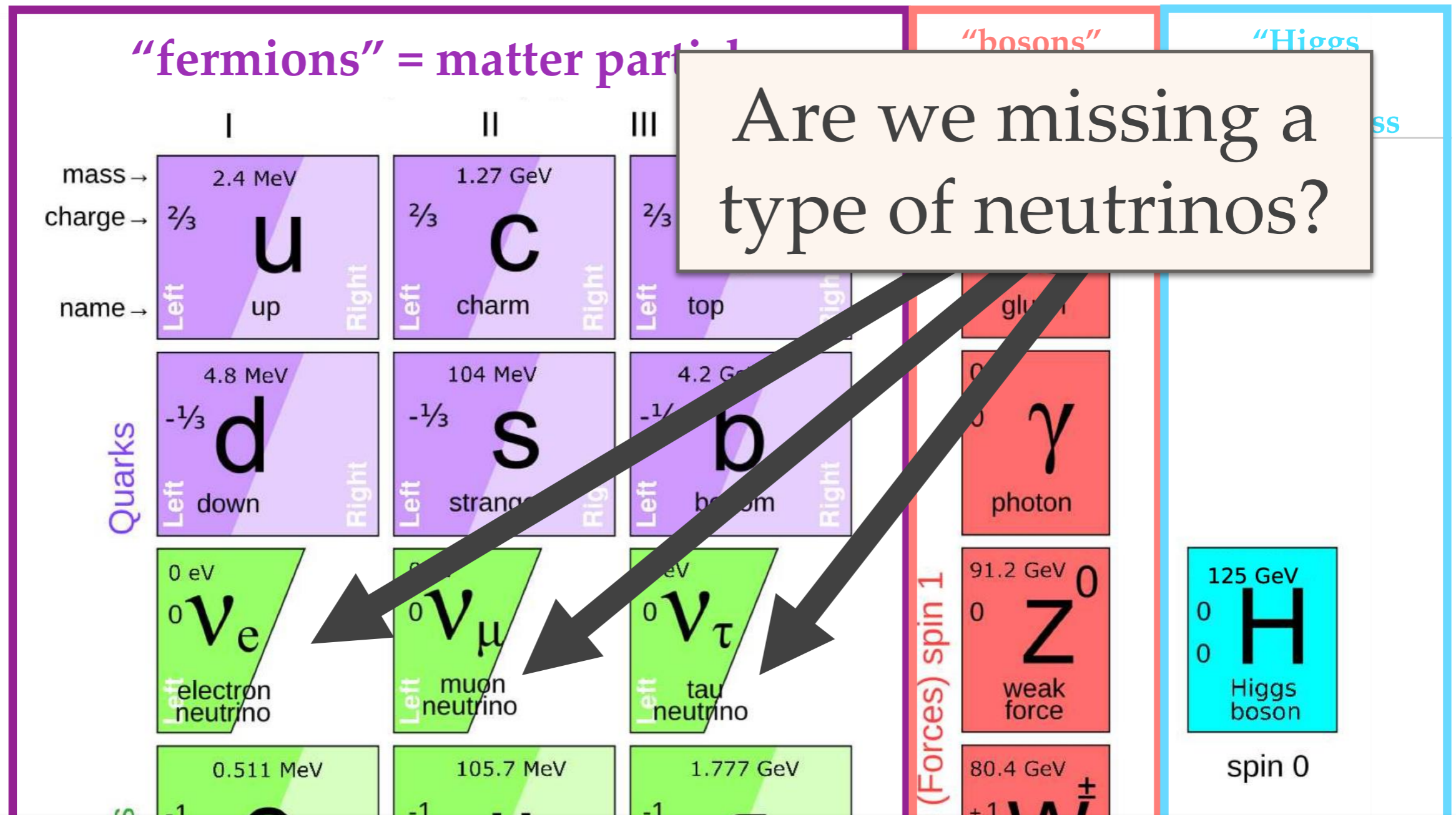
**Bosons (Forces) spin 1**

**Are we missing a type of neutrinos?**

The "periodic table" of elementary particles - who is missing?



# The Standard Model of Particle Physics



Are we missing a type of neutrinos?

**Ordinary "left handed" neutrinos**

- Are massless in the Standard Model
- Can feel the weak nuclear force

**Sterile "right handed" neutrinos**

- Can have a (Majorana) mass.
- Do not feel any of the forces of nature (except gravity)

# The Seesaw Mechanism

$$H = (\nu, 0)$$

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \not{\partial} \nu_R - \bar{L}_L F \nu_R \tilde{H} - \tilde{H}^\dagger \bar{\nu}_R F^\dagger L$$

$$- \frac{1}{2} (\bar{\nu}_R^c M_M \nu_R + \bar{\nu}_R M_M^\dagger \nu_R^c)$$

electroweak symmetry breaking generates a Dirac mass term

$$m_D \equiv F \nu$$

$$\overline{\nu}_L m_D \nu_R + h.c.$$

The complete mass term can then be written as

$$\frac{1}{2} (\overline{\nu}_L \quad \overline{\nu}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.,$$

Three Generations of Matter (Fermions) spin 1/2

	I	II	III		
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0	125 GeV
charge →	2/3	2/3	2/3	0	0
name →	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> Higgs boson
Quarks	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b>γ</b> photon	Bosons (Forces) spin 1
	<b>ν<sub>e</sub></b> electron neutrino	<b>ν<sub>μ</sub></b> muon neutrino	<b>ν<sub>τ</sub></b> tau neutrino	<b>Z<sup>0</sup></b> weak force	
	<b>e</b> electron	<b>μ</b> muon	<b>τ</b> tau	<b>W<sup>±</sup></b> weak force	
Leptons	0 eV	0 eV	0 eV	91.2 GeV	spin 0



---

# The Seesaw Mechanism

---

$$\frac{1}{2}(\overline{\nu}_L \quad \overline{\nu}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.,$$

Full mass term is diagonalised as

$$\mathcal{U}^\dagger \mathcal{M} \mathcal{U}^* = \begin{pmatrix} m_\nu^{\text{diag}} & \\ & M_N^{\text{diag}} \end{pmatrix} \quad \text{with} \quad \begin{aligned} M_N^{\text{diag}} &= U_N^T M_N U_N \\ m_\nu^{\text{diag}} &= U_\nu^\dagger m_\nu U_\nu^* \end{aligned}$$

The rotation matrix is given by

$$\mathcal{U} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta^\dagger) & \cos(\theta^\dagger) \end{pmatrix} \begin{pmatrix} U_\nu \\ U_N^* \end{pmatrix} \quad \text{with} \quad \begin{aligned} \sin(\theta) &= \sum_{n=0}^{\infty} \frac{(-\theta\theta^\dagger)^n \theta}{(2n+1)!} \\ \cos(\theta) &= \sum_{n=0}^{\infty} \frac{(-\theta\theta^\dagger)^n}{(2n)!} \end{aligned}$$

# The Seesaw Mechanism

At linear order in  $\theta \equiv m_D M_M^{-1}$   
we find two sets of mass eigenstates

**light “active” neutrinos**

$$\nu \simeq U_\nu^\dagger (\nu_L - \theta \nu_R^c) + \text{c.c.}$$

with mass matrix

$$m_\nu = -v^2 F M_M^{-1} F^T = -m_D M_M^{-1} m_D^T = -\theta M_M \theta^T$$

**heavy sterile neutrinos**

$$N \simeq U_N^\dagger (\nu_R + \theta^T \nu_L^c) + \text{c.c.}$$

with mass matrix  $M_N = M_M + \frac{1}{2} (\theta^\dagger \theta M_M + M_M^T \theta^T \theta^*)$



# Right Handed Neutrino Mass Scale



**Neutrino Physics**

LSND etc

Explain Light Neutrino Masses  
("Seesaw Mechanism")

**Cosmology**

Dark Radiation

Dark Matter

Origin of Matter  
("Leptogenesis")

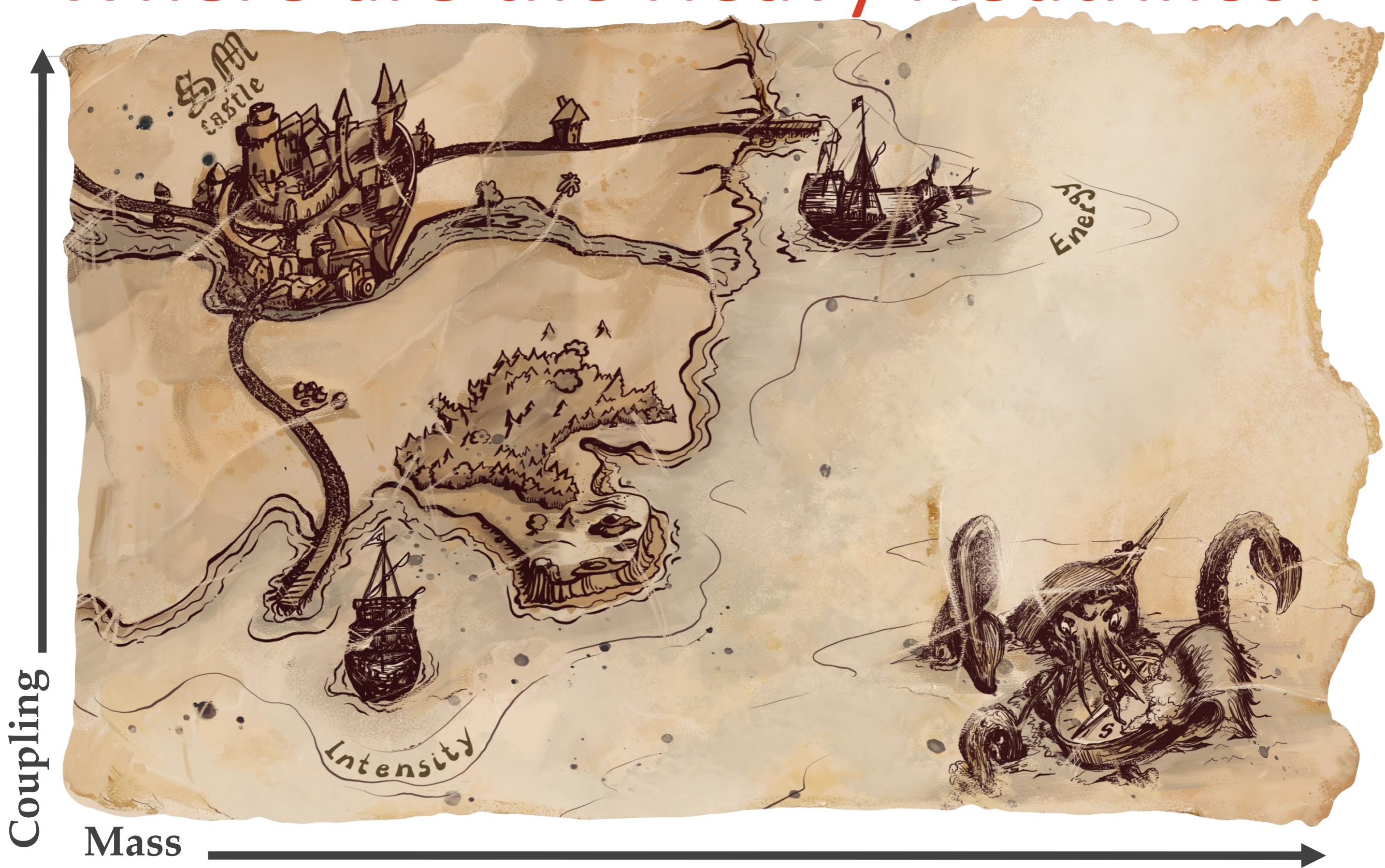
**High Energy Physics**

Direct Searches

Indirect Searches



# Where are the Heavy Neutrinos?





# Right Handed Neutrino Mass Scale

nuclear  
decay spectra



TRISTAN,  
ECHO

fixed target  
experiments



SHiP

Search for Hidden Particles



b factories



far detector



lepton colliders

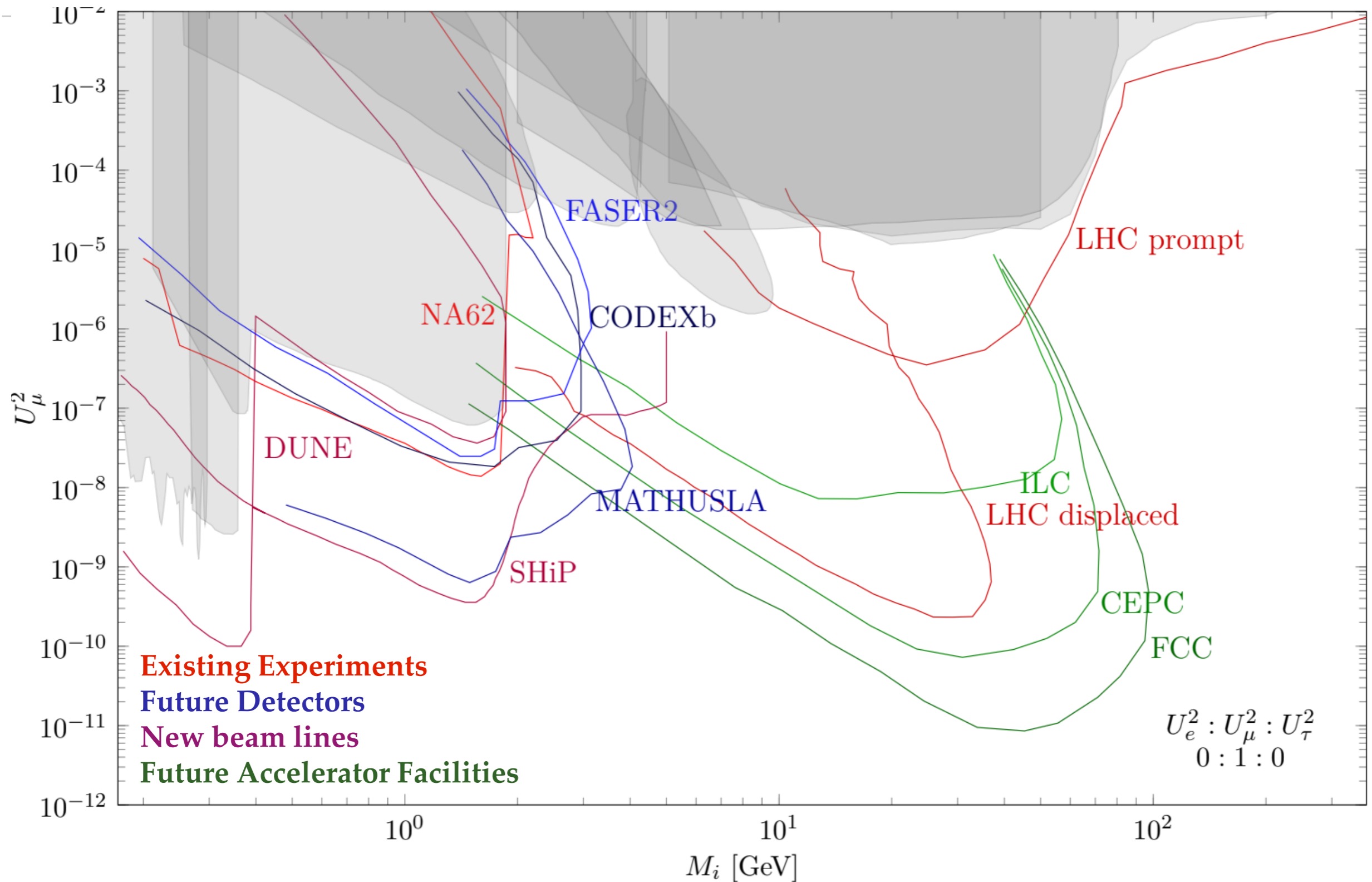


Direct Searches

Indirect Searches

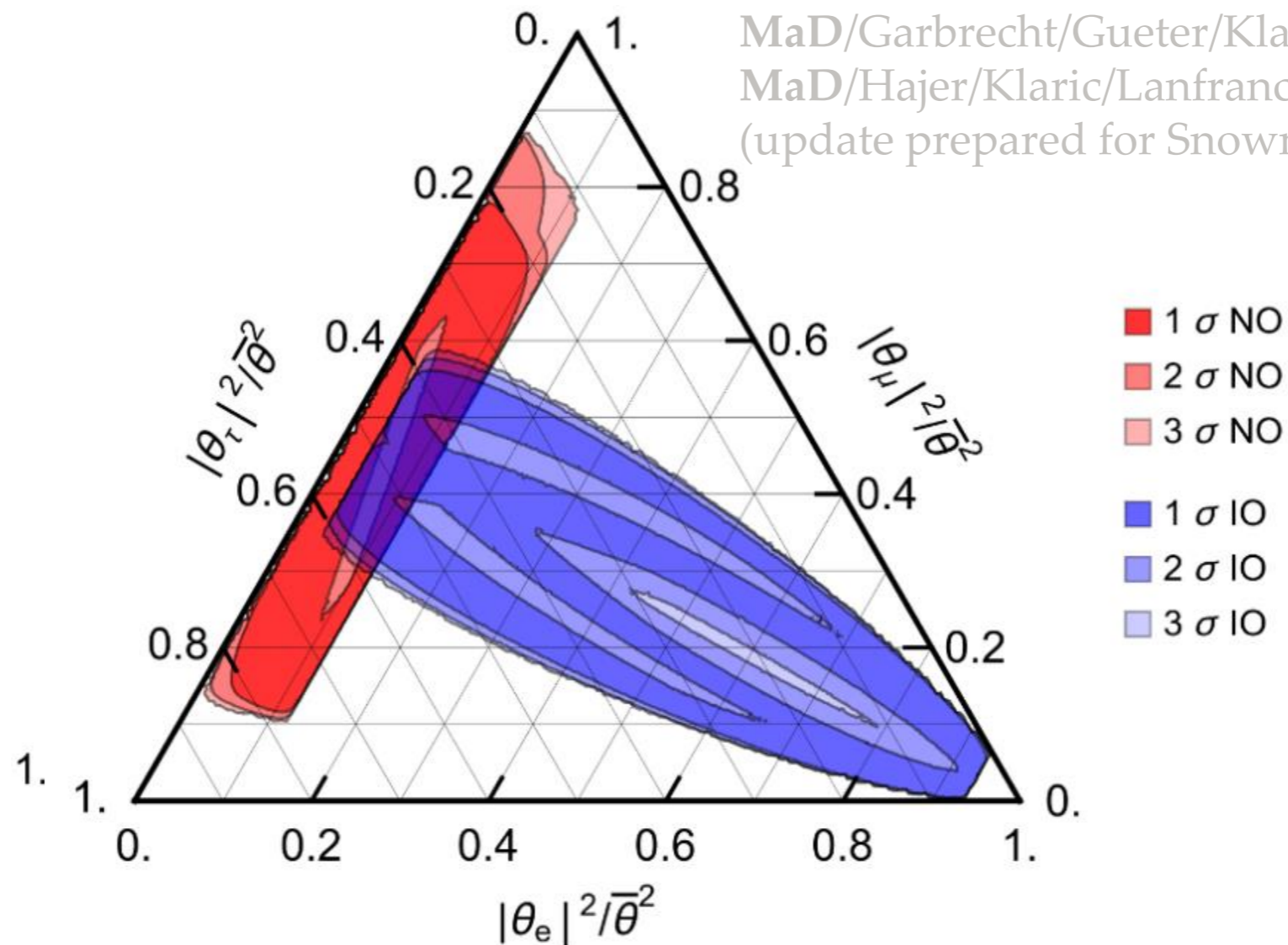
High Energy  
Physics

# Heavy Neutrino Searches



# Branching ratios from $\nu$ -Oscillation Data

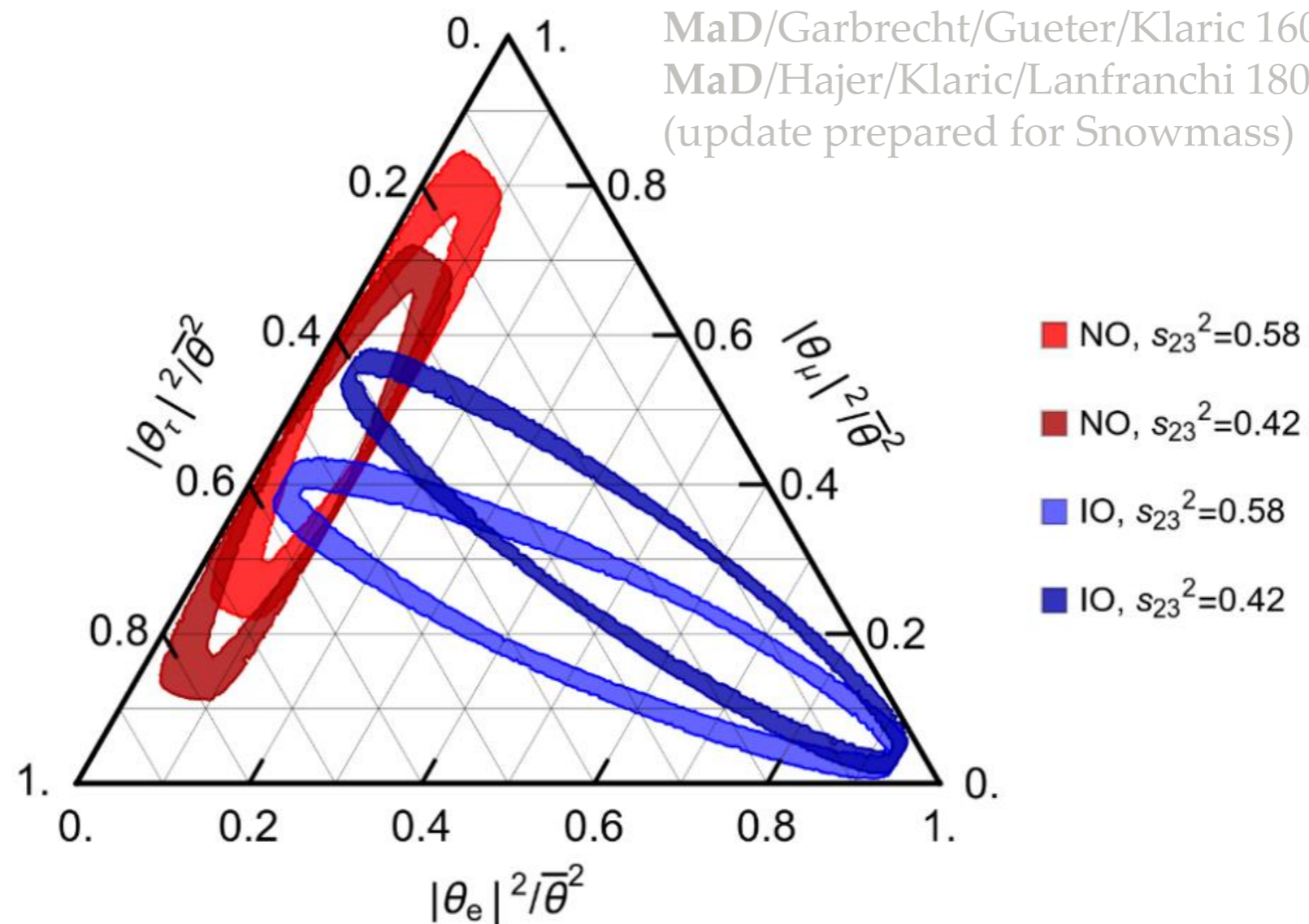
- If RH neutrinos generate light neutrino masses, requirement to reproduce neutrino oscillation data constrains their properties
- In particular: branching ratio in their decays into SM flavours





# Forecast with DUNE

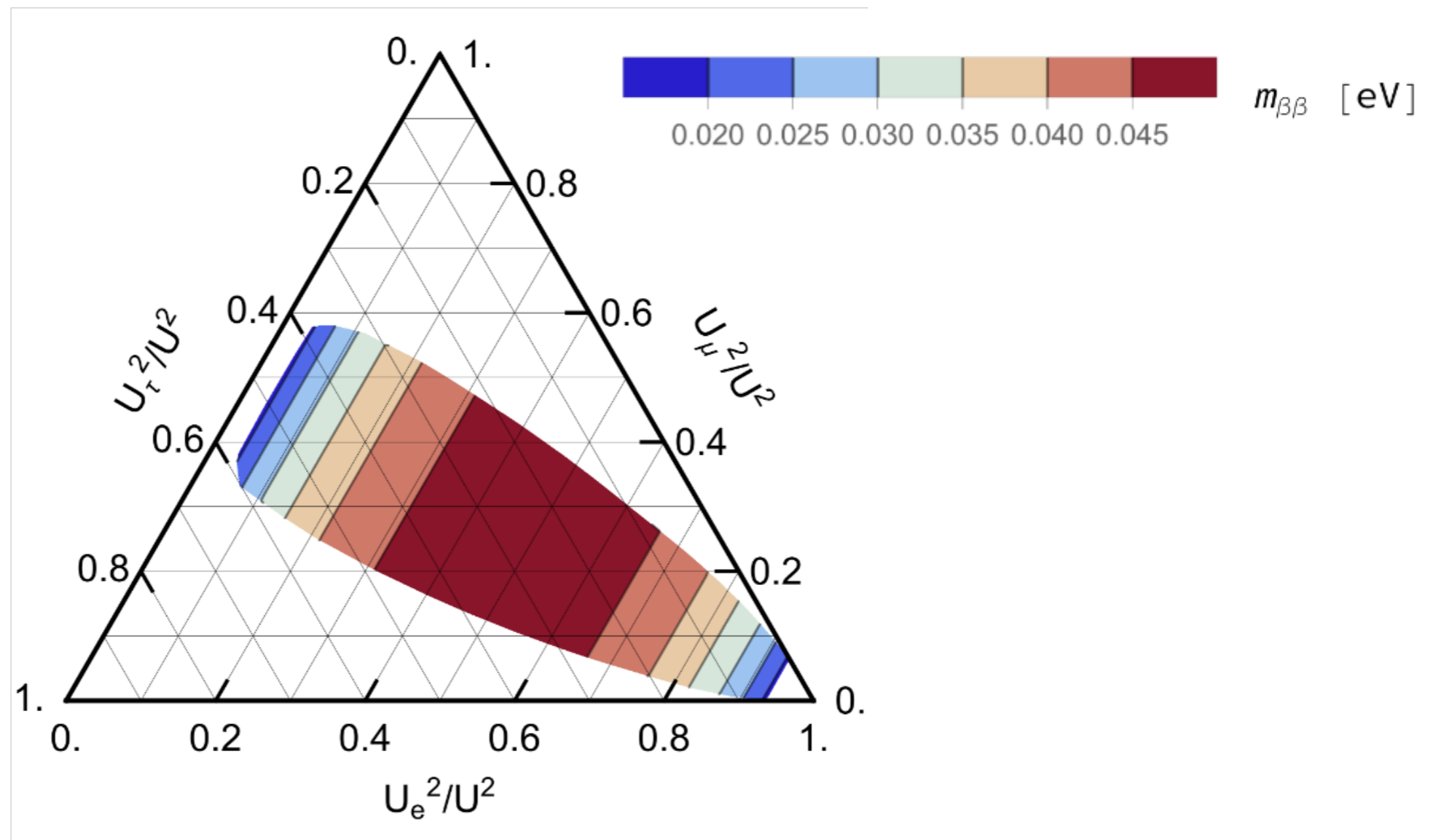
- If RH neutrinos generate light neutrino masses, requirement to reproduce neutrino oscillation data constrains their properties
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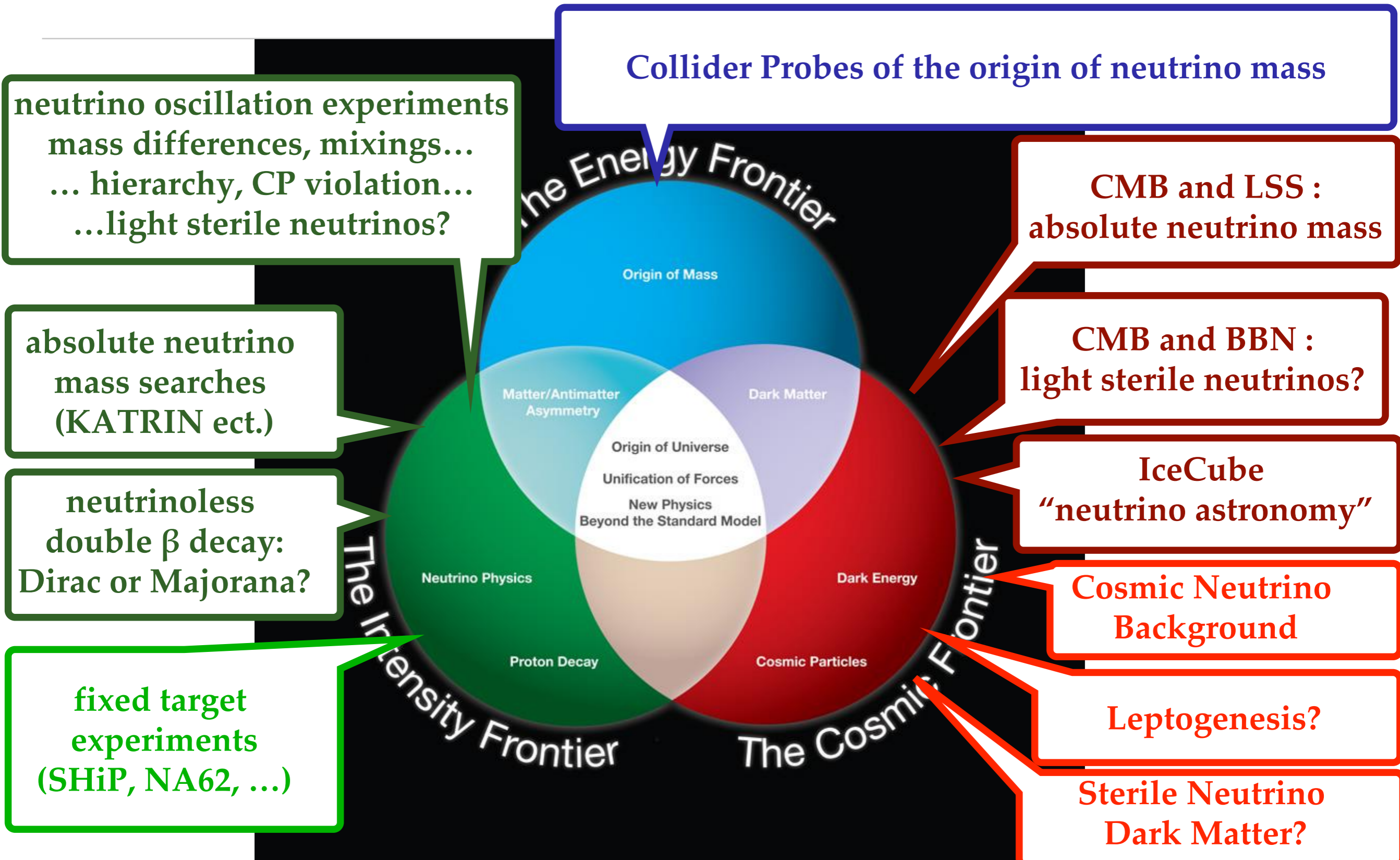


# Connection to Neutrinoless Double $\beta$ Decay

- If RH neutrinos generate light neutrino masses, requirement to reproduce neutrino oscillation data constrains their properties
- In particular: branching ratio in their decays into SM flavours



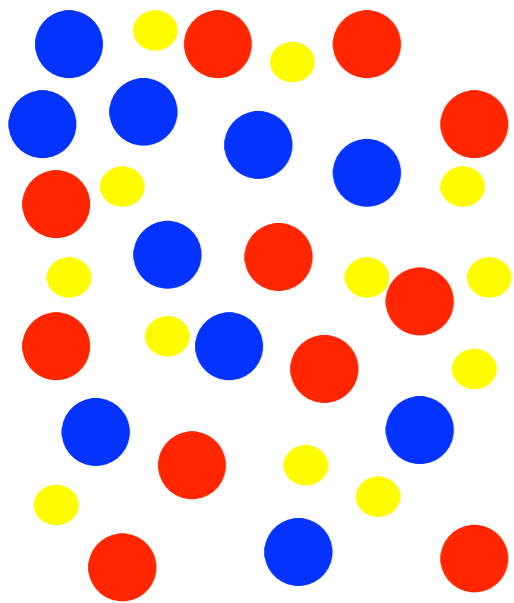
# A Multi-Frontier Problem



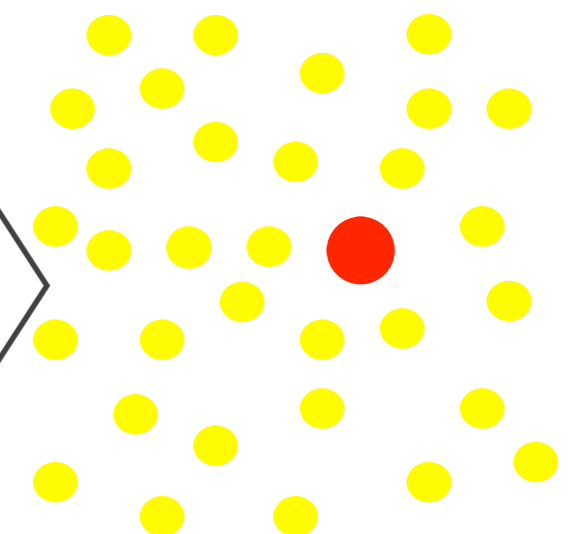
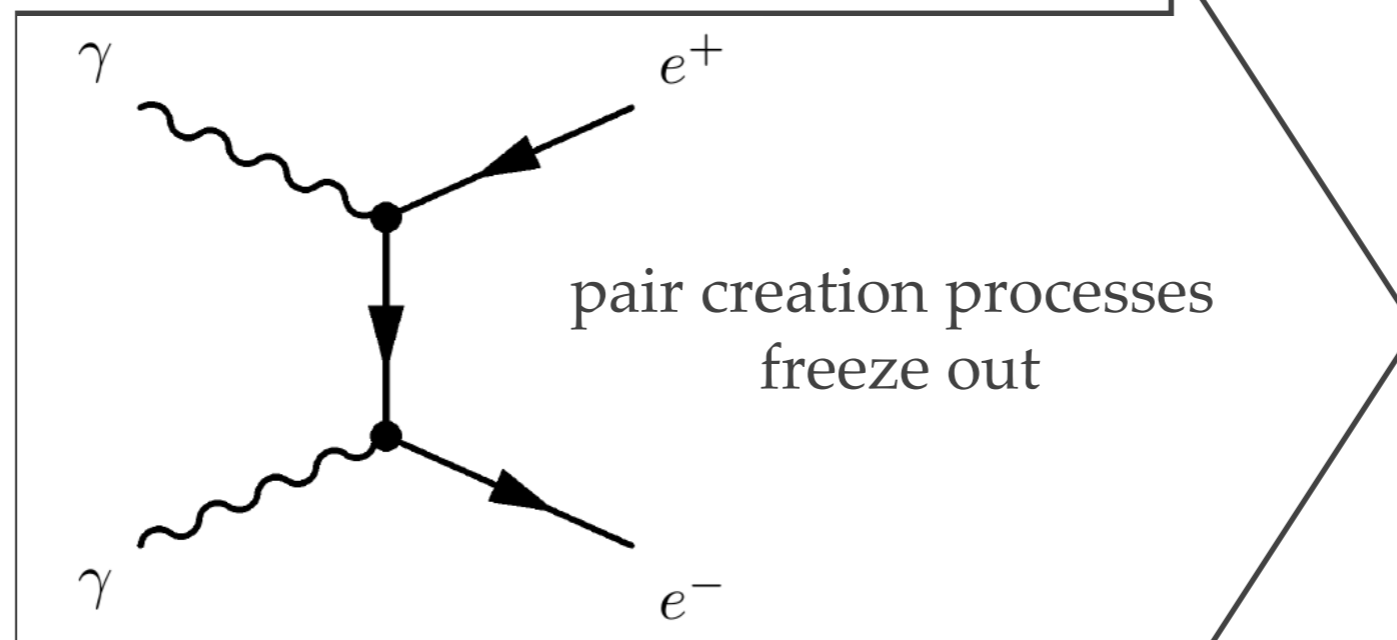
# Leptogenesis

# Baryon Asymmetry of the Universe

The observable universe contains almost no antimatter and a lot more photons than baryons.



$$T > 2 mc^2$$



$$T < 2 mc^2$$

CMB constraint on  
baryon-to-photon ratio  $\eta$ :  
 $6.03 \times 10^{-10} < \eta < 6.15 \times 10^{-10}$

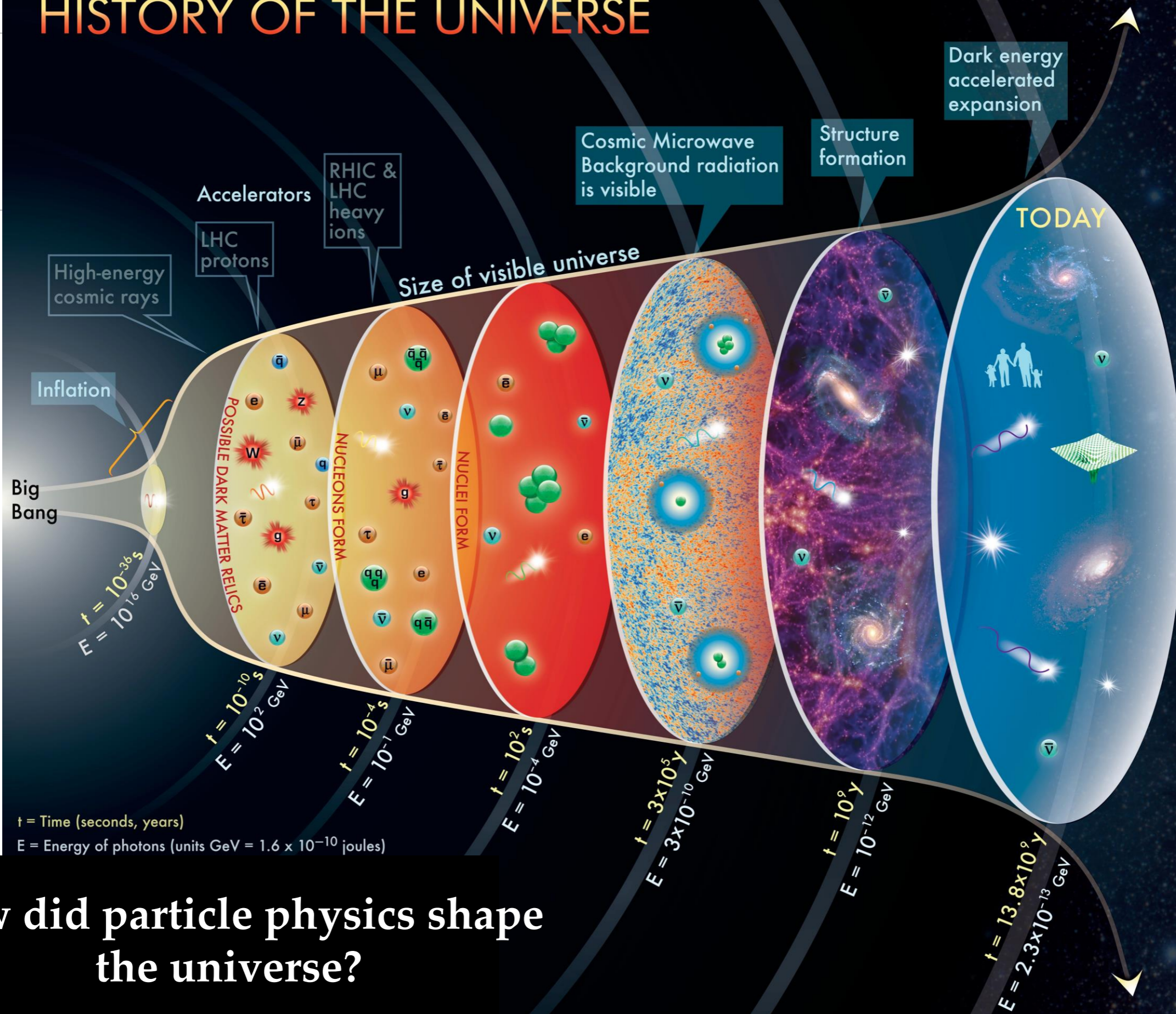
(Planck Collaboration)

BBN constraint on baryon-to-  
photon ratio  $\eta$ :  
 $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$

(PDG)



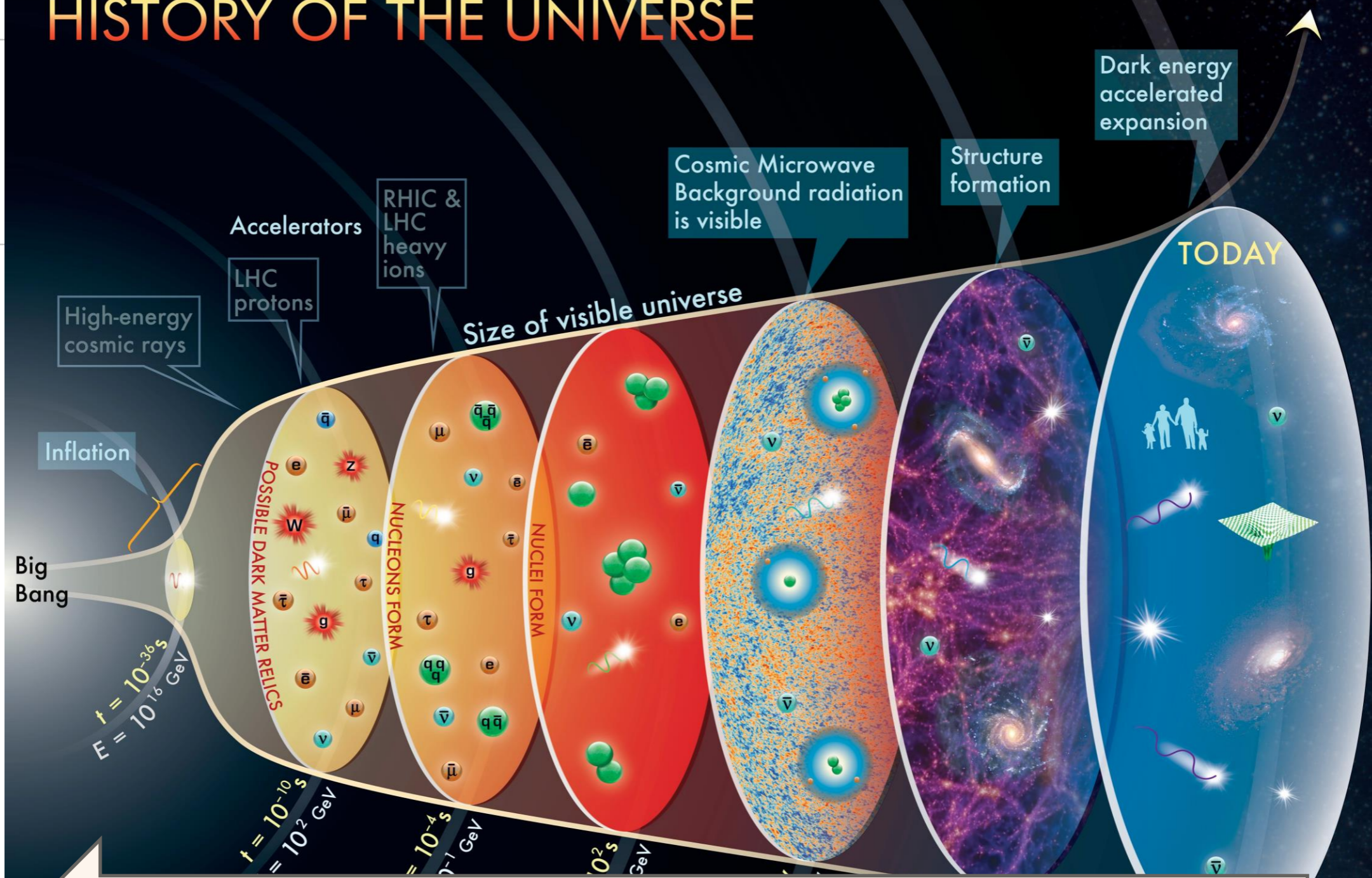
# HISTORY OF THE UNIVERSE



How did particle physics shape the universe?



# HISTORY OF THE UNIVERSE



energy density, temperature

cosmic time

The concept for the above figure originated in a 1986 paper by Michael Turner.

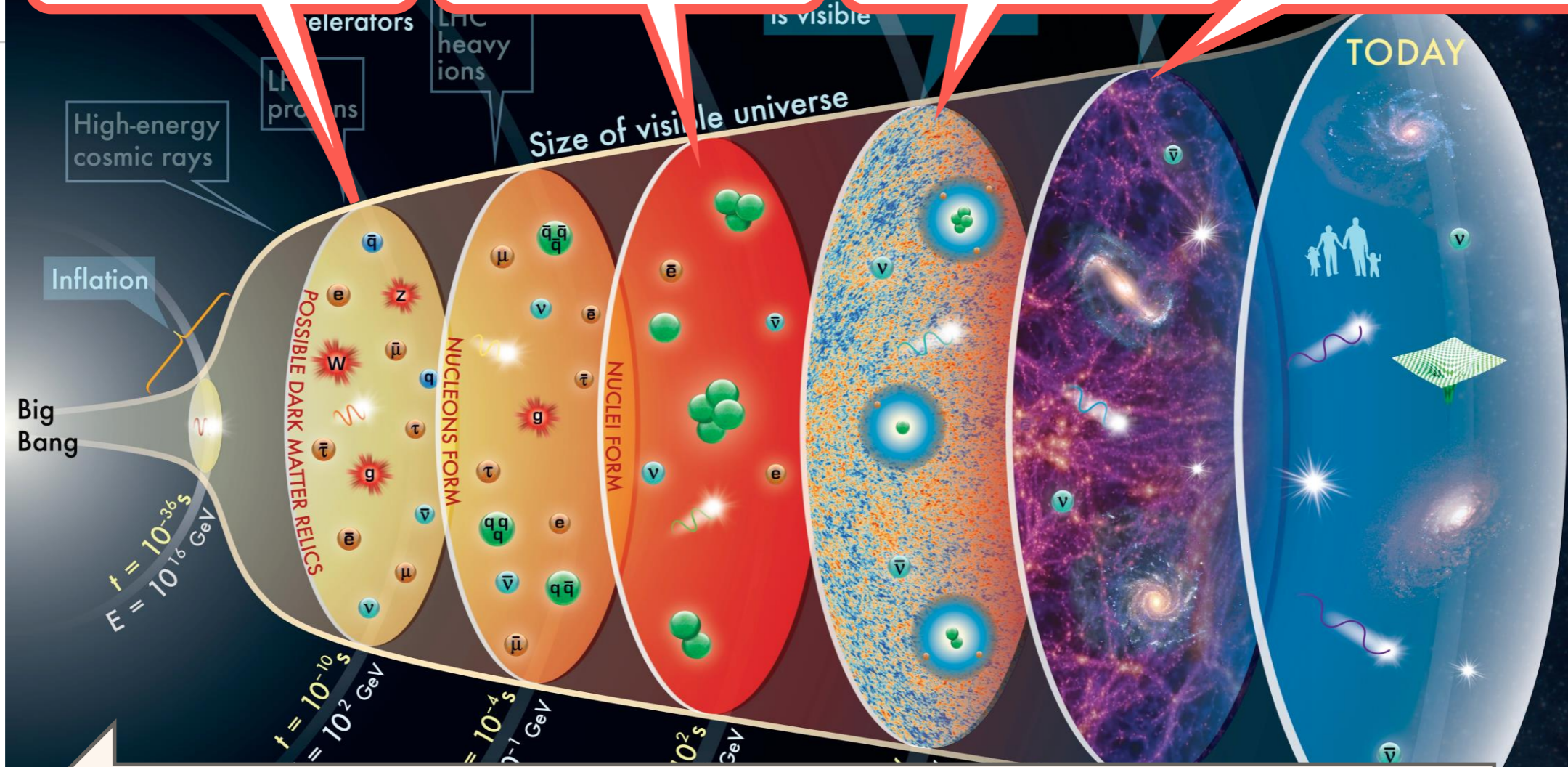


Large Hadron Collider

light element abundances

Cosmic Microwave Background

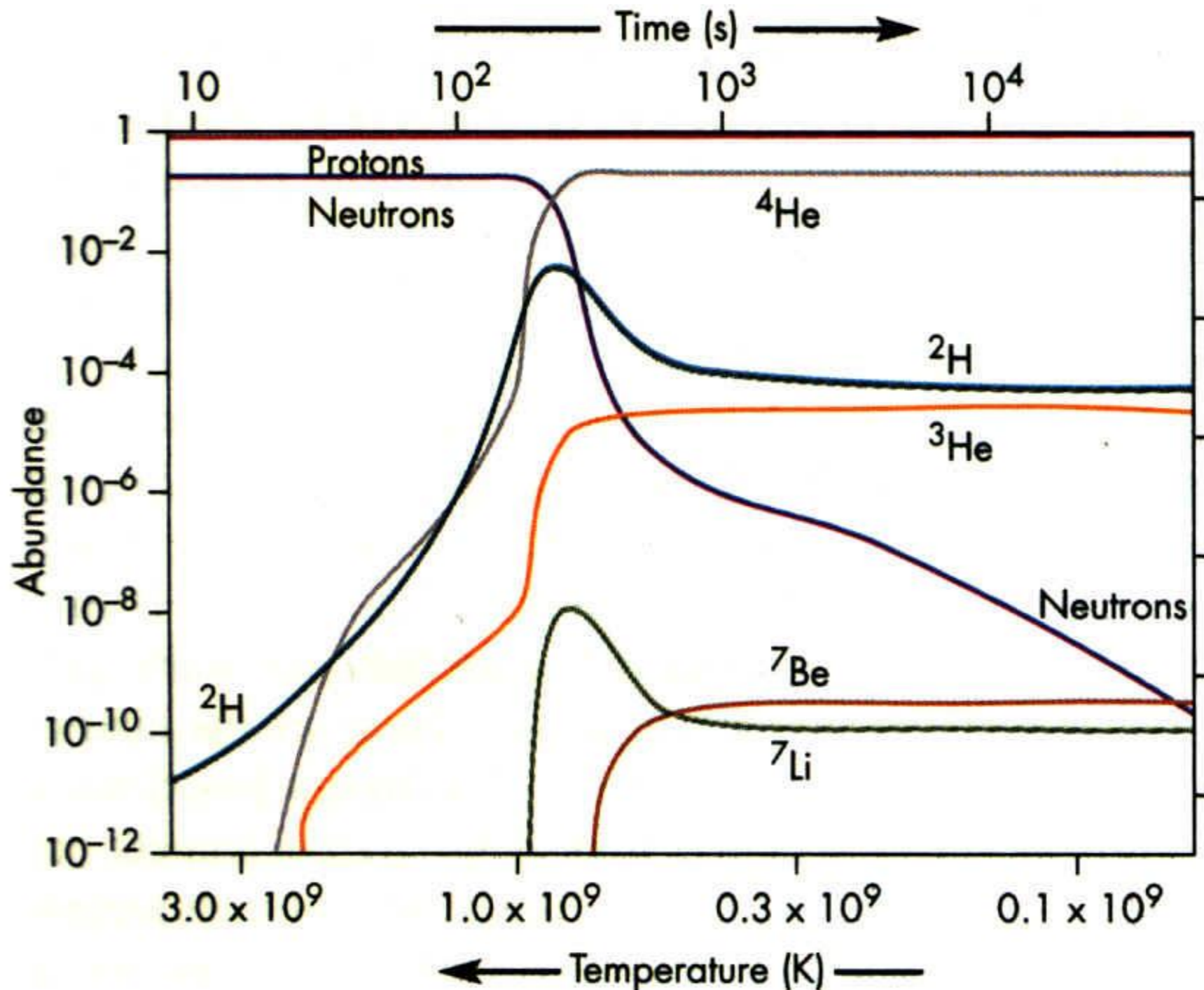
optical astronomy



energy density, temperature

cosmic time

# Big Bang Nucleosynthesis



Light elements are produced in a chain of nuclear reactions.

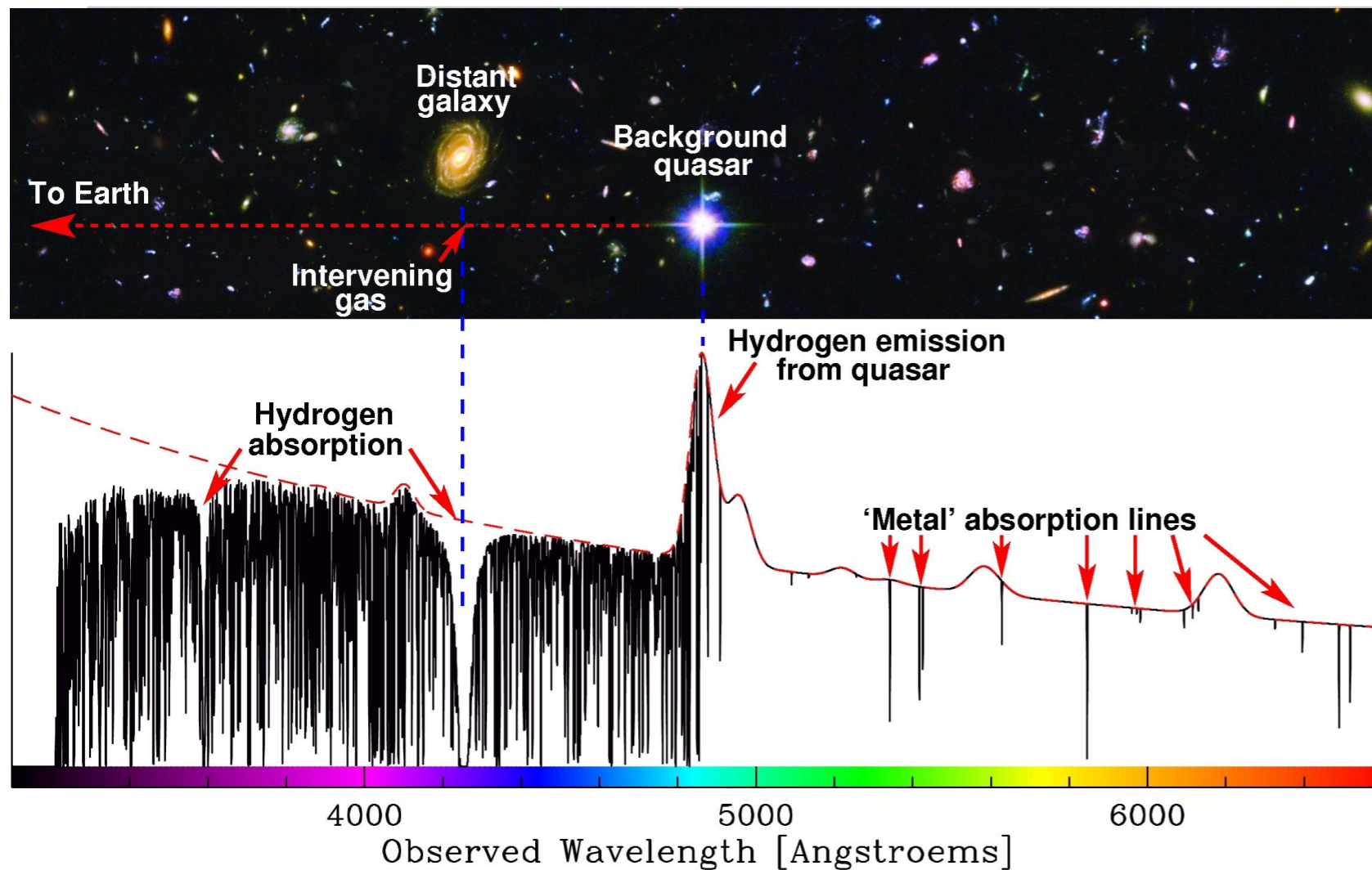
The only unknown parameter is the baryon-to-photon ratio

**Primordial light element abundances measure the baryon asymmetry!**





# Big Bang Nucleosynthesis



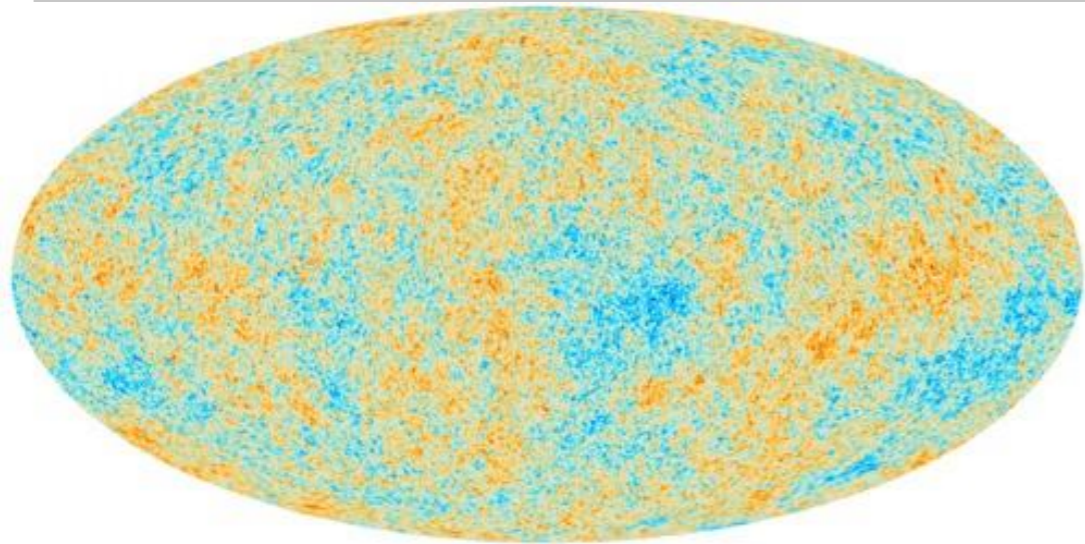
❖ light element abundances in intergalactic medium can be measured in quasar spectra

❖ Deuterium is sensitive to baryon asymmetry and not produced in stars

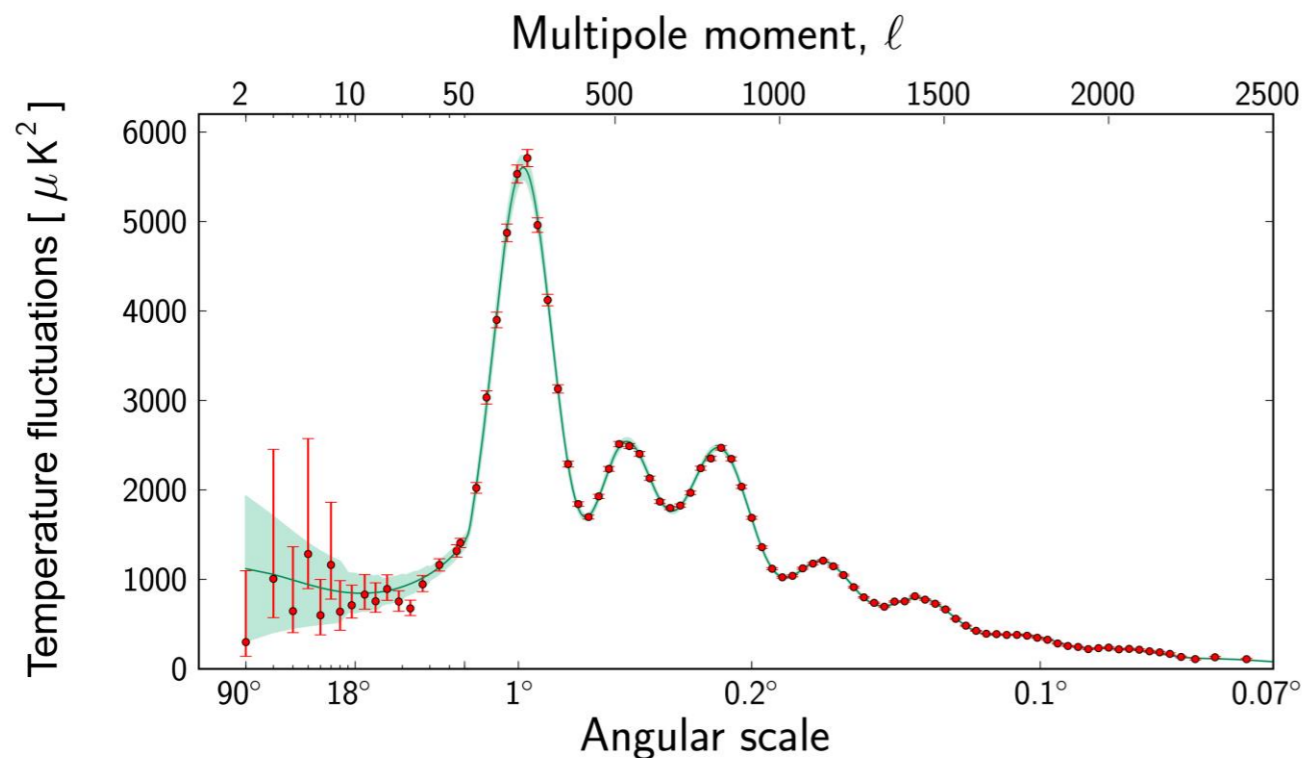
**Constraint on baryon-to-photon ratio  $\eta$ :**

$$5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10} \quad \text{PDG 2016}$$

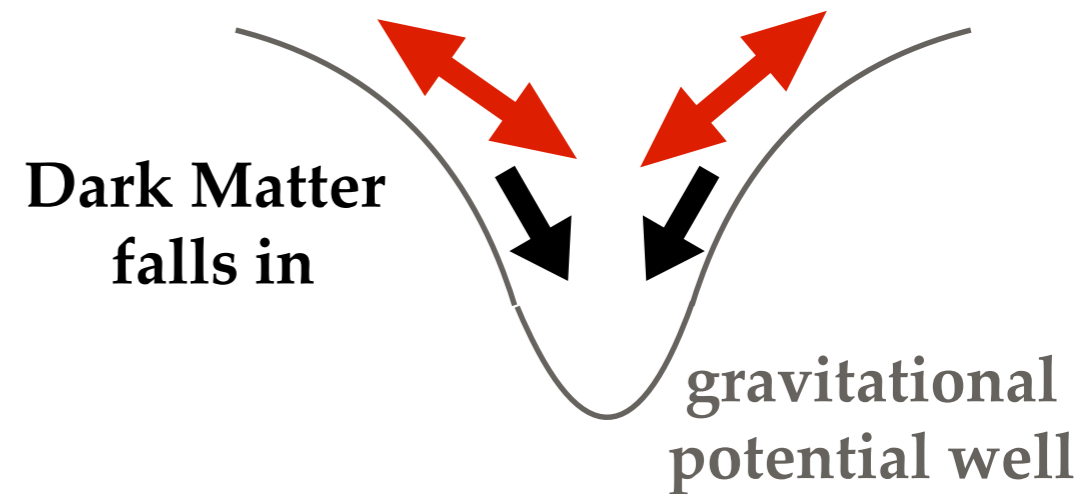
# Baryon Acoustic Oscillations



Radiation, baryons and Dark Matter affect the acoustic oscillations in the primordial plasma in different ways...

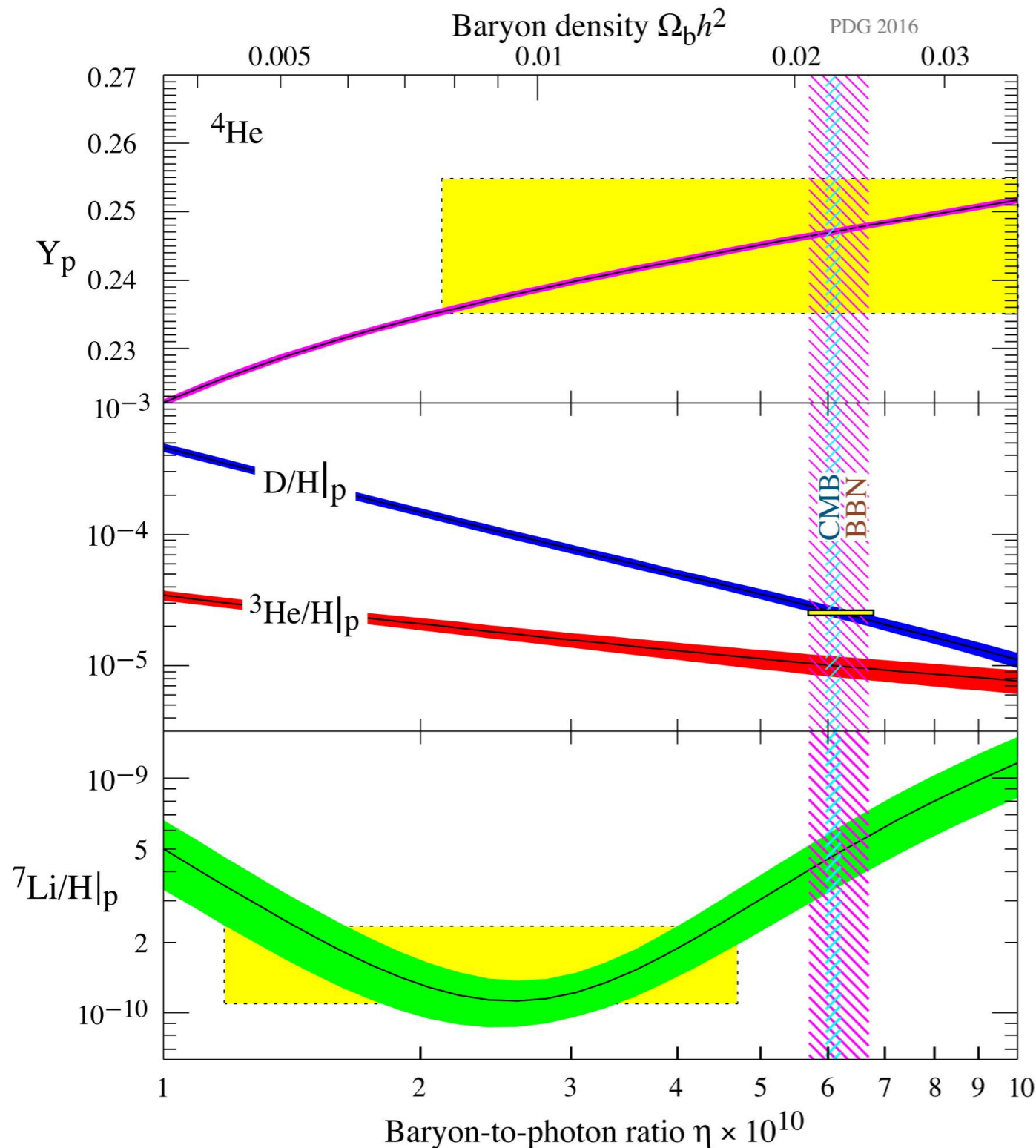


**ordinary matter/radiation oscillates due to radiation pressure**



**neutrinos stream freely until they become non-relativistic**

# Cosmic Microwave Background



**CMB constraint on baryon-to-photon ratio  $\eta$ :**  
 $6.03 \times 10^{-10} < \eta < 6.15 \times 10^{-10}$

**BBN constraint on baryon-to-photon ratio  $\eta$ :**  
 $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$



# Where does the asymmetry come from?

## Sakharov Conditions (1967)

- ❖ Baryon number violation
- ❖ C and CP violation
- ❖ Deviation from thermal equilibrium



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Exists in Standard Model  
(sphaleron)



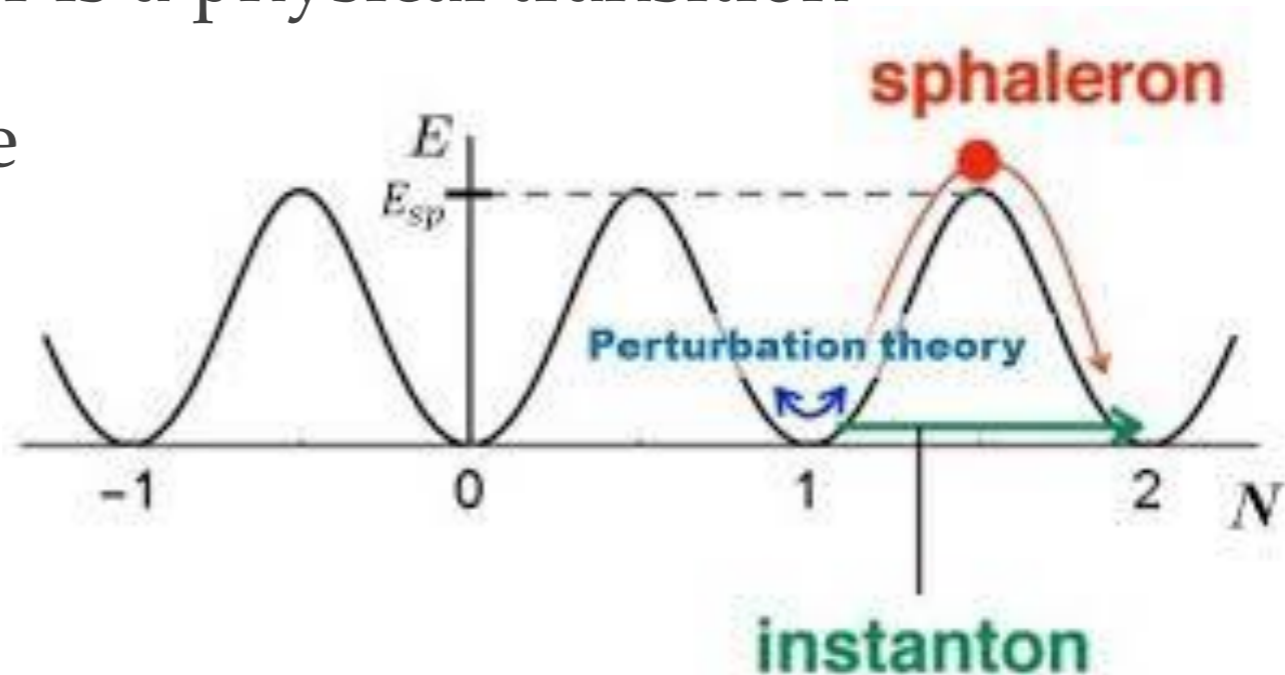


# B-L Violation in the SM

- B and L are conserved in the SM at the perturbative level
- But B-L is violated by the quantum anomaly

$$\partial^\mu j_\mu^B = \frac{n_f}{32\pi^2} \left( -g^2 \text{tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) + g'^2 F'_{\mu\nu} \tilde{F}'^{\mu\nu} \right)$$

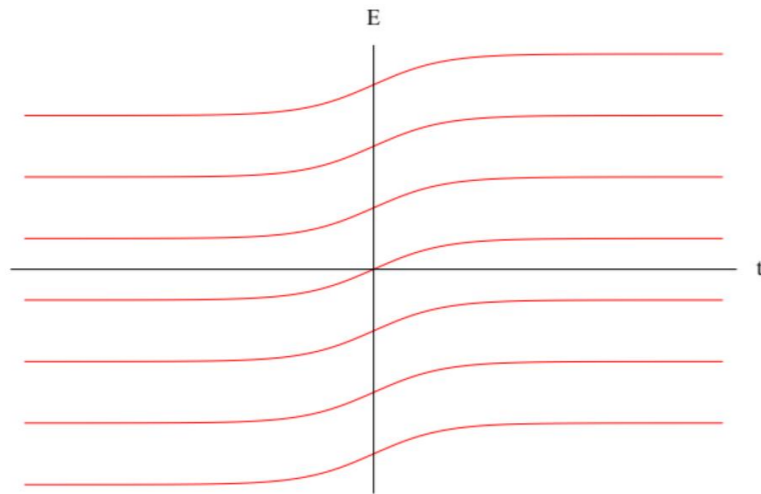
- EW theory has infinitely many degenerate vacua that are related by gauge transitions
- It is impossible to distinguish in what vacuum one is, but going from one to another is a physical transition
- At vanishing temperature the transition can only happen via tunnelling
- At  $T > 0$  thermal fluctuations can induce a transition



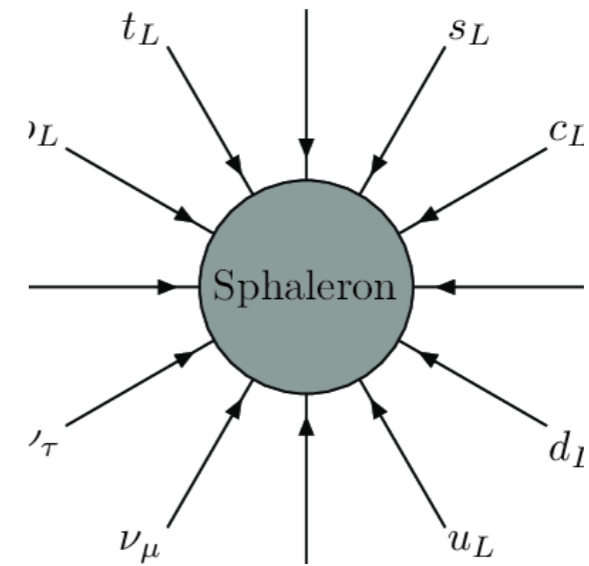


# Sphaleron

- Doing the transition corresponds to a change in the background gauge field configuration that is felt by the fermions



- Intuitively this lifts up the levels in the Dirac sea, leading to a production of particles...



- The rate is given by:

$$\Gamma_{\text{sph}} = (25.4 \pm 2.0) \alpha_W^5 T^4 \quad (\text{symmetric phase})$$

$$\Gamma_{\text{sph}} = A (\alpha_W T)^4 \left( \frac{M_{\text{sph}}}{T} \right)^7 \exp \left( - \frac{M_{\text{sph}}}{T} \right) \quad (\text{Higgs phase})$$

- This equals the Hubble rate at roughly  $T = 131 \text{ GeV}$

# Where does the asymmetry come from?

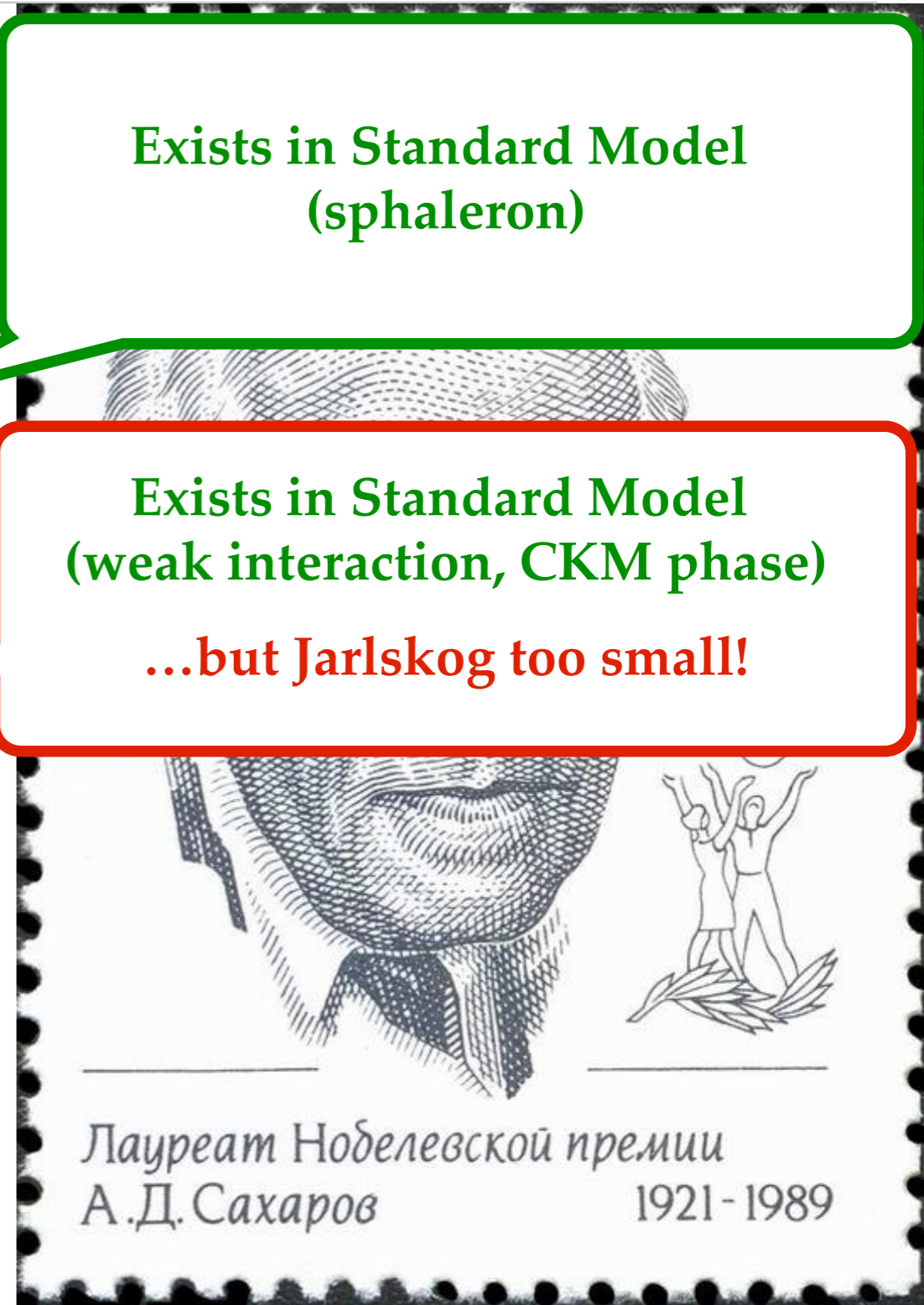
## Sakharov Conditions (1967)

- ❖ Baryon number violation
- ❖ C and CP violation
- ❖ Deviation from thermal equilibrium

Exists in Standard Model  
(sphaleron)

Exists in Standard Model  
(weak interaction, CKM phase)

...but Jarlskog too small!



# CKM Matrix

- CKM matrix mixes quark mass- and interaction eigenstates

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} = \begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix}$$

$$U_\nu = V^{(23)} U_\delta V^{(13)} U_{-\delta} V^{(12)} \quad U_{\pm\delta} = \text{diag}(e^{\mp i\delta/2}, 1, e^{\pm i\delta/2})$$

$$V^{(23)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \quad V^{(13)} = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \quad V^{(12)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- But relevant CP violation in the early universe is given by Jarlkog determinant in units of the temperature  $T$

$$D = \sin(\theta_{12}) \sin(\theta_{23}) \sin(\theta_{13}) \delta_{KM} (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2).$$

- Since  $T \sim 131 \text{ GeV}$  greatly exceeds the quark masses, this is small!



# Where does the asymmetry come from?

## Sakharov Conditions (1967)

- ❖ Baryon number violation
- ❖ C and CP violation
- ❖ Deviation from thermal equilibrium

Exists in Standard Model  
(sphaleron)

Exists in Standard Model  
(weak interaction, CKM phase)

...but Jarlskog too small!

Exists in Standard Model  
(Hubble expansion of the universe)

...but deviation too small!

А.Д. Сахаров

1921-1989

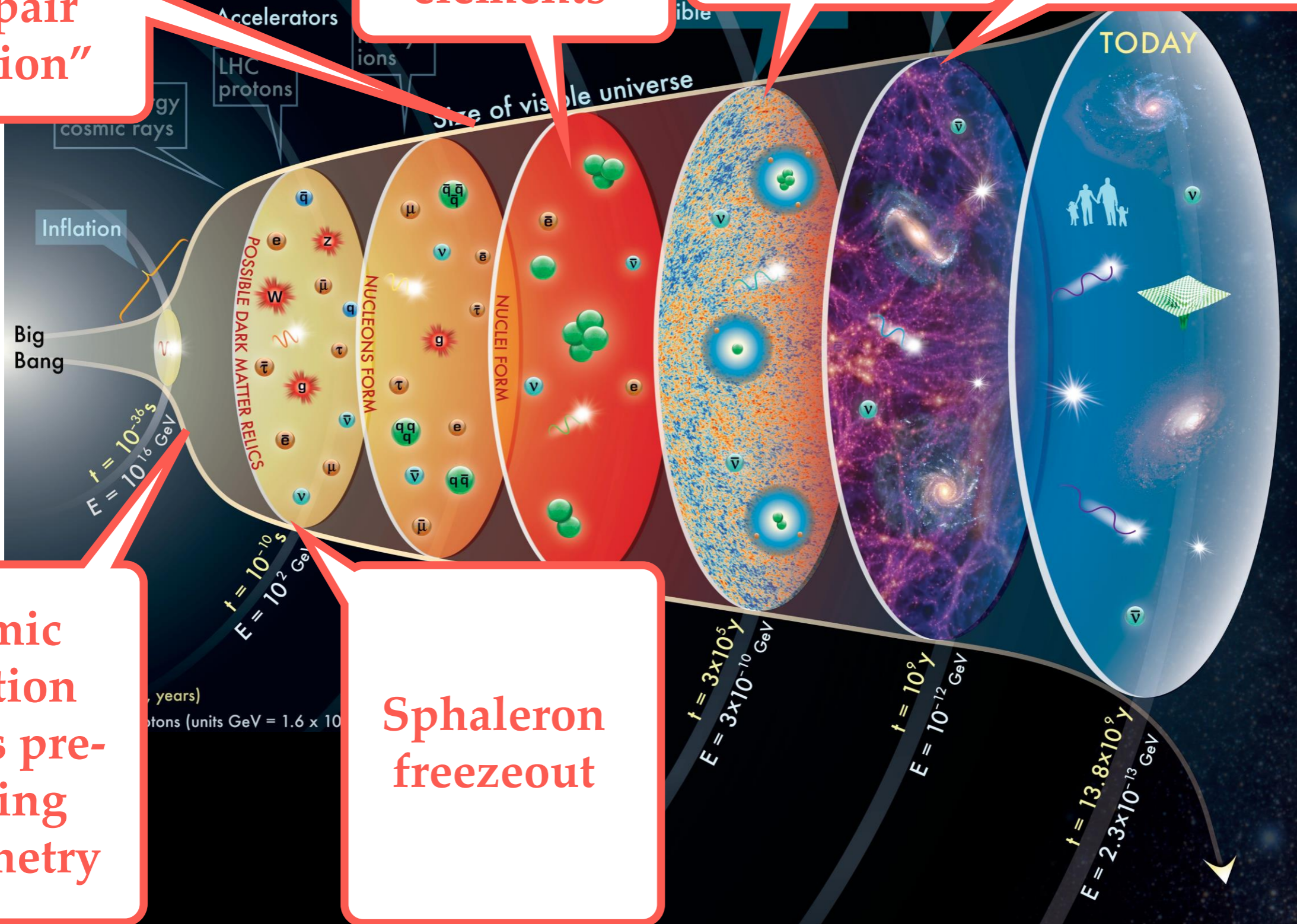
# COSMOLOGY OF

Hot enough to produce antimatter in "pair creation"

nuclear reactions form light elements

Cosmic Microwave Background

optical astronomy



Cosmic Inflation dilutes pre-existing asymmetry

Sphaleron freezeout

The concept for the above figure originated in a 1986 paper by Michael Turner.



# ORY OF

Hot enough to produce antimatter in "pair creation"

nuclear reactions form light elements

Cosmic Microwave Background

optical astronomy

**Baryon number diluted by inflation**

**Baryon number conserved**

Cosmic Inflation dilutes pre-existing asymmetry

Sphaleron freezeout



The concept for the above figure originated in a 1986 paper by Michael Turner.



ORY OF

Hot enough to produce antimatter in "pair creation"

nuclear reactions form light elements

Cosmic Microwave Background

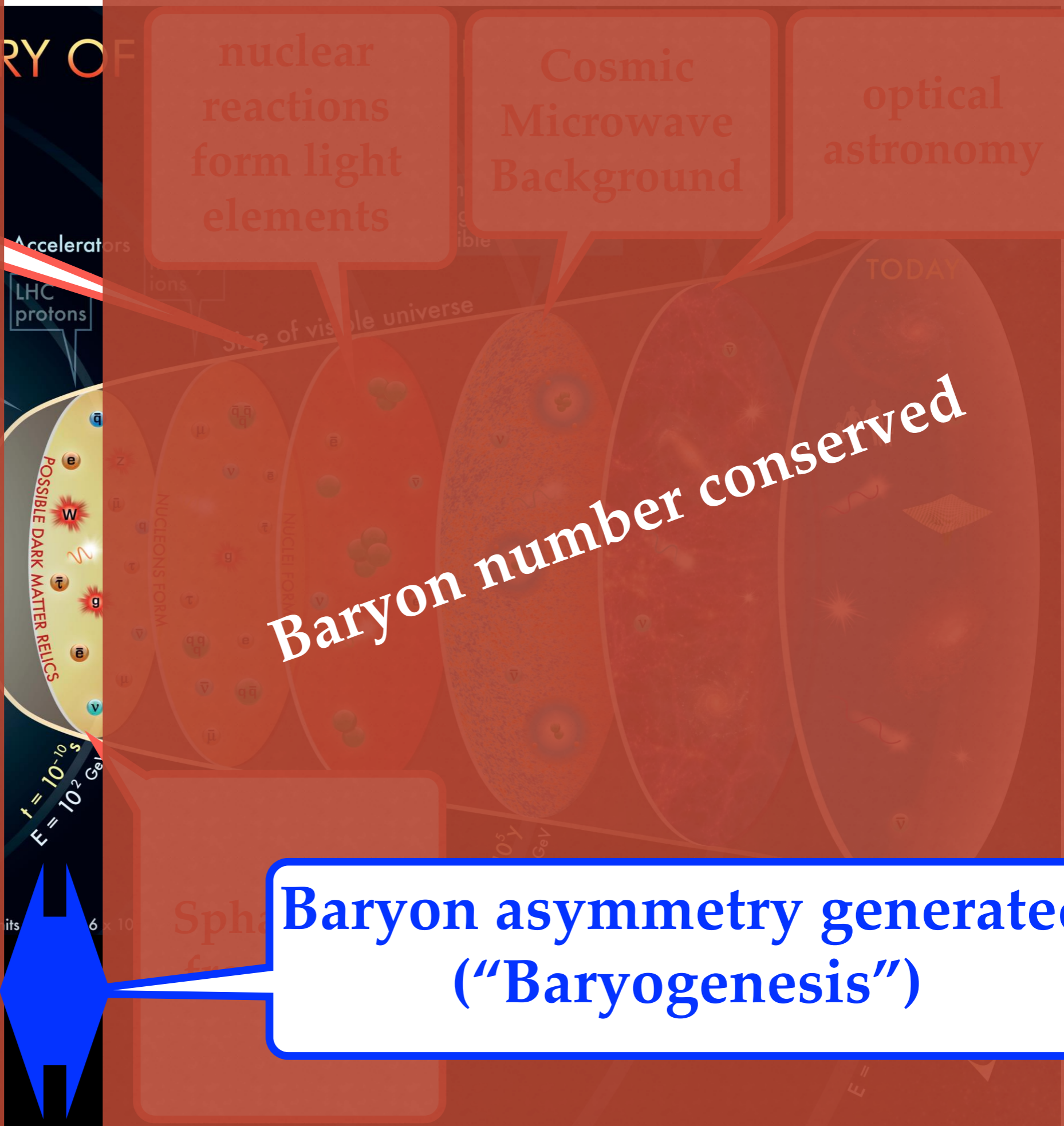
optical astronomy

Baryon number diluted by inflation

Baryon number conserved

Cosmic Inflation dilutes pre-existing asymmetry

Baryon asymmetry generated ("Baryogenesis")



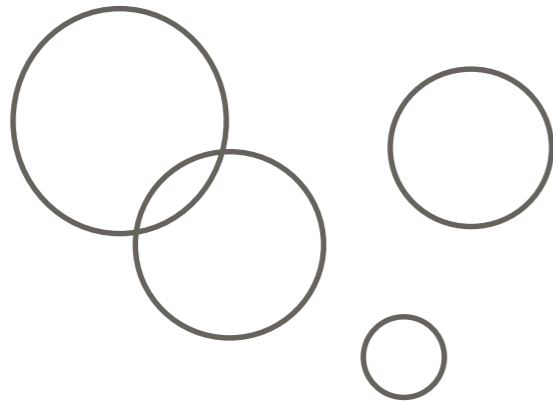
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# Where does the asymmetry come from?

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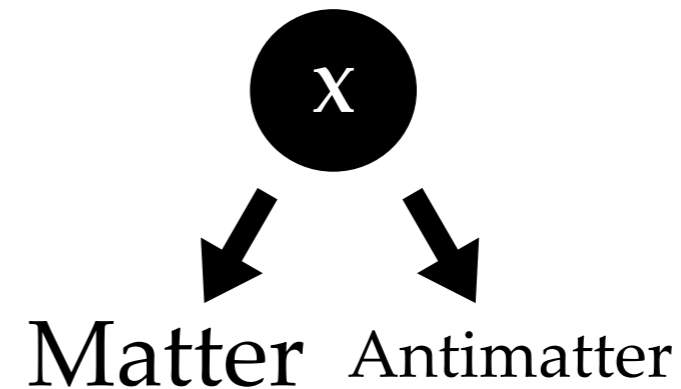
## Baryogenesis requires New Physics!

Cosmic phase transition?



Electroweak baryogenesis,  
...

Decay of a heavy particle?



GUT baryogenesis,  
leptogenesis,  
...

	Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 4.14$ )		Any Ordering
	bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range	$3\sigma$ range
$\sin^2 \theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \rightarrow 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \rightarrow 0.346$	$0.272 \rightarrow 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \rightarrow 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \rightarrow 36.06$	$31.42 \rightarrow 36.05$
$\sin^2 \theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \rightarrow 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \rightarrow 0.616$	$0.418 \rightarrow 0.613$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \rightarrow 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \rightarrow 51.7$	$40.3 \rightarrow 51.5$
$\sin^2 \theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \rightarrow 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \rightarrow 0.02452$	$0.01981 \rightarrow 0.02436$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \rightarrow 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \rightarrow 9.01$	$8.09 \rightarrow 8.98$
$\delta_{CP}/^\circ$	$234^{+43}_{-31}$	$144 \rightarrow 374$	$278^{+26}_{-29}$	$192 \rightarrow 354$	$144 \rightarrow 374$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \rightarrow 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \rightarrow 8.02$	$6.80 \rightarrow 8.02$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \rightarrow +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \rightarrow -2.369$	$\left[ +2.399 \rightarrow +2.593 \right]$ $\left[ -2.536 \rightarrow -2.395 \right]$

❖ C and CP violation

❖ Deviation from thermal equilibrium



---

# Leptogenesis

---

## General idea:

The matter-antimatter asymmetry was created amongst leptons and then transferred to baryons in the early universe.

## Original framework: Fukugita/Yanagida 86

Right handed neutrinos (type I seesaw)

## Keep in mind:

Many other neutrino mass models exist. And many of them can accommodate leptogenesis!

# Thermal Leptogenesis

## Basic idea

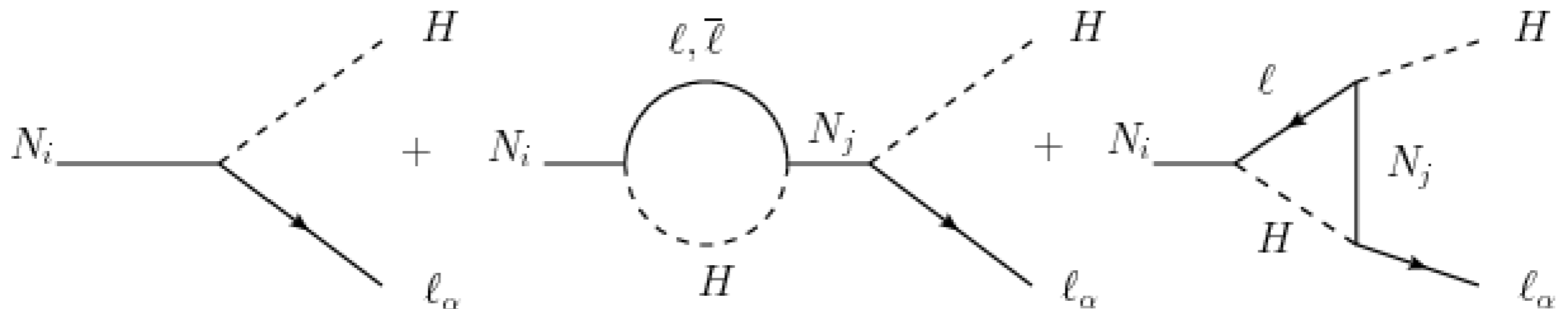
- $N$  are around in the early universe
- Yukawas  $F$  are CP violating
- $N$  may preferably decay into matter

CP violating parameter  $\epsilon$

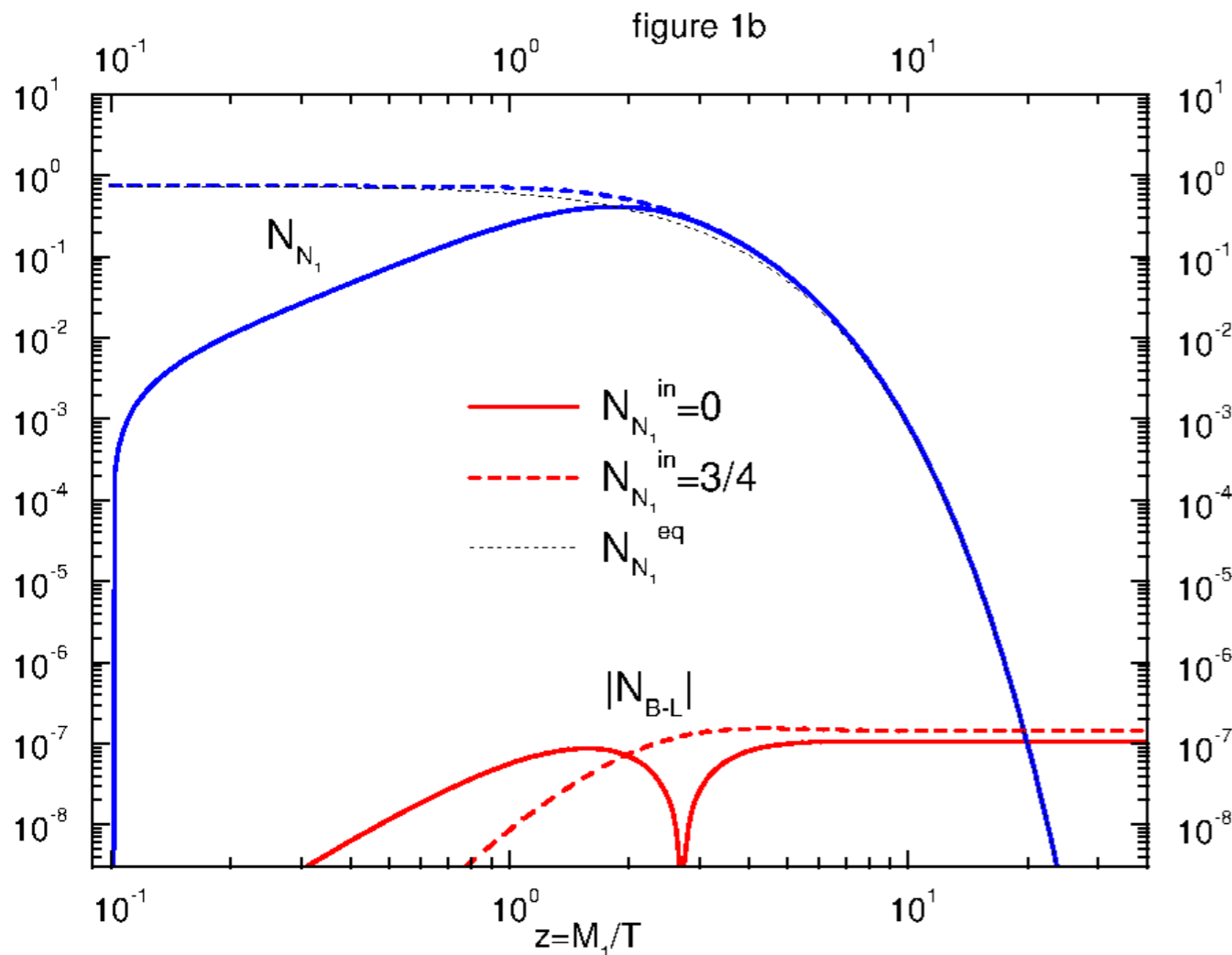
$$\epsilon = \frac{\Gamma_{N \rightarrow \ell H} - \Gamma_{N \rightarrow \bar{\ell} H^*}}{\Gamma_{N \rightarrow \ell H} + \Gamma_{N \rightarrow \bar{\ell} H^*}}$$

final asymmetry

$$Y_{B-L} \propto \epsilon/g_*$$



# Boltzmann Equation



$$\epsilon = \frac{\Gamma_{N \rightarrow \ell H} - \Gamma_{N \rightarrow \bar{\ell} H^*}}{\Gamma_{N \rightarrow \ell H} + \Gamma_{N \rightarrow \bar{\ell} H^*}}$$

Evolution of heavy neutrinos and asymmetries is described by Boltzmann equations

$$xH \frac{dY_N}{dx} = -\Gamma_N (Y_N - Y_N^{eq}) \quad x = M/T$$

$$xH \frac{dY_{B-L}}{dx} = \underbrace{\epsilon \Gamma_N (Y_N - Y_N^{eq})}_{\text{"source"}} - \underbrace{c_W \Gamma_N Y_{B-L}}_{\text{"washout"}}$$



# “Vanilla Leptogenesis”

Temperature  $T > 10^{12}$  GeV

- gauge interactions in equilibrium
- charged lepton Yukawa interactions slower than cosmic expansion

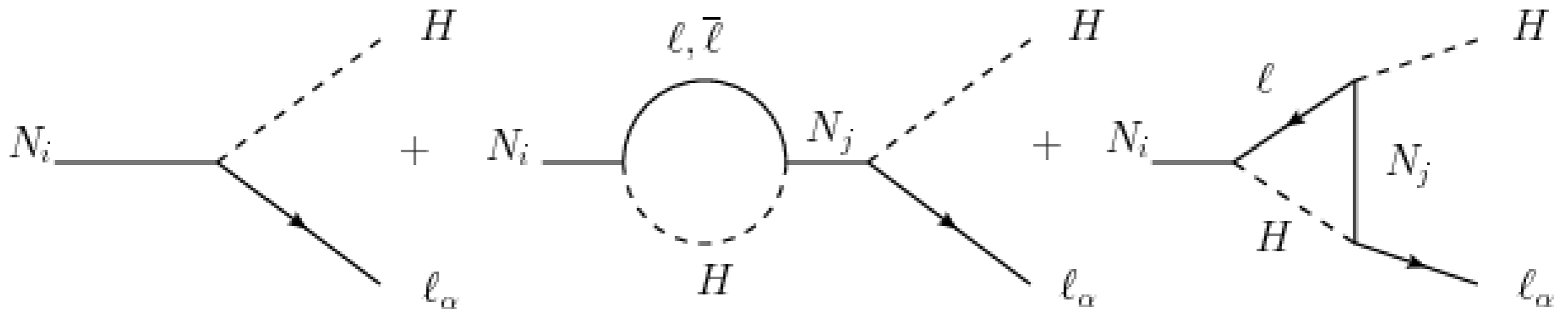
SM flavours indistinguishable!

CP violating parameter  $\epsilon$

$$\epsilon = \frac{\Gamma_{N \rightarrow \ell H} - \Gamma_{N \rightarrow \bar{\ell} H^*}}{\Gamma_{N \rightarrow \ell H} + \Gamma_{N \rightarrow \bar{\ell} H^*}}$$

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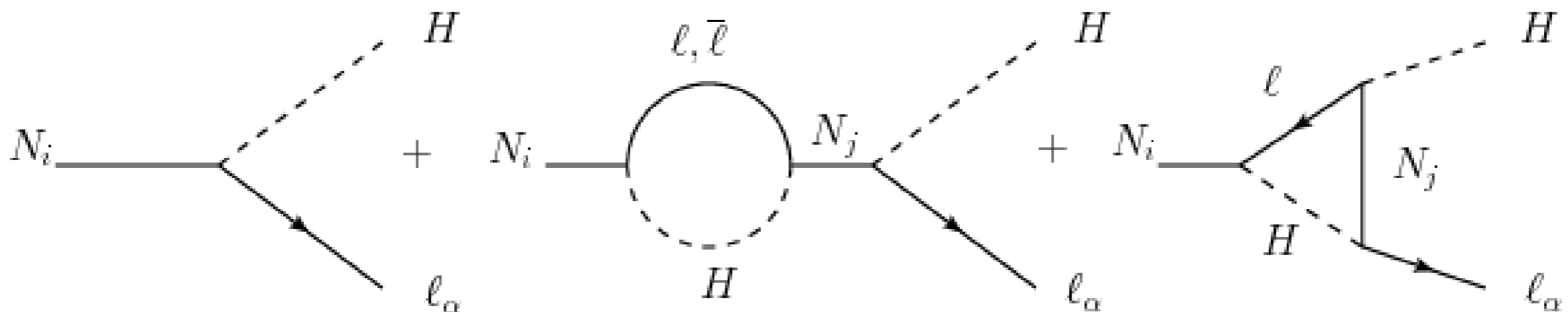
final asymmetry

$$Y_{B-L} \propto \epsilon/g_*$$

Casas-Ibarra parameters for Yukawa coupling

$$F = \frac{1}{v} U_\nu \sqrt{m_\nu^{\text{diag}}} \mathcal{R} \sqrt{M^{\text{diag}}} \begin{matrix} \text{heavy neutrino} \\ \text{parameters} \end{matrix}$$

light neutrino parameters



# “Vanilla Leptogenesis”

Temperature  $T > 10^{12}$  GeV

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- charged lepton Yukawa interactions slower than cosmic expansion

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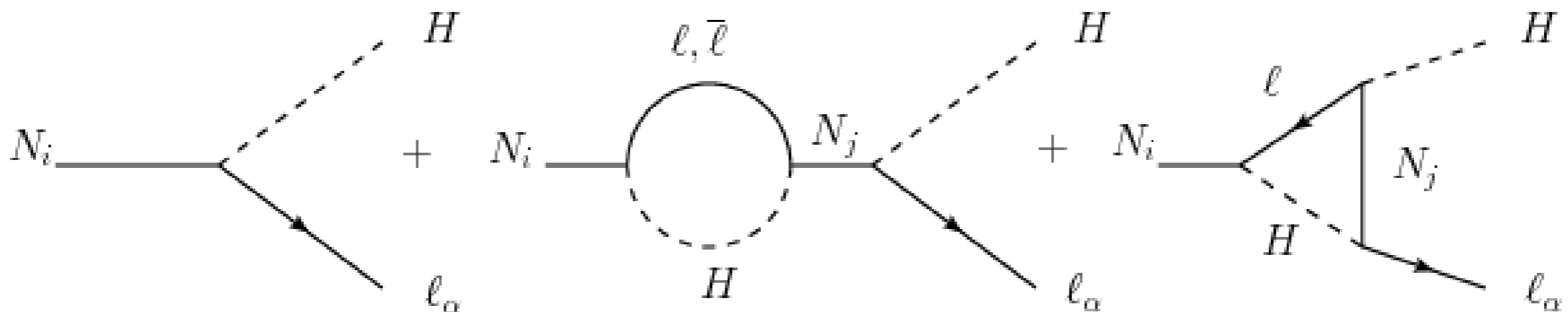
CP violating parameter  $\epsilon$

$$\epsilon = \frac{\Gamma_{N \rightarrow \ell H} - \Gamma_{N \rightarrow \bar{\ell} H^*}}{\Gamma_{N \rightarrow \ell H} + \Gamma_{N \rightarrow \bar{\ell} H^*}}$$

final asymmetry

$$Y_{B-L} \propto \epsilon/g_*$$

Asymmetry only depends on the combination  $F^\dagger F$   $\epsilon \simeq -\frac{3}{16\pi} \frac{1}{(F^\dagger F)_{11}} \sum_I \text{Im} [(F^\dagger F)_{I1}]^2 \frac{M_1}{M_I}$





# “Vanilla Leptogenesis”

Temperature  $T > 10^{12}$  GeV

- gauge interactions in equilibrium
- charged lepton Yukawa interactions slower than cosmic expansion

SM flavours

indistinguishable!

CP violating parameter  $\epsilon$

$$\epsilon = \frac{\Gamma_{N \rightarrow \ell H} - \Gamma_{N \rightarrow \bar{\ell} H^*}}{\Gamma_{N \rightarrow \ell H} + \Gamma_{N \rightarrow \bar{\ell} H^*}}$$

final asymmetry

$$Y_{B-L} \propto \epsilon/g_*$$

Asymmetry only depends on the combination  $F^\dagger F$

$$\epsilon \simeq -\frac{3}{16\pi} \frac{1}{(F^\dagger F)_{11}} \sum_I \text{Im} \left[ (F^\dagger F)_{I1} \right]^2 \frac{M_1}{M_I}$$

But  $F = \frac{1}{v} U_\nu \sqrt{m_\nu^{\text{diag}}} \mathcal{R} \sqrt{M^{\text{diag}}}$

gives  $F^\dagger F = \sqrt{M^{\text{diag}}} \mathcal{R}^\dagger \frac{m_\nu^{\text{diag}}}{v^2} \mathcal{R} \sqrt{M^{\text{diag}}}$

$\epsilon$  is independent of the PMNS matrix!

So is the asymmetry...

# “Flavoured Leptogenesis”

$10^{12} \text{ GeV} > T > 1 \text{ TeV}$

- gauge interactions in equilibrium
- charged lepton Yukawa interactions faster than cosmic expansion

**SM flavours distinguishable!**

CP violating parameters  $\epsilon_{\alpha\beta}$

Lepton number matrix  $(Y_{\text{B-L}})_{\alpha\beta}$

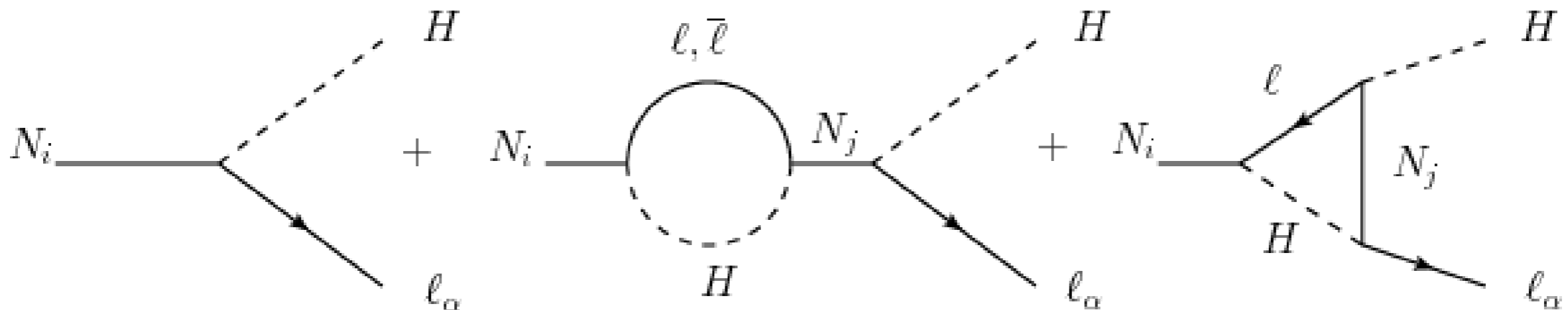
$(Y_{\text{B-L}})_{\alpha\alpha}$  is charge in flavour  $\alpha$

Flavoured asymmetries depend on individual  $F_{\alpha I}$

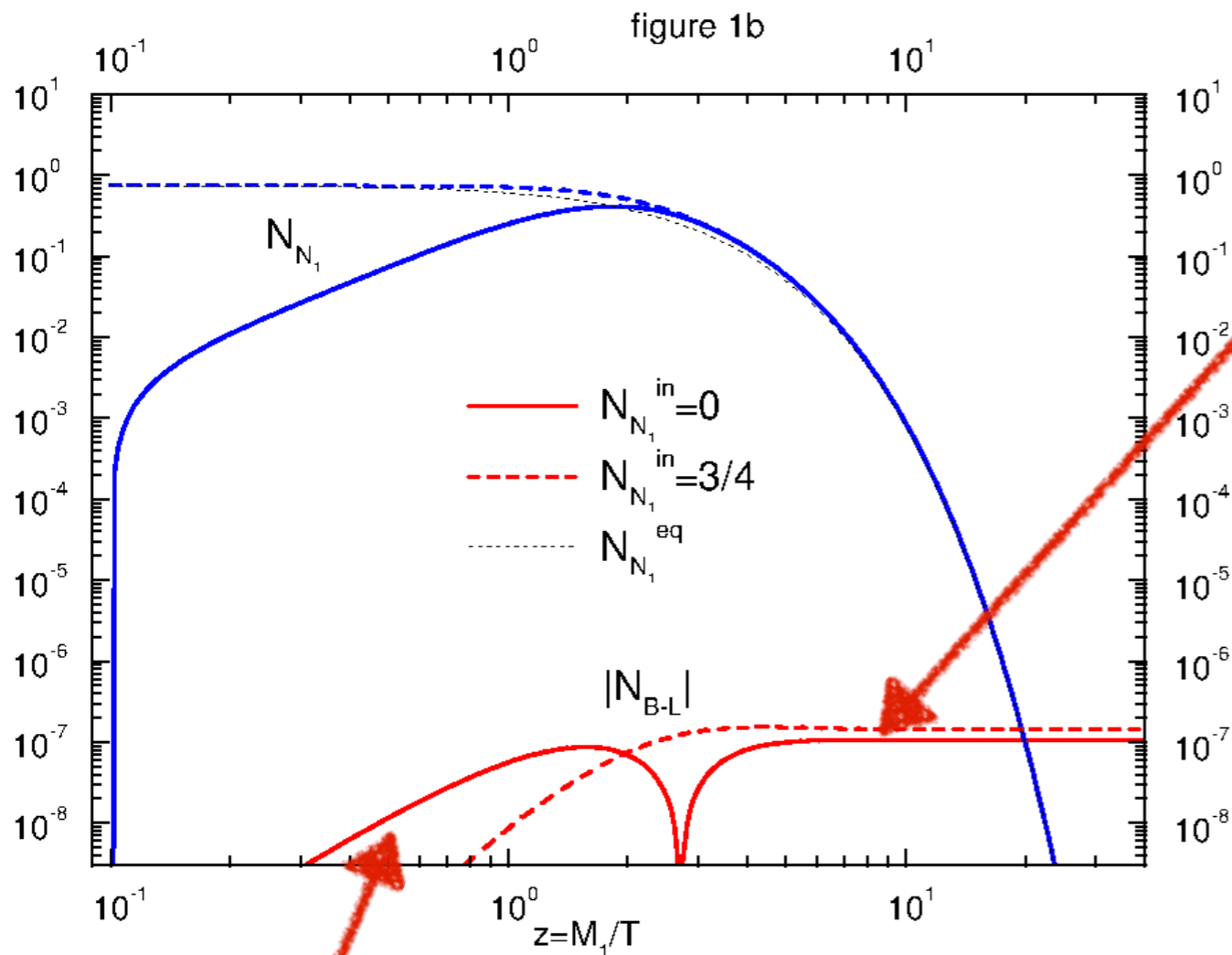


Baryon asymmetry depends on  $\delta$ !

... $\delta$  alone is even sufficient



# Leptogenesis with small M ?



asymmetry generated during  $N$  decay ("freeze-out scenario")

Evolution of heavy neutrinos and asymmetries is described by Boltzmann equations

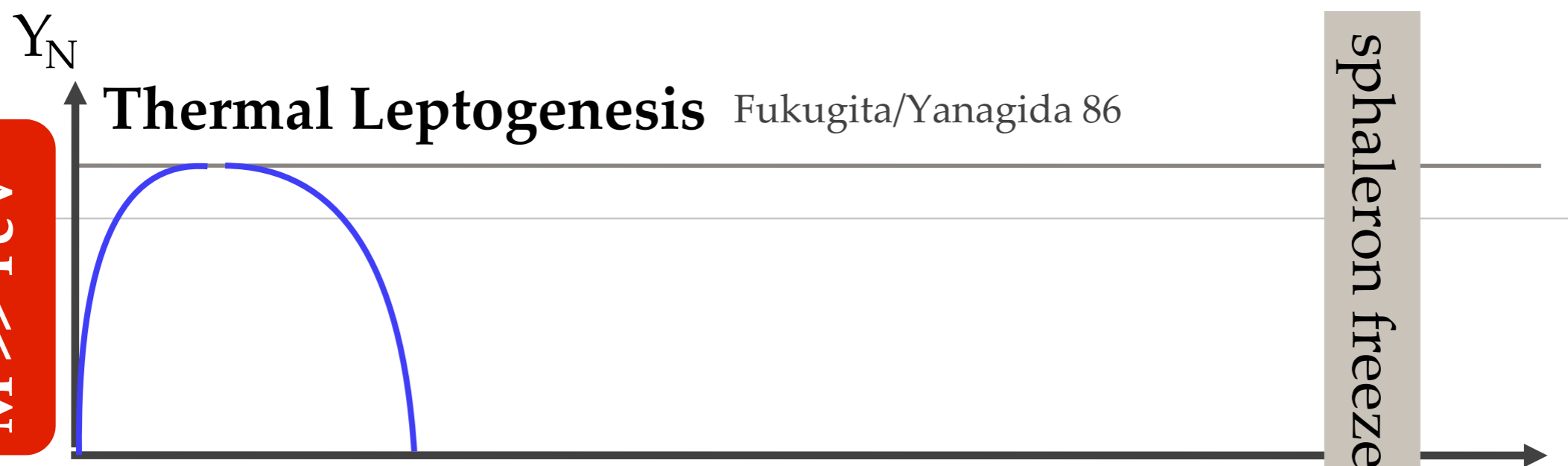
asymmetry generated during  $N$  production ("freeze-in scenario")

$$xH \frac{dY_N}{dx} = -\Gamma_N (Y_N - Y_N^{eq}) \quad x = M/T$$

$$xH \frac{dY_{B-L}}{dx} = \underbrace{\epsilon \Gamma_N (Y_N - Y_N^{eq})}_{\text{"source"}} - \underbrace{c_W \Gamma_N Y_{B-L}}_{\text{"washout"}}$$

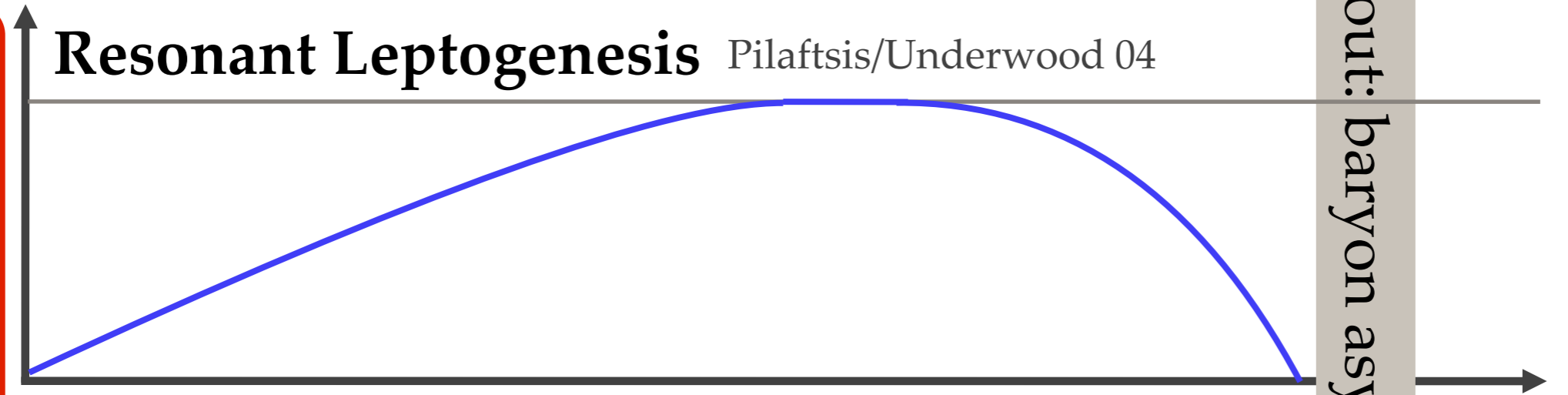


high scale  
 $M \gg \text{TeV}$

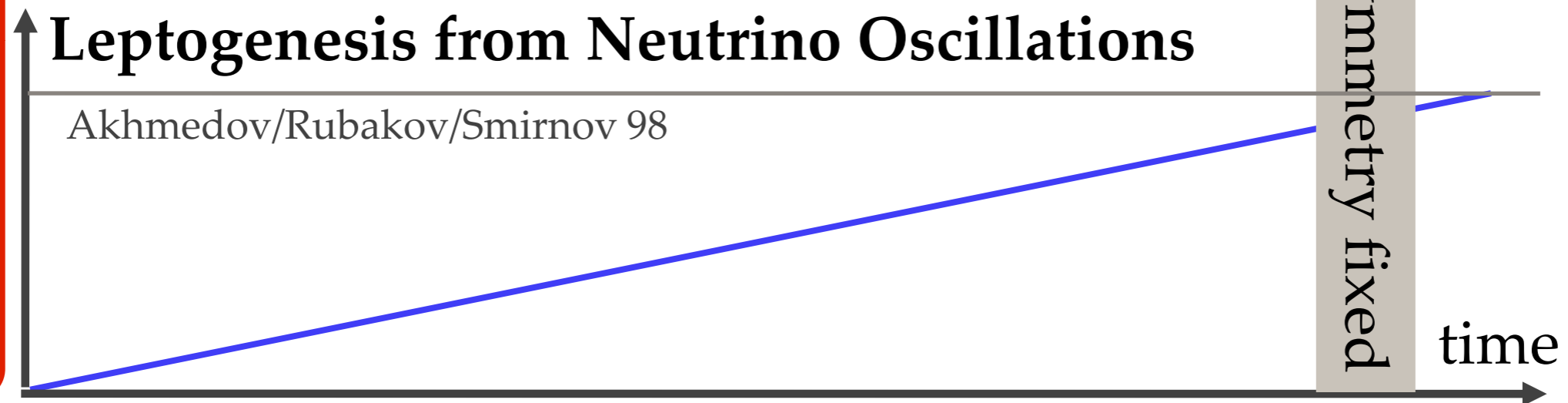


asymmetry generated in  
freeze-out and decay

low scale  
 $M < \text{TeV}$



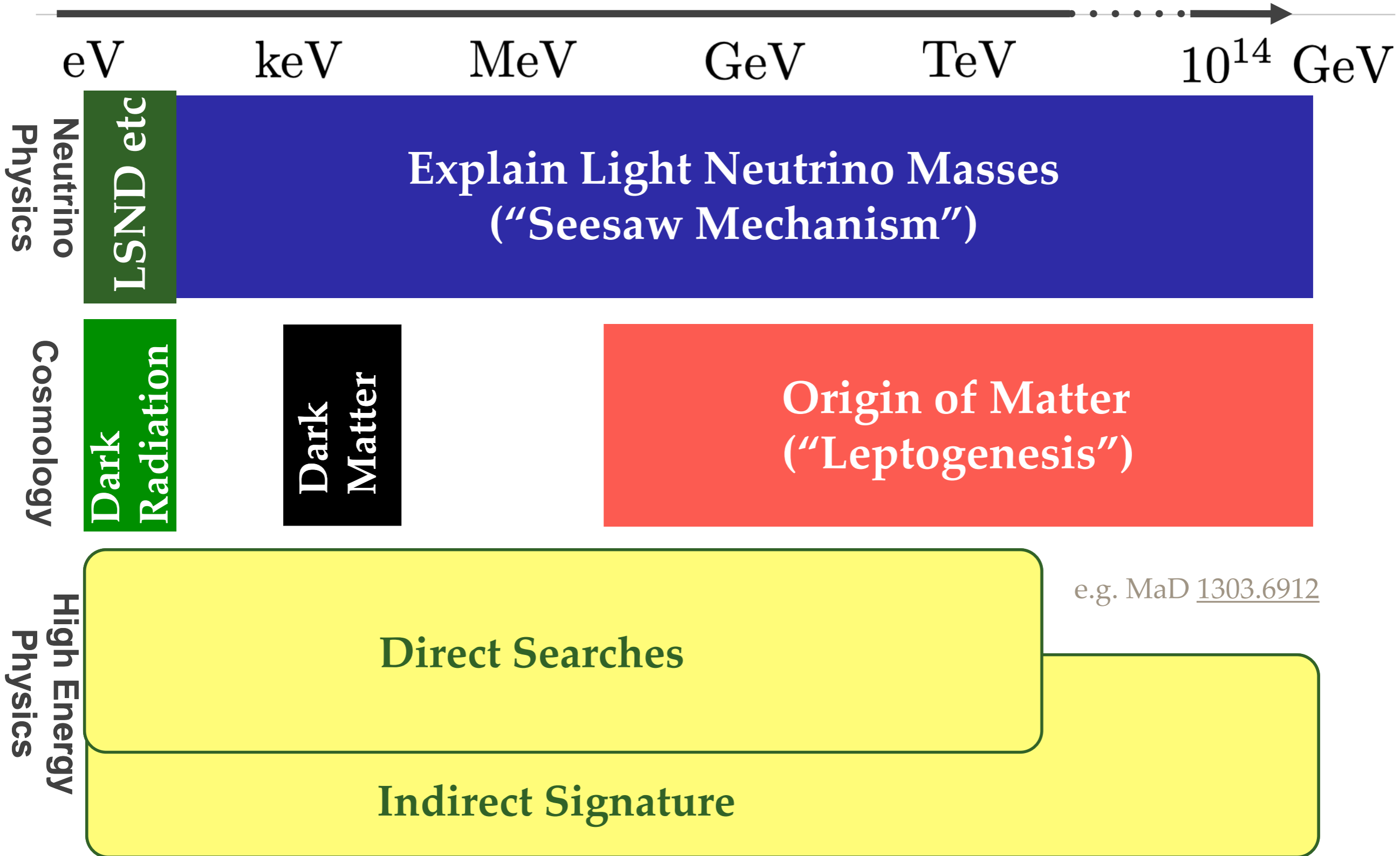
asymmetry  
generated in  
freeze-in



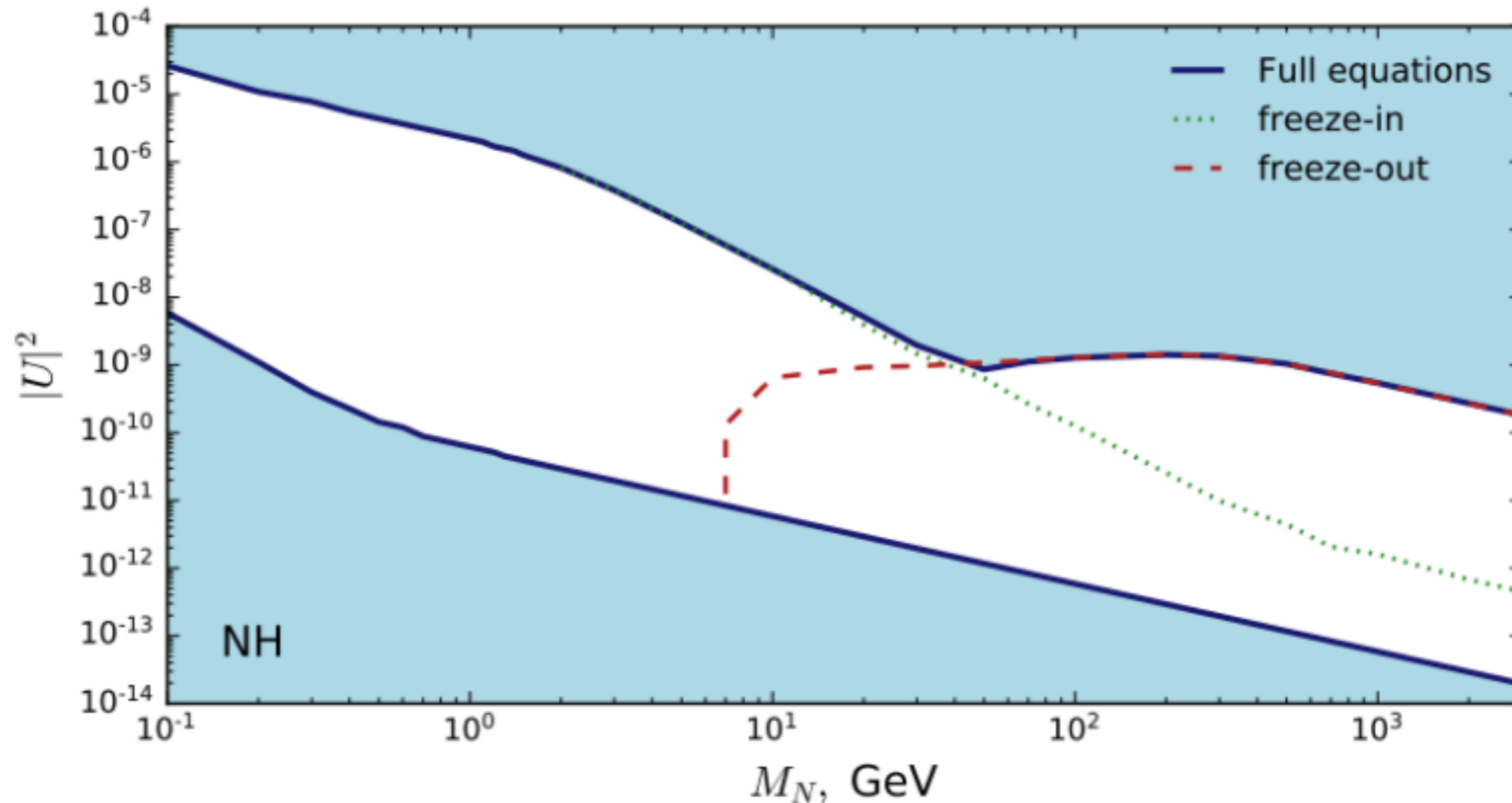
"big bang"

$T = 130 \text{ GeV}$

# Heavy Neutrino Mass Scale



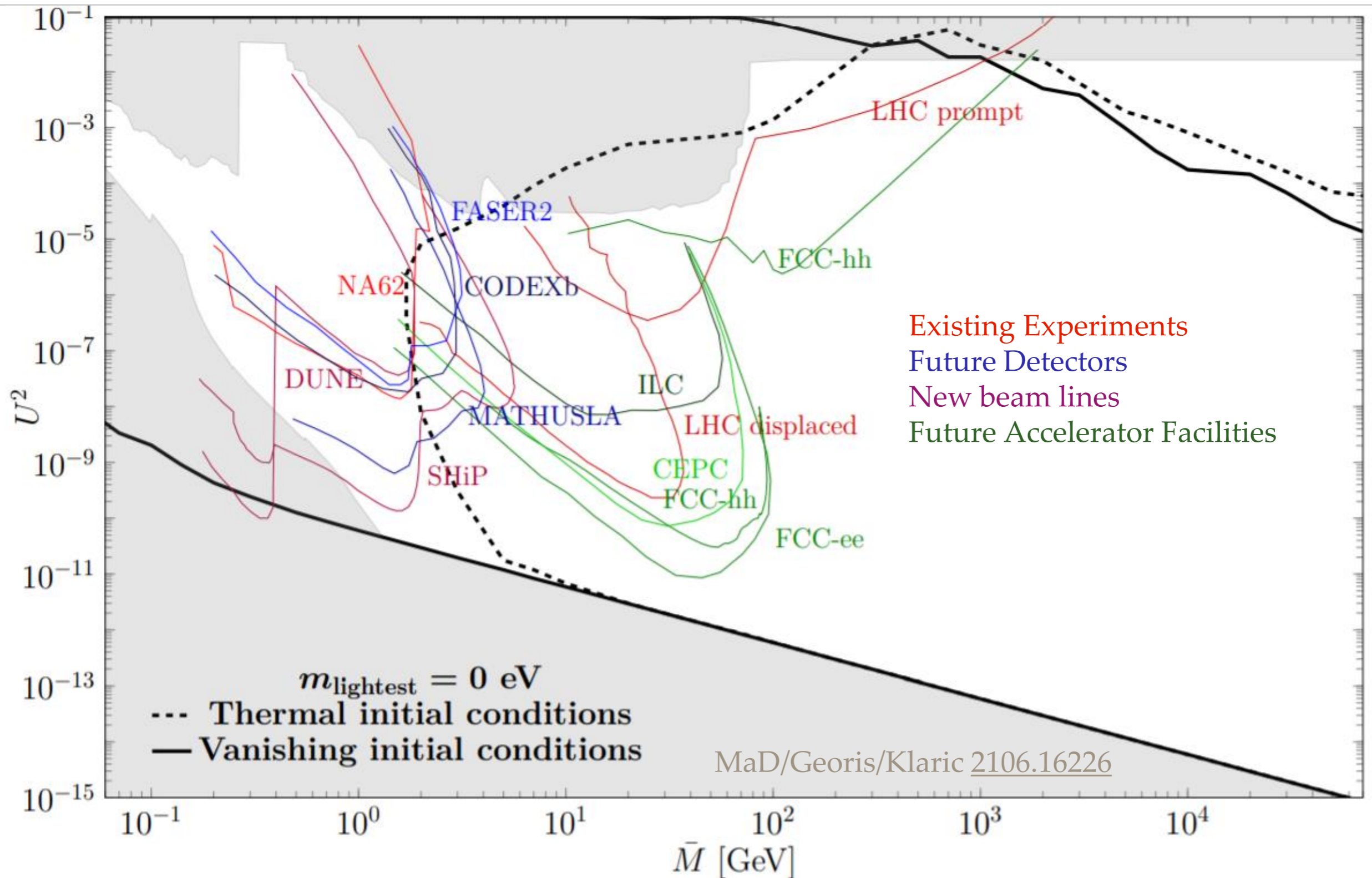
# Leptogenesis with 2 RH Neutrinos



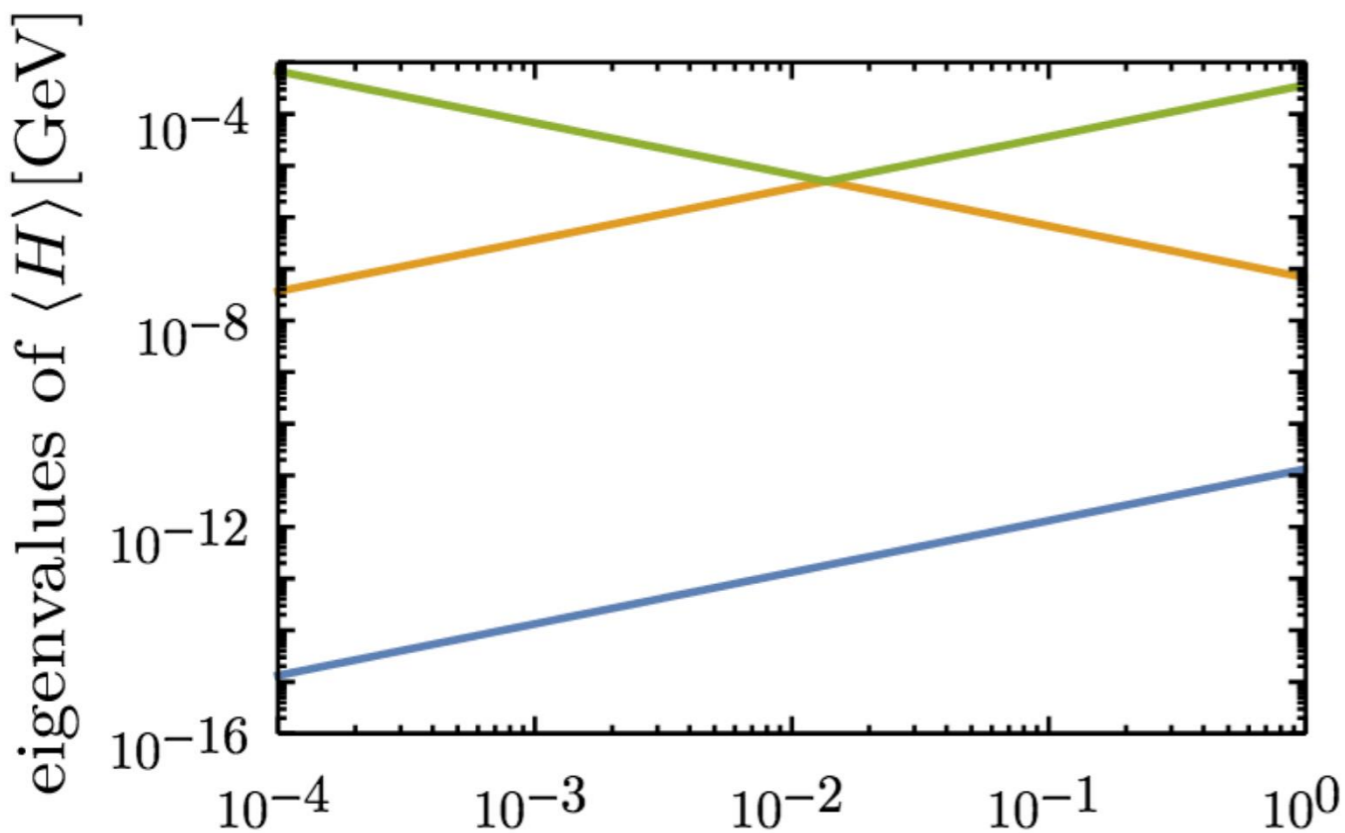
The region in which the freeze-out scenario (“resonant leptogenesis”) and freeze-in scenario (“ARS leptogenesis”) work overlap!



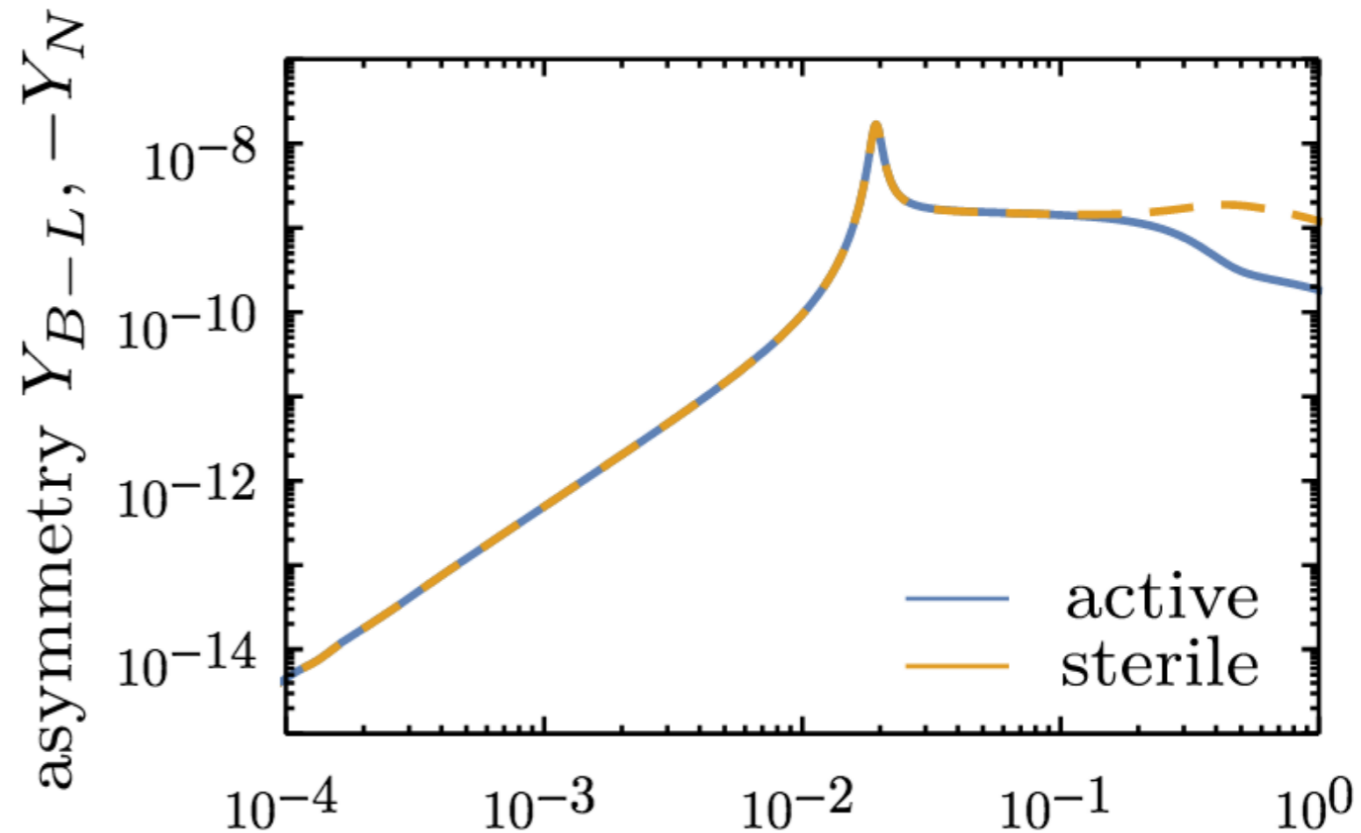
# Leptogenesis with 3 RH Neutrinos



# Dynamical Generation of Resonance



$$x = T_{EW}/T$$



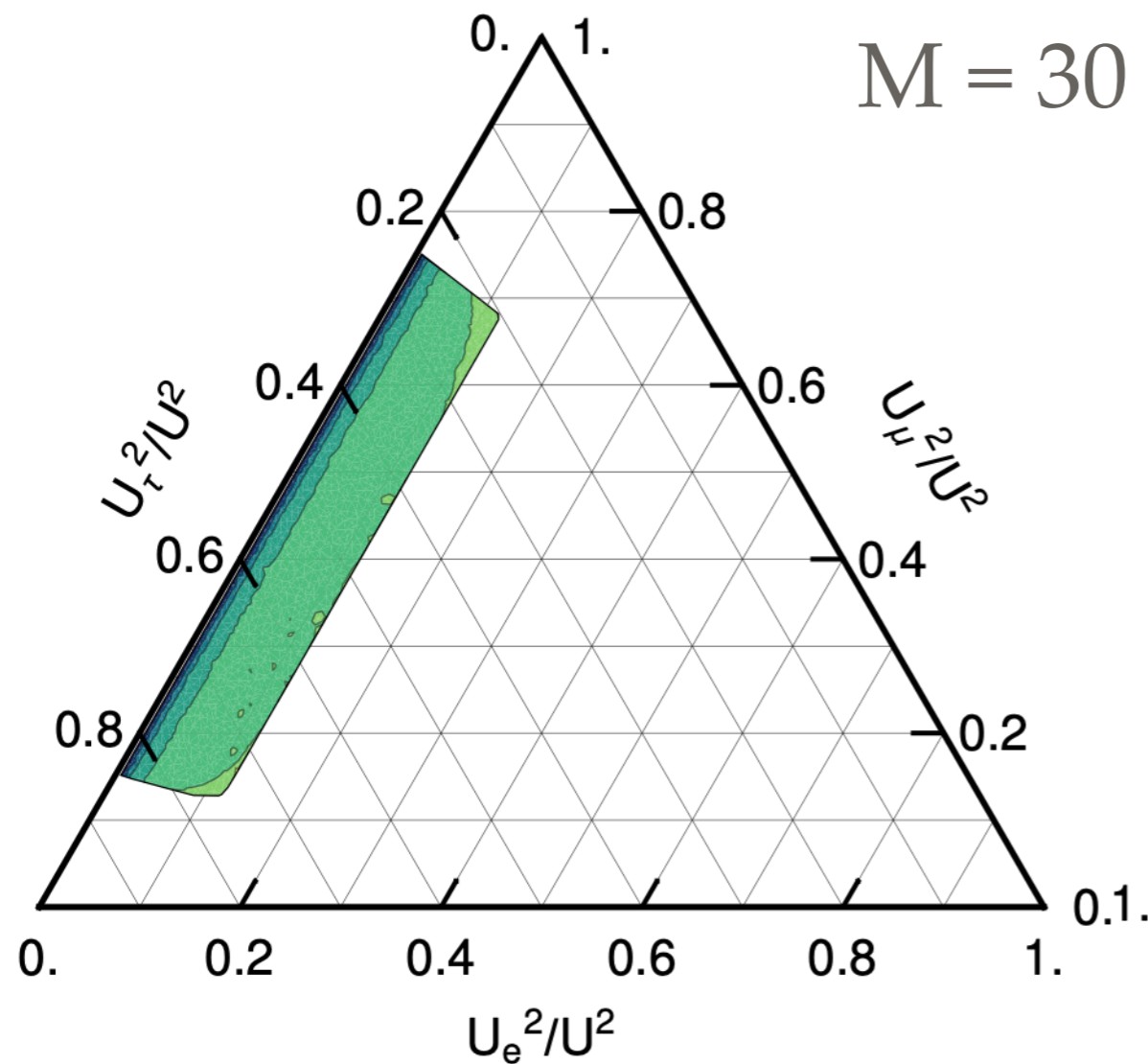
$$x = T_{EW}/T$$

Abada et al [1810.12463](https://arxiv.org/abs/1810.12463)

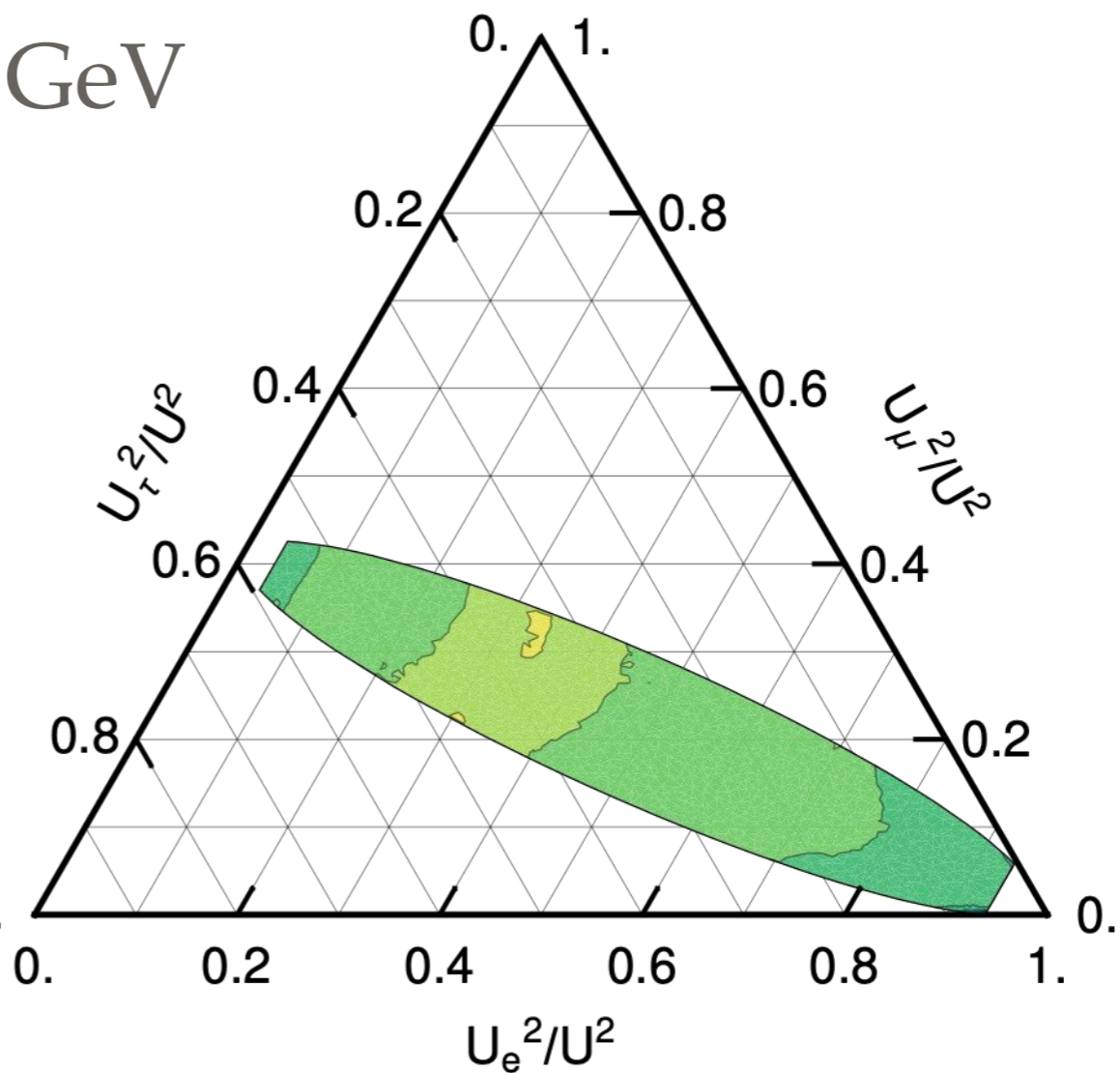
- quantum kinetic equations describe screening, quantum statistics and (de)coherence etc.
- level crossing between the quasiparticle dispersion relations in the plasma (“thermal masses”) can dynamically generate a resonance

# Current Status: Constraints from Leptogenesis

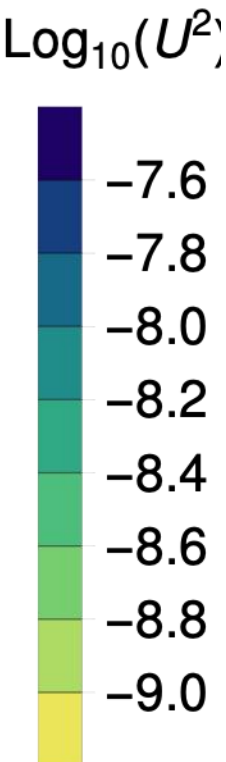
$M = 30 \text{ GeV}$



normal neutrino mass ordering



inverted neutrino mass ordering



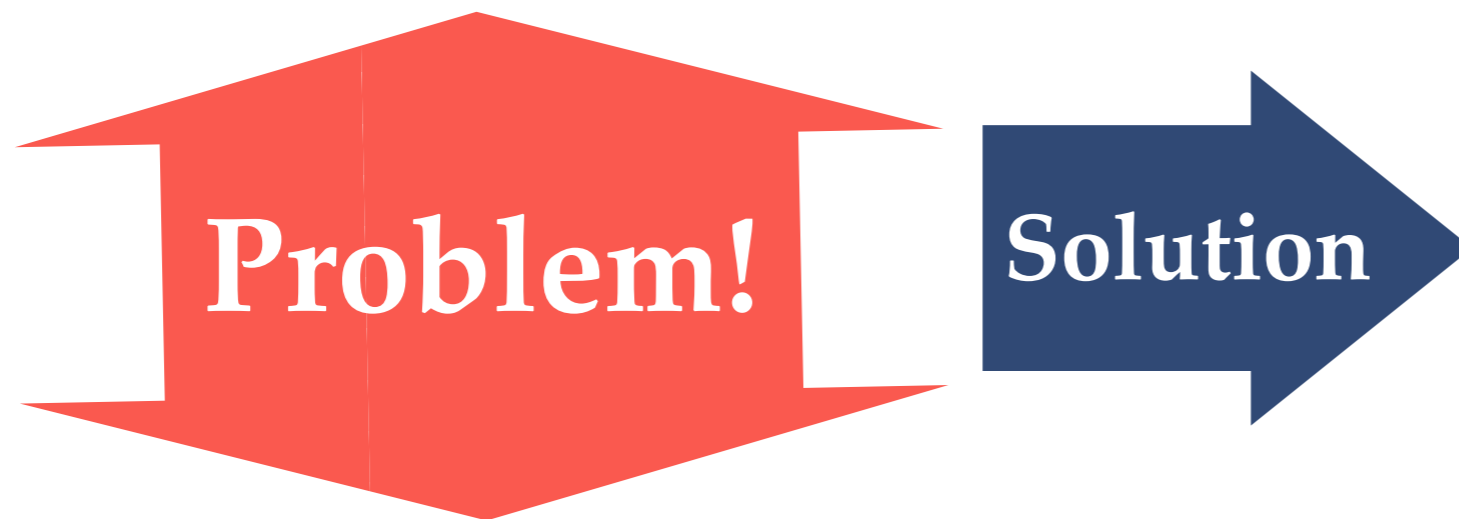
The dark regions are preferred for testable leptogenesis



# Neutrino masses vs collider searches

neutrino masses  $m_i$  are small (sub eV)

→ active-sterile mixing angle  $\theta$  must be small



approximate  
B-L  
conservation

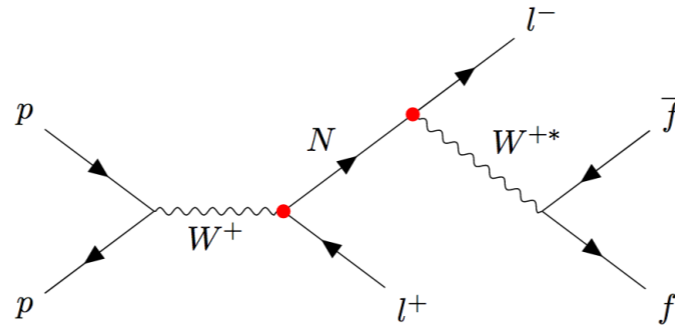
Shaposhnikov [0605047](#)  
Kersten/Smirnov [0705.3221](#)

colliders rely on branching ratio

→ active-sterile mixing angle  $\theta$  must be large

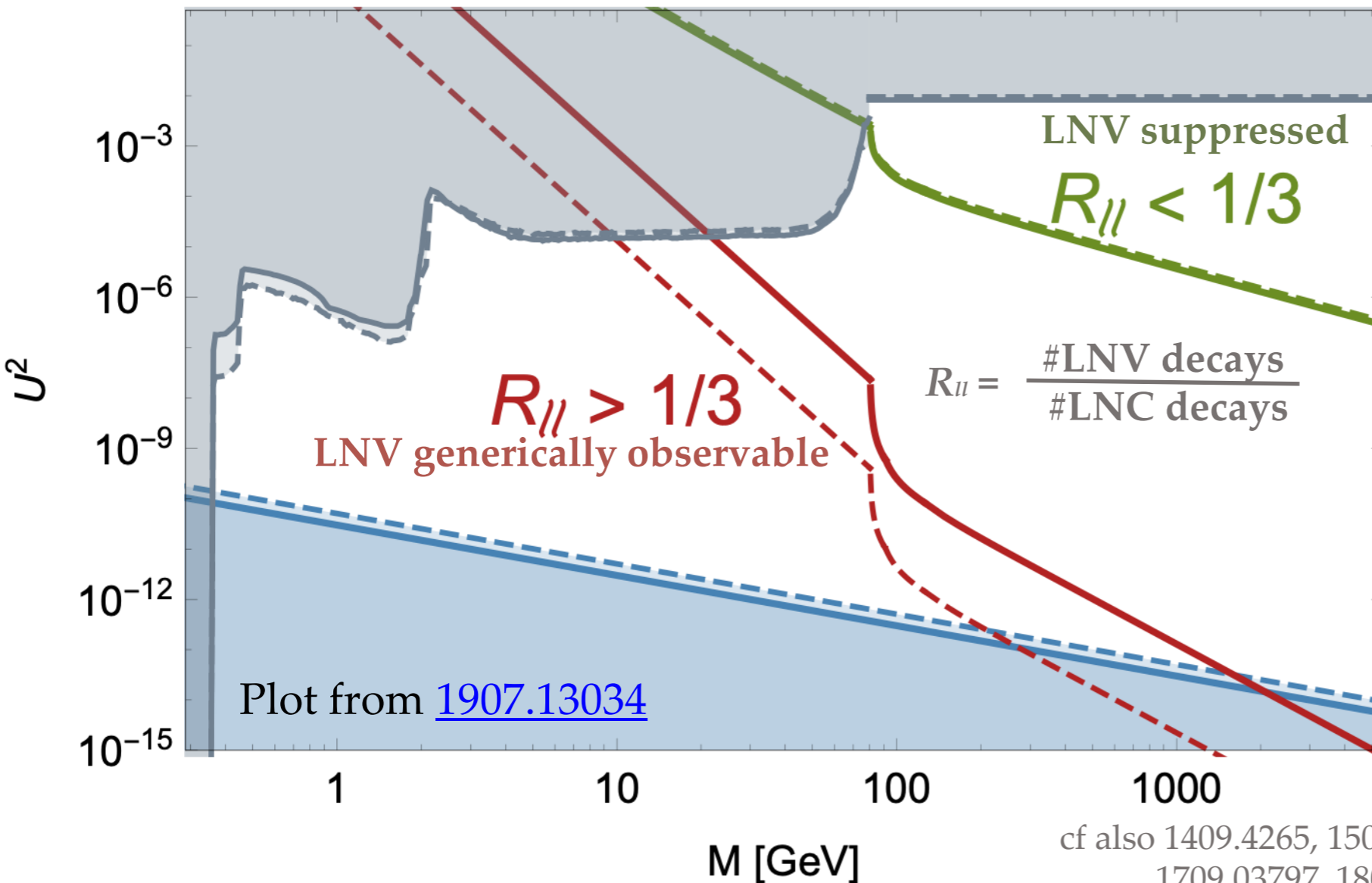
# Can LNV be observed?

**B-L symmetry: destructive interference amongst contributions from different HNL flavour**



(a) Charged current decay

**But: B-L is broken to generate neutrino mass. Is this enough???**



**HNL oscillations destroy coherence and make LNV observable!**

Anamiati et al [1607.05641](#)

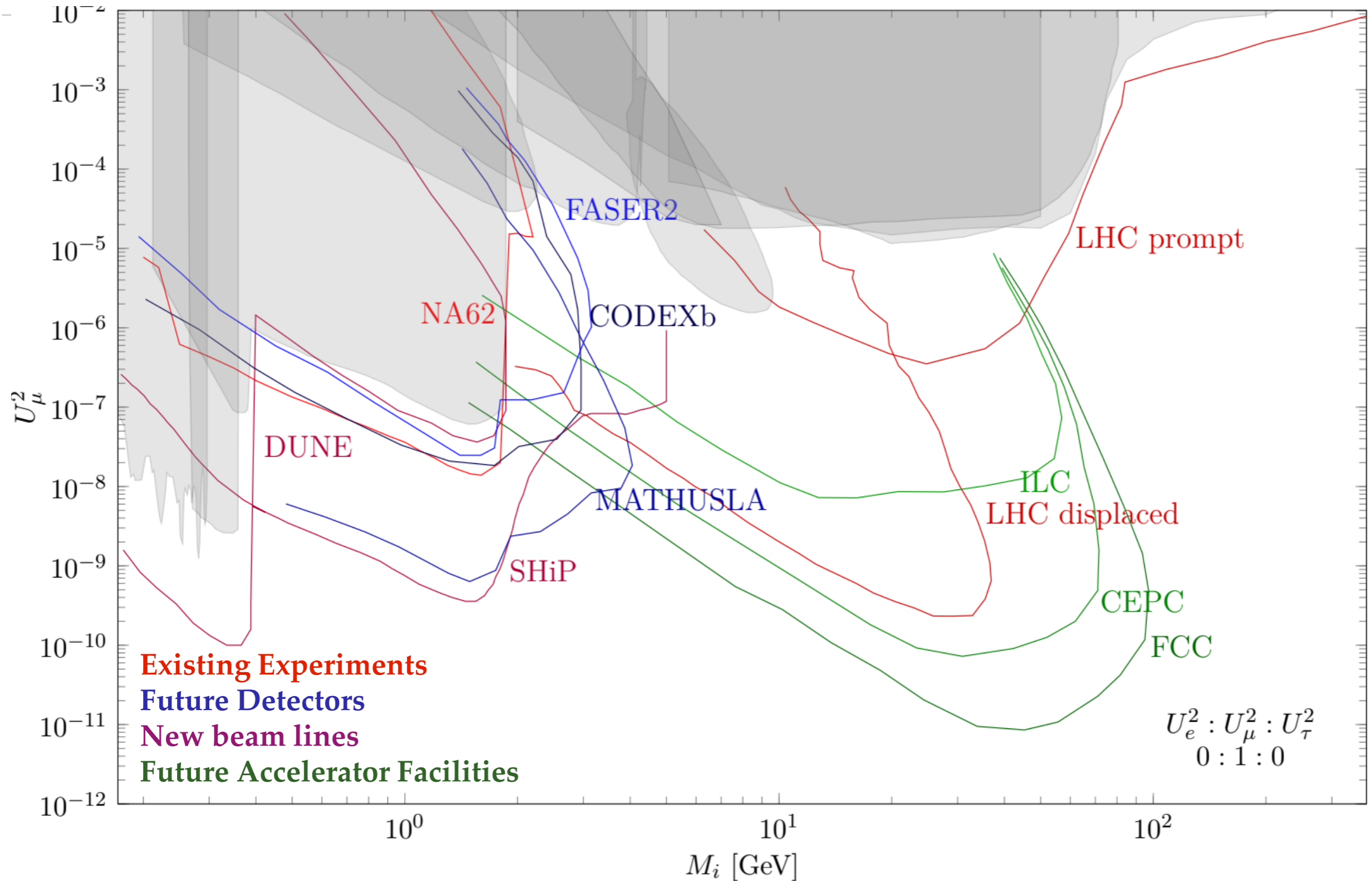
$$\mathcal{R}_{ll} = \frac{\Delta M_{\text{phys}}^2}{2\Gamma_N^2 + \Delta M_{\text{phys}}^2}$$

**Does neutrino osc. data allow for this without fine tuning? It depends**

MaD/Klaric/Klose [1907.13034](#)

cf also [1409.4265](#), [1505.04749](#), [1605.01123](#), [1709.06553](#), [1703.01934](#), [1709.03797](#), [1805.00070](#), [1810.07210](#), [1905.03097](#), [1904.05367](#)

# Heavy Neutrino Searches



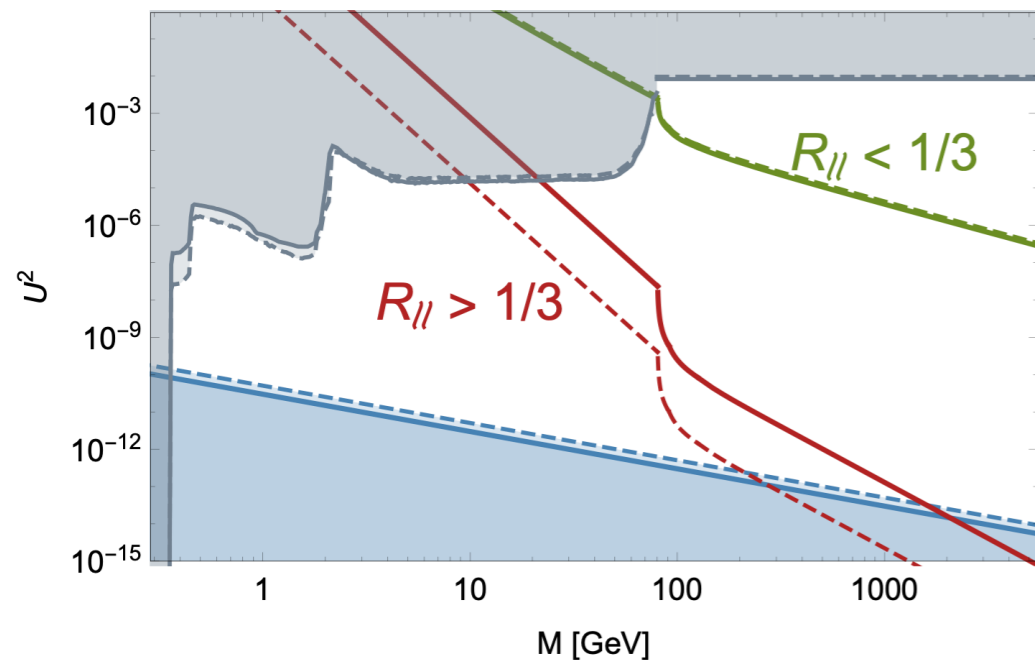


# How to measure $\Delta M$ ?

ratio of LNV to LNC decays  
is sensitive to  $\Delta M$

$$\mathcal{R}_{ll} = \frac{\Delta M_{\text{phys}}^2}{2\Gamma_N^2 + \Delta M_{\text{phys}}^2}$$

Anamiati et al [1607.05641](#)



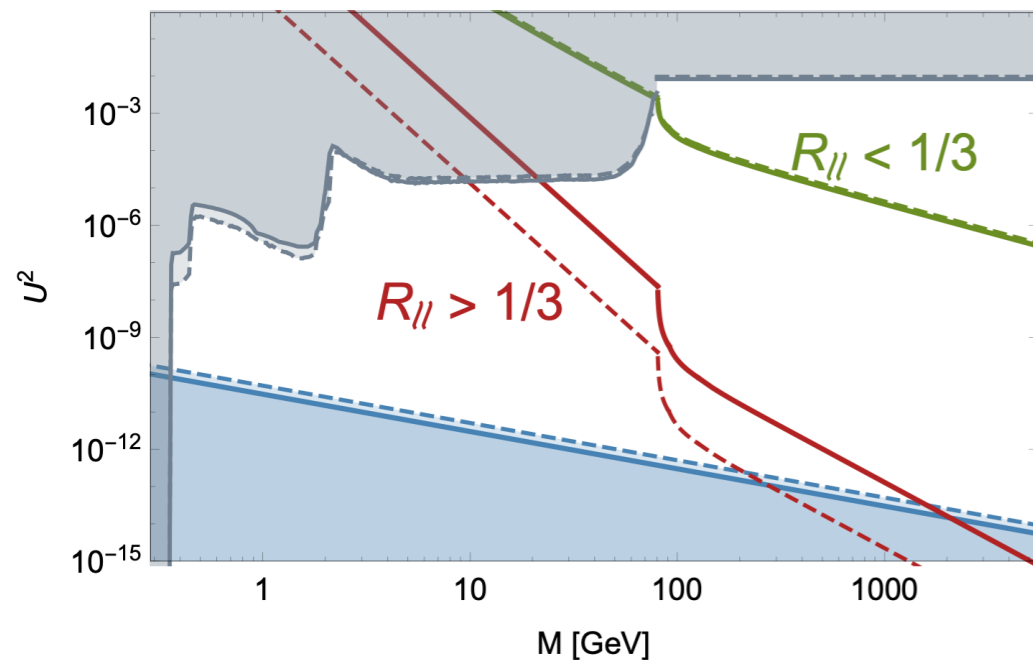
MaD/Klaric/Klose [1907.13034](#)

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$$\mathcal{R}_{ll} = \frac{\Delta M_{\text{phys}}^2}{2\Gamma_N^2 + \Delta M_{\text{phys}}^2}$$

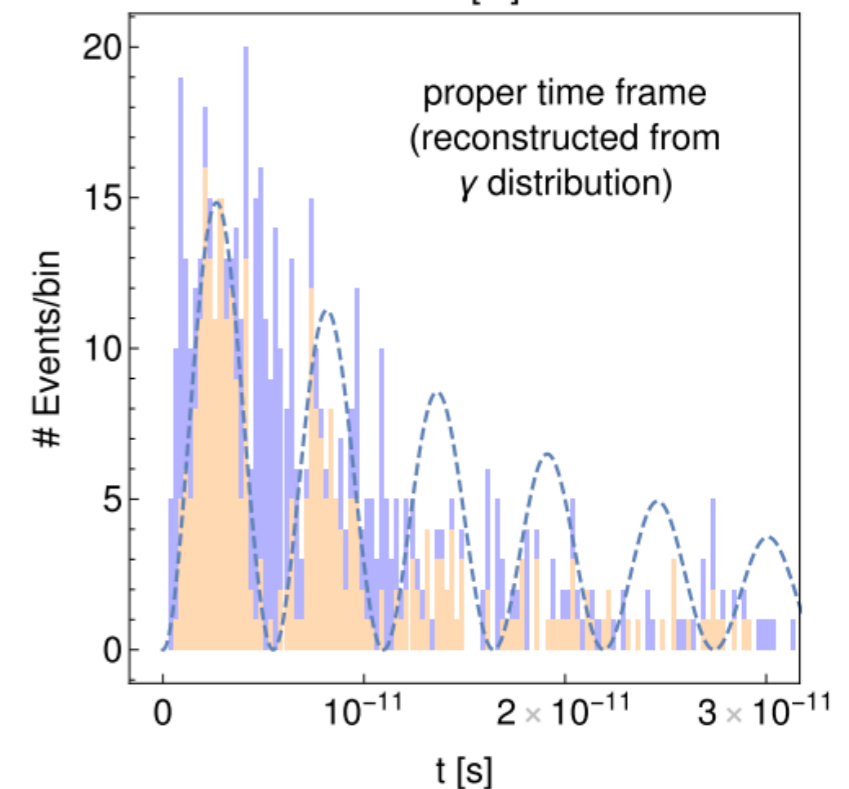
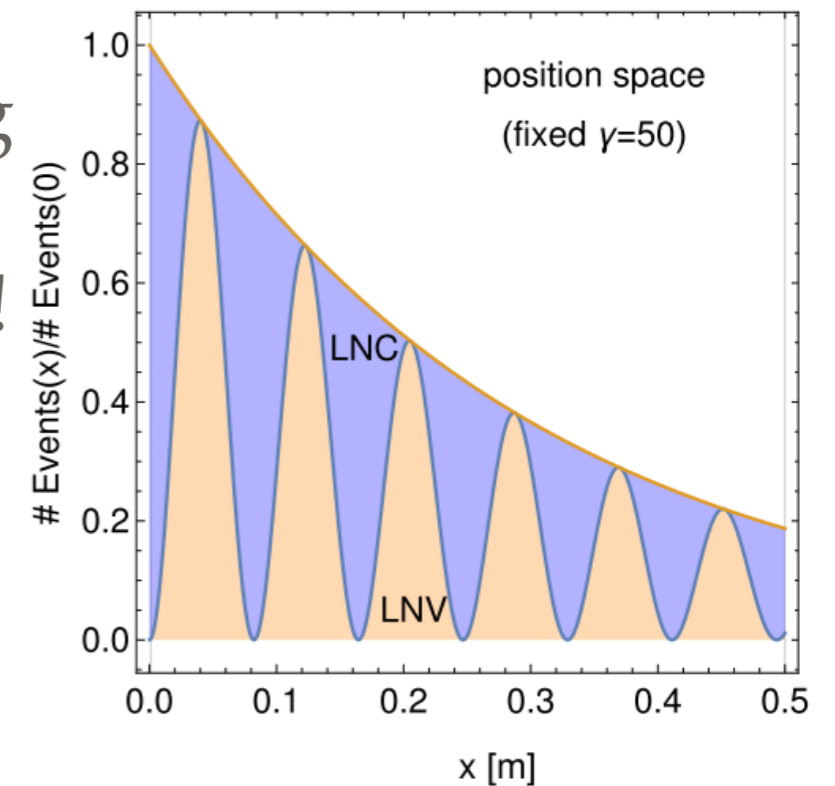
Anamiati et al [1607.05641](#)



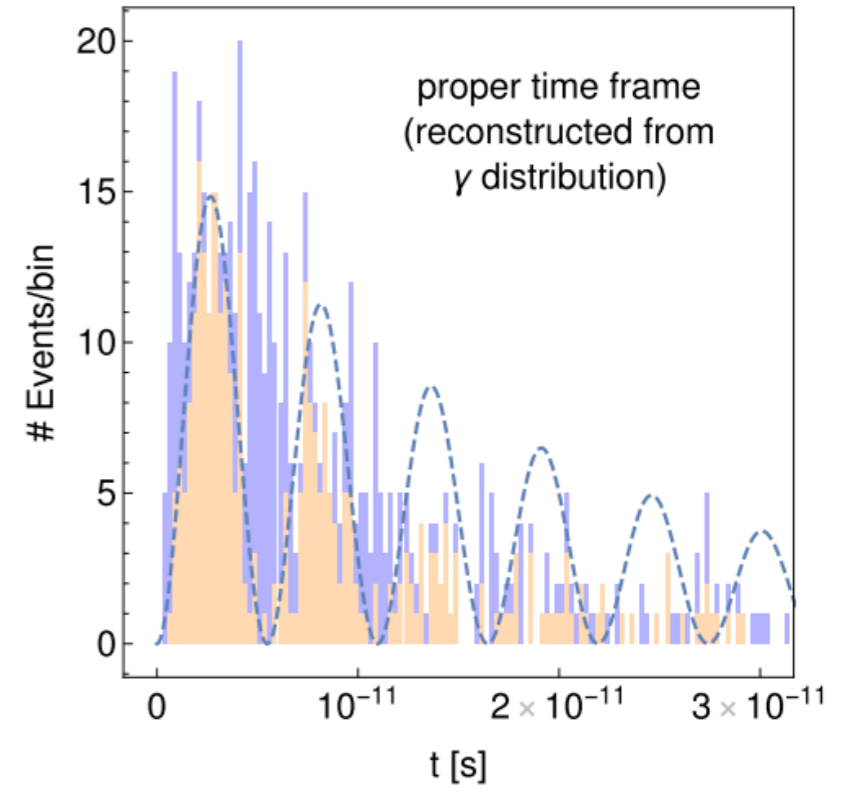
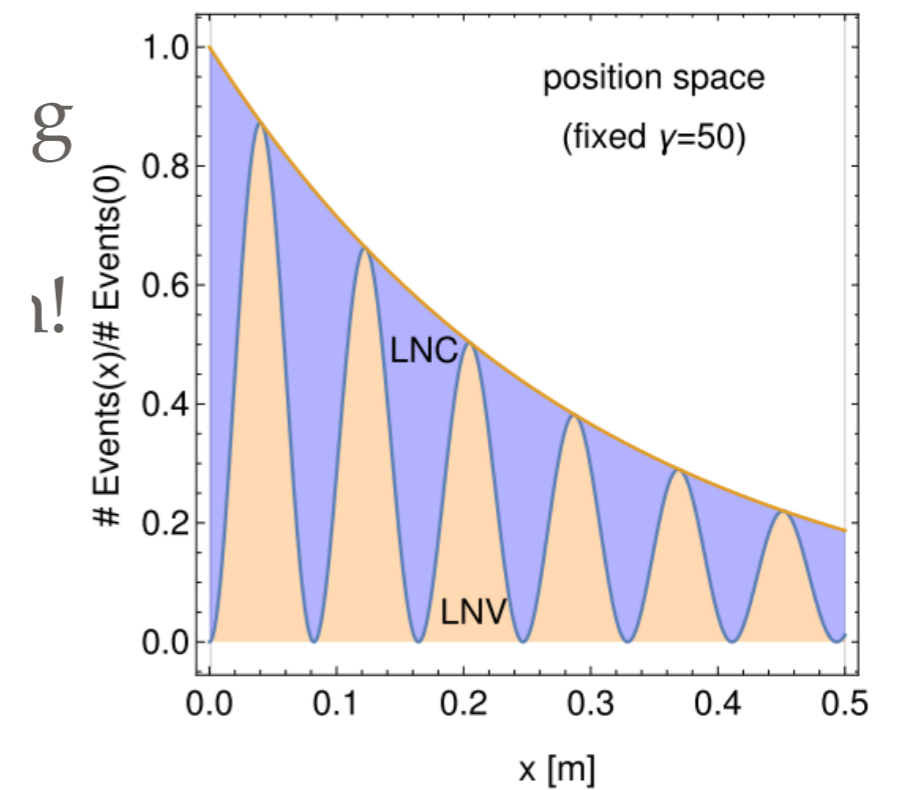
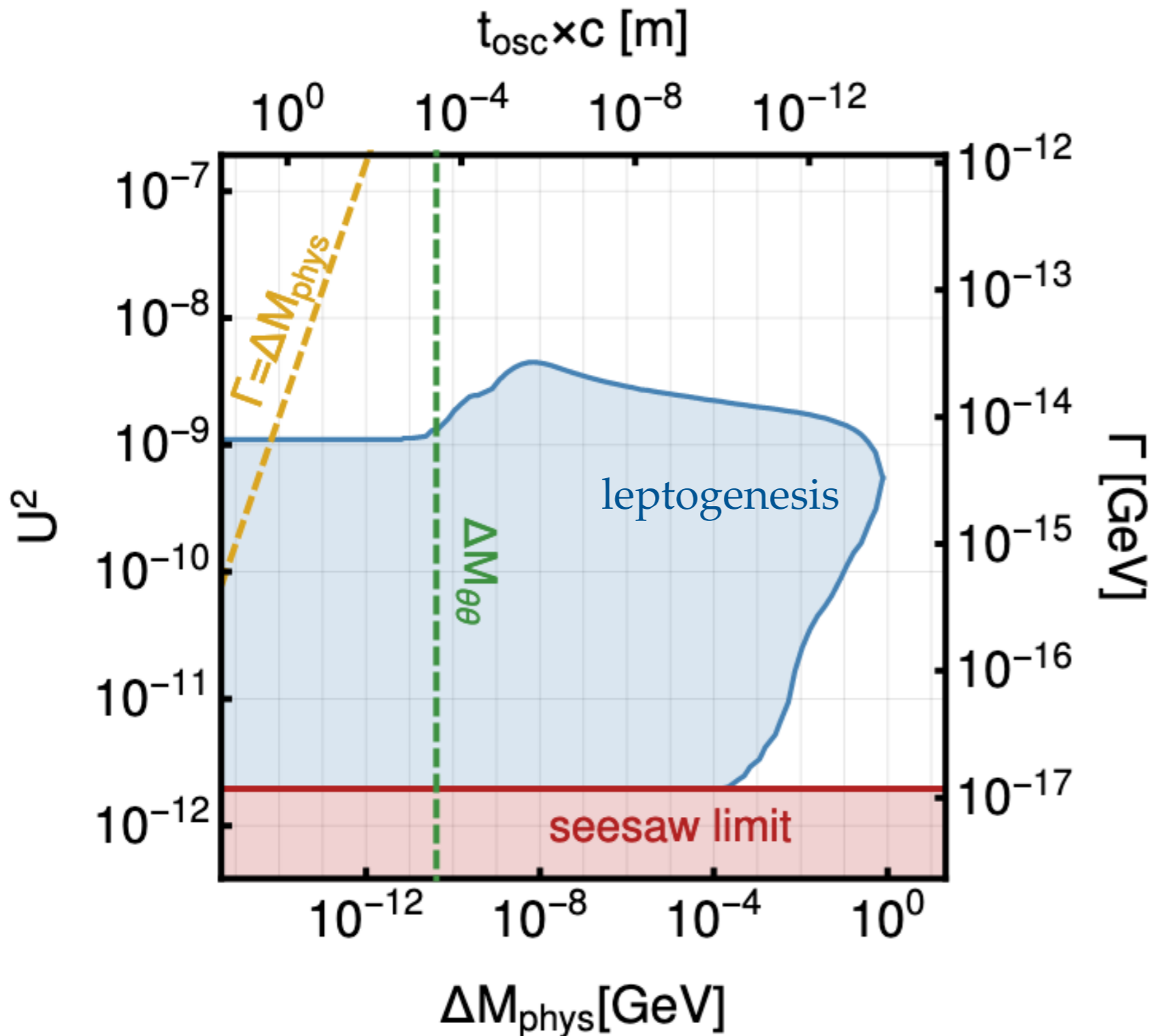
MaD/Klaric/Klose [1907.13034](#)

spatially resolving this ratio gives more information!

Antusch et al [1709.03797](#)



# How to measure $\Delta M$ ?



# The $0\nu\beta\beta$ Connection

Heavy neutrino exchange can dominate  $0\nu\beta\beta$ ...  
...even in the leptogenesis region  
 $\Rightarrow$  additional probe of  $\text{Re}\omega$  !

Bezrukov [0505247](#)

Blennow et al [1005.3240](#)

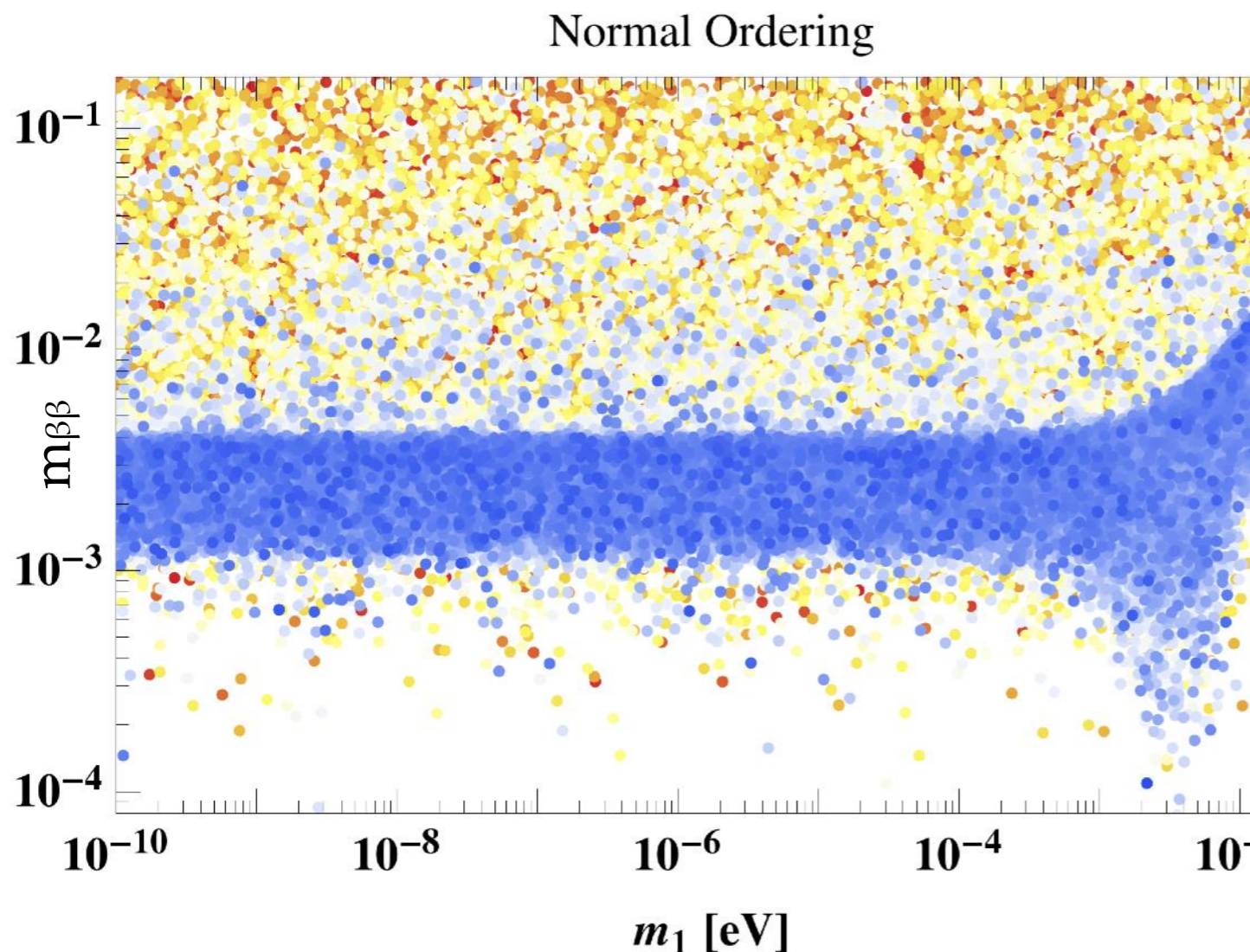
Lopez Pavon et al [1209.5342](#)

MaD/Eijima [1606.06221](#)

Hernandez et al [1606.06719](#)

Asaka et al [1606.06686](#)

Abada et al [1810.12463](#)

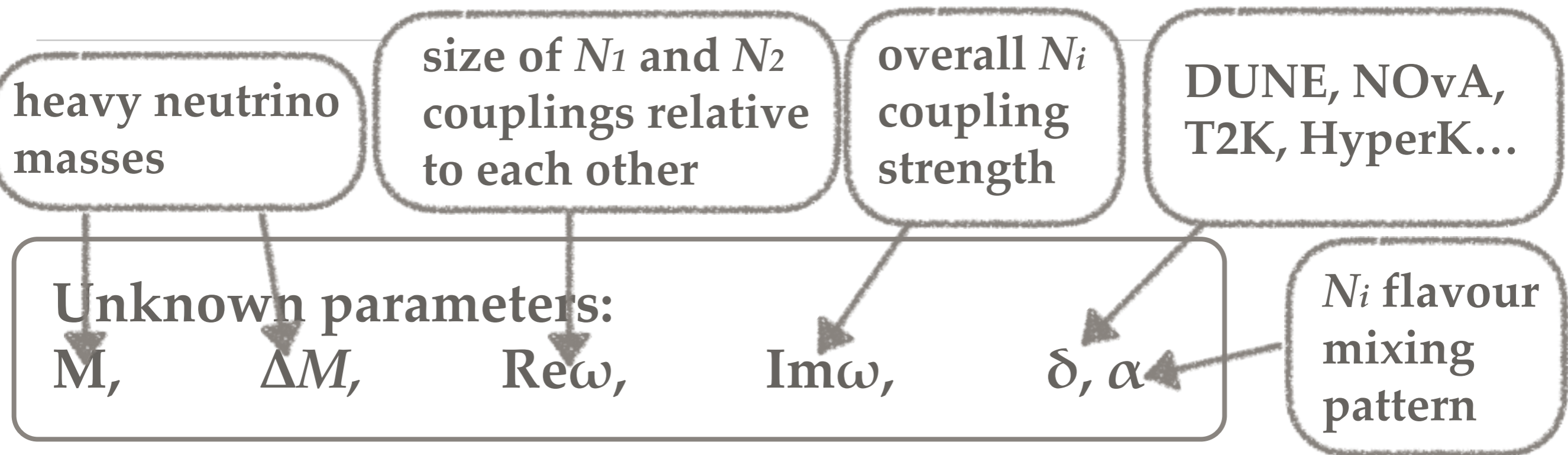


Abada et al [1810.12463](#)

- **colourful points: can explain baryon asymmetry and neutrino mass**
- **colour code measures the degree of fine tuning**
- **points outside the standard  $0\nu\beta\beta$  band are possible...**
- **.....observing a non-standard value provides a probe of RH neutrino properties!**

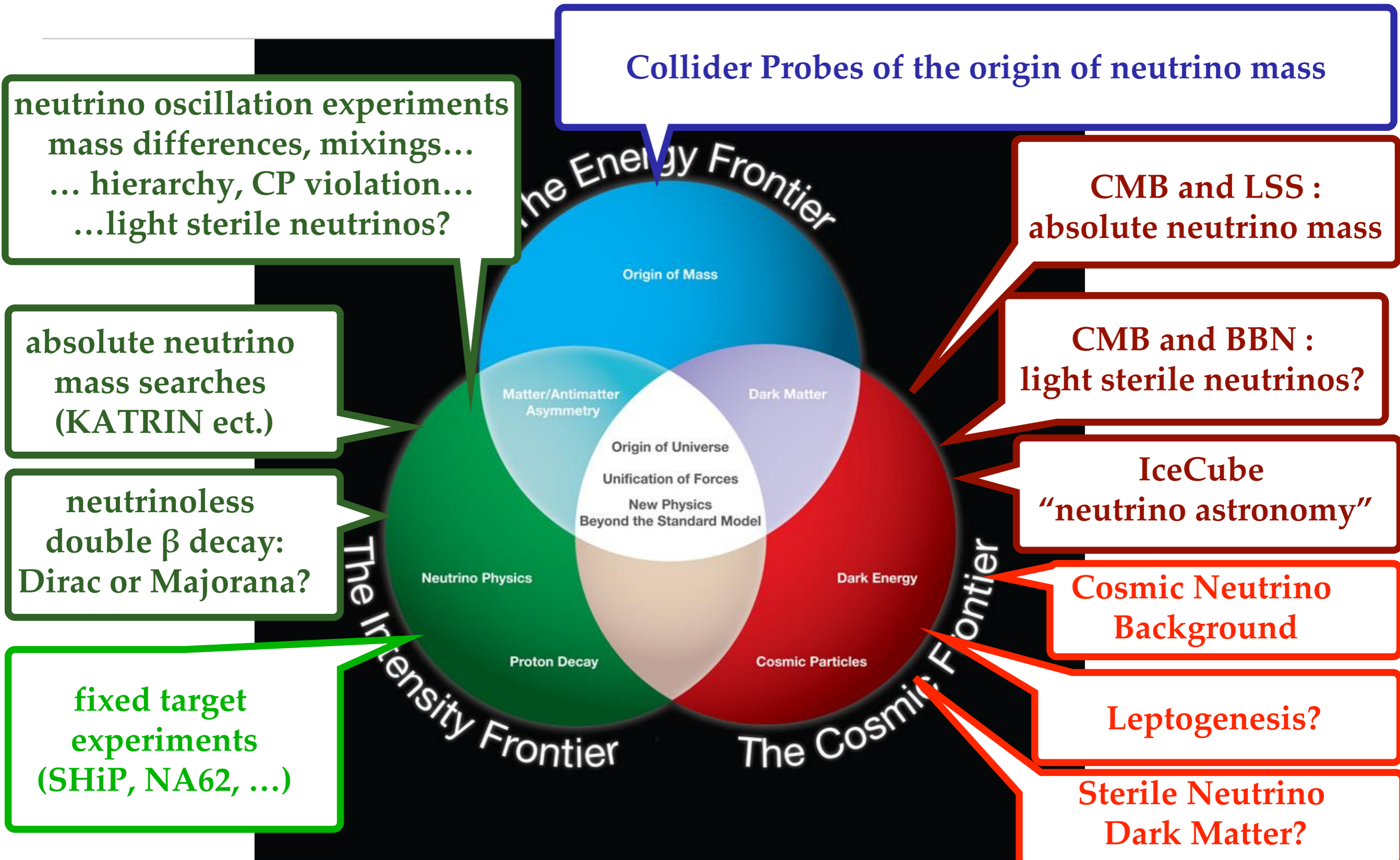


# Full Testability



- In principle all parameters can be measured  
 **$\Rightarrow$  fully testable model of neutrino masses and baryogenesis**
- This requires a combination of collider/fixed target experiment data and  $\nu$ -osc. data (and possibly  $0\nu\beta\beta$ )  
 **$\Rightarrow$  poster child example for synergy between collider and long baseline programs!**

# A Multi-Frontier Problem

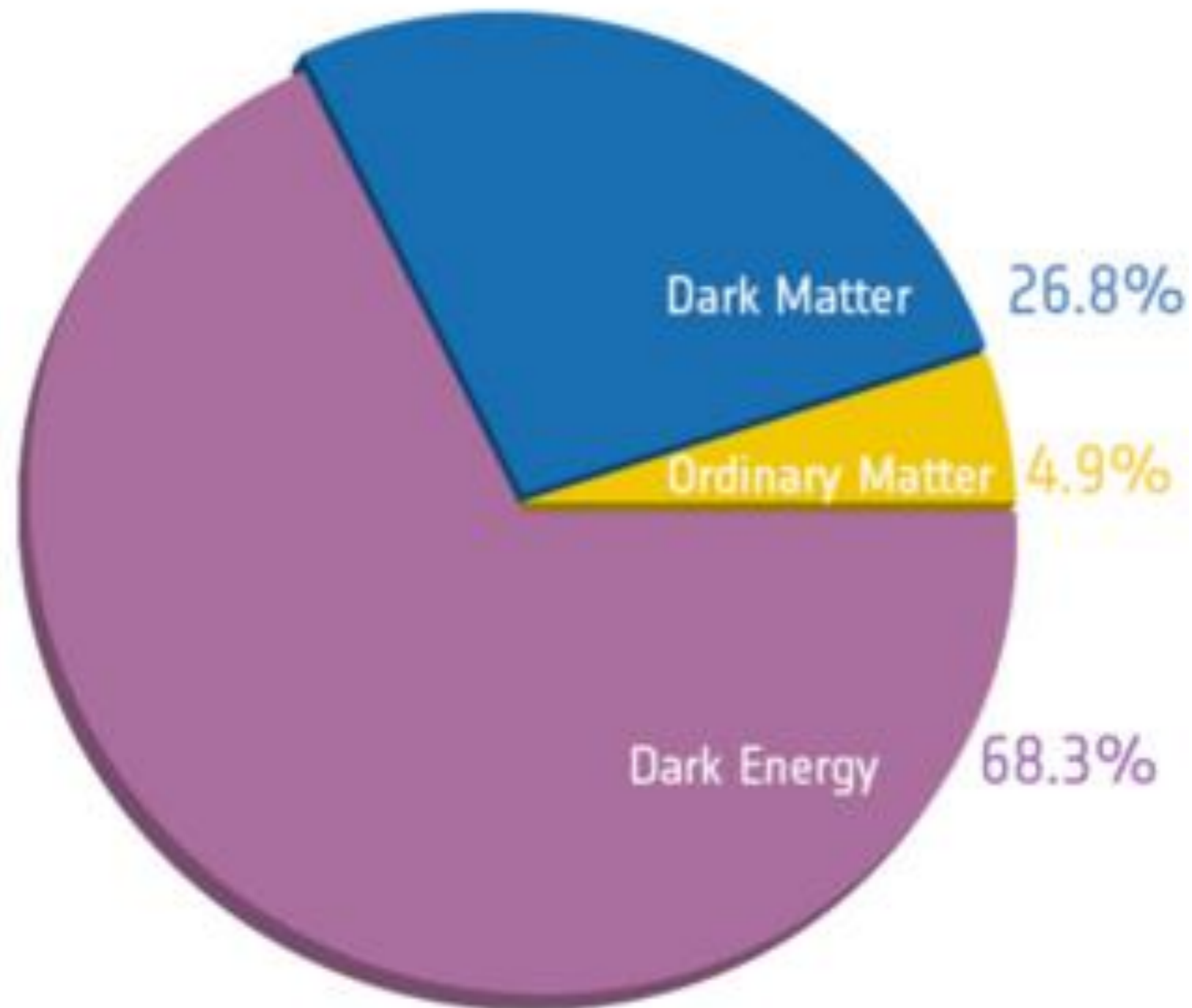


Dark Matter

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# Energy Content of the Universe

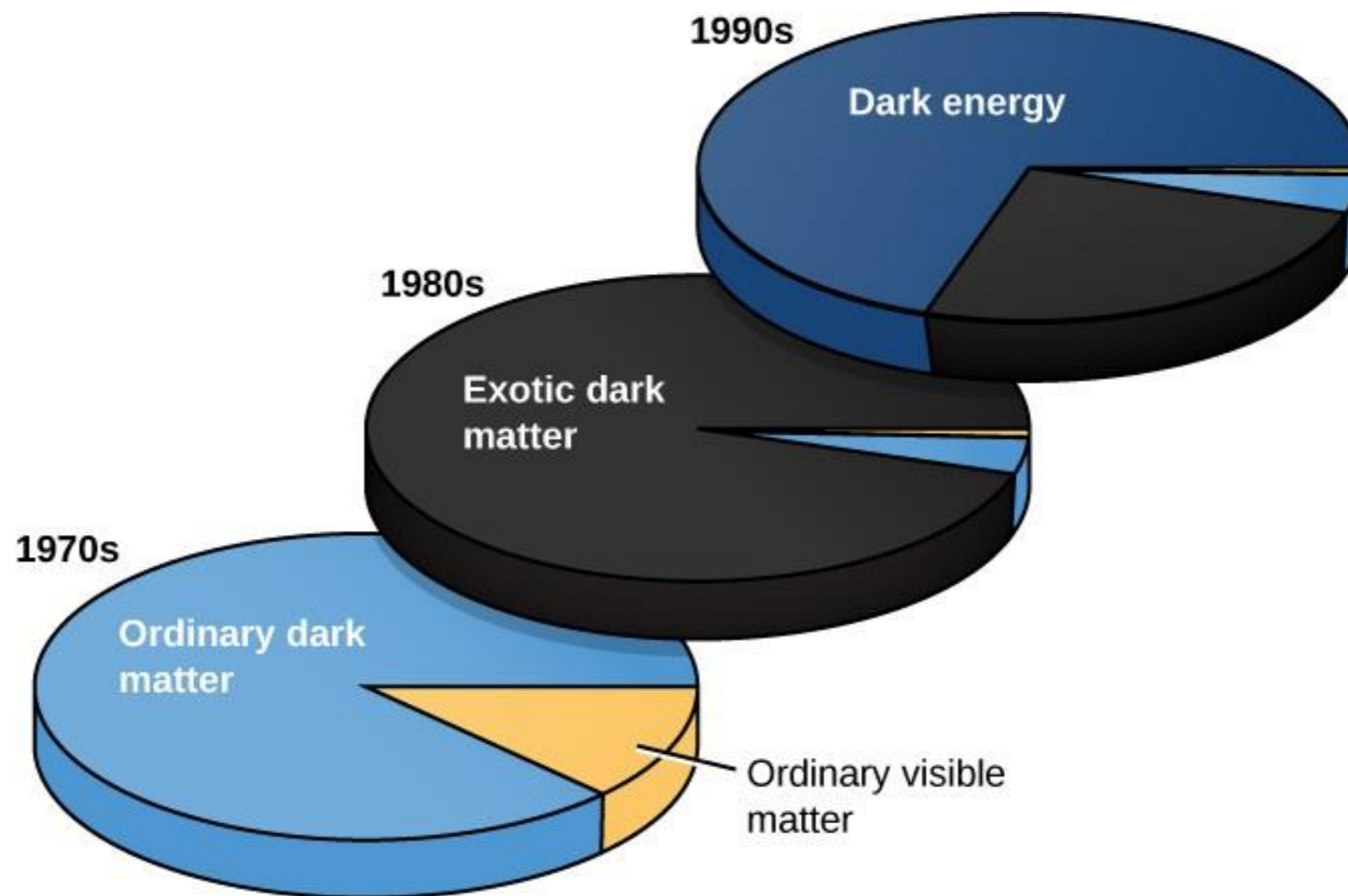
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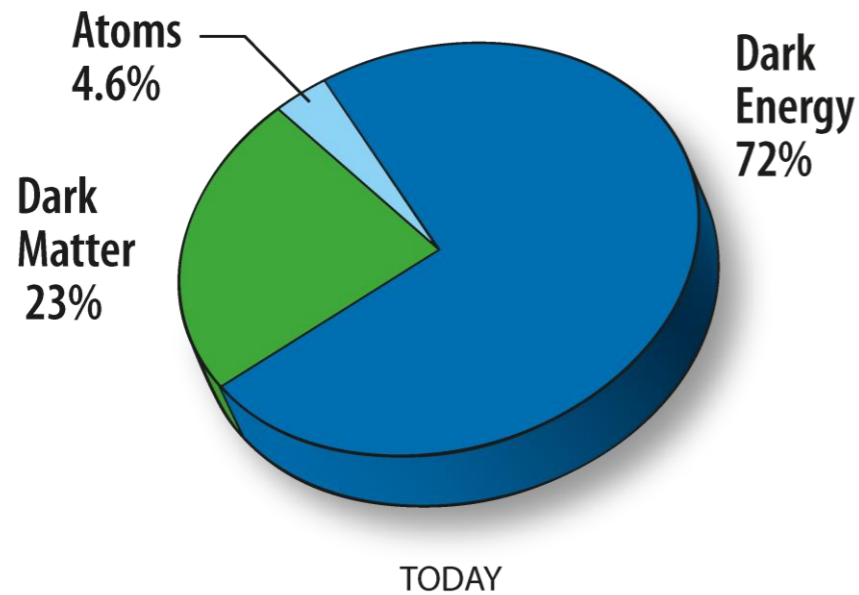




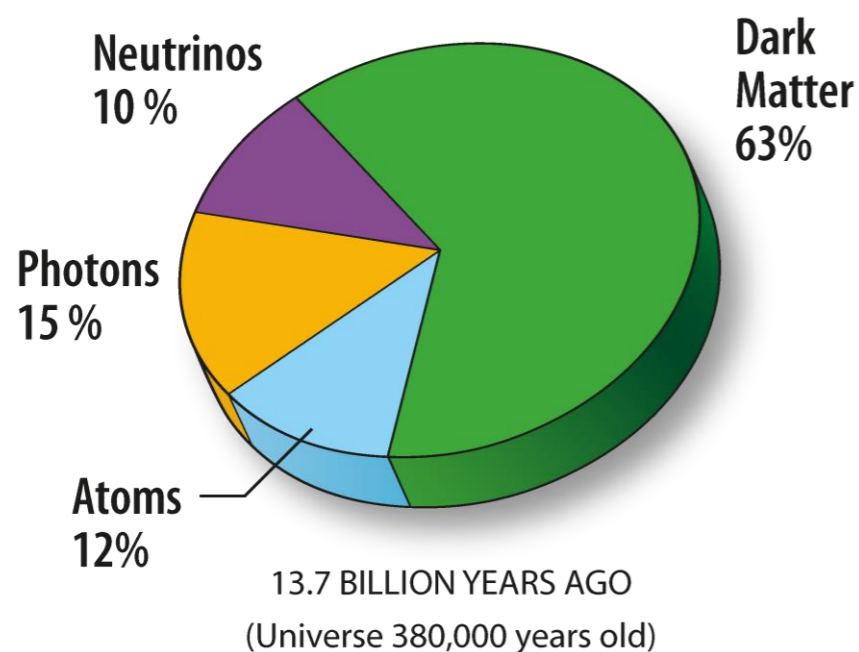
# Energy Content of the Universe



# Energy Content of the Universe



These fractions change as a function of time because different components get diluted by the expansion of the universe in a different way:



$$\rho_{\text{matter}} \propto a^{-3} \quad \text{dilution of number density}$$

$$\rho_{\text{radiation}} \propto a^{-4} \quad \text{dilution of number density, redshifting of frequencies}$$

$$\rho_{\text{vacuum}} = \text{constant} \quad \text{no dilution}$$

---

# Energy Content of the Universe

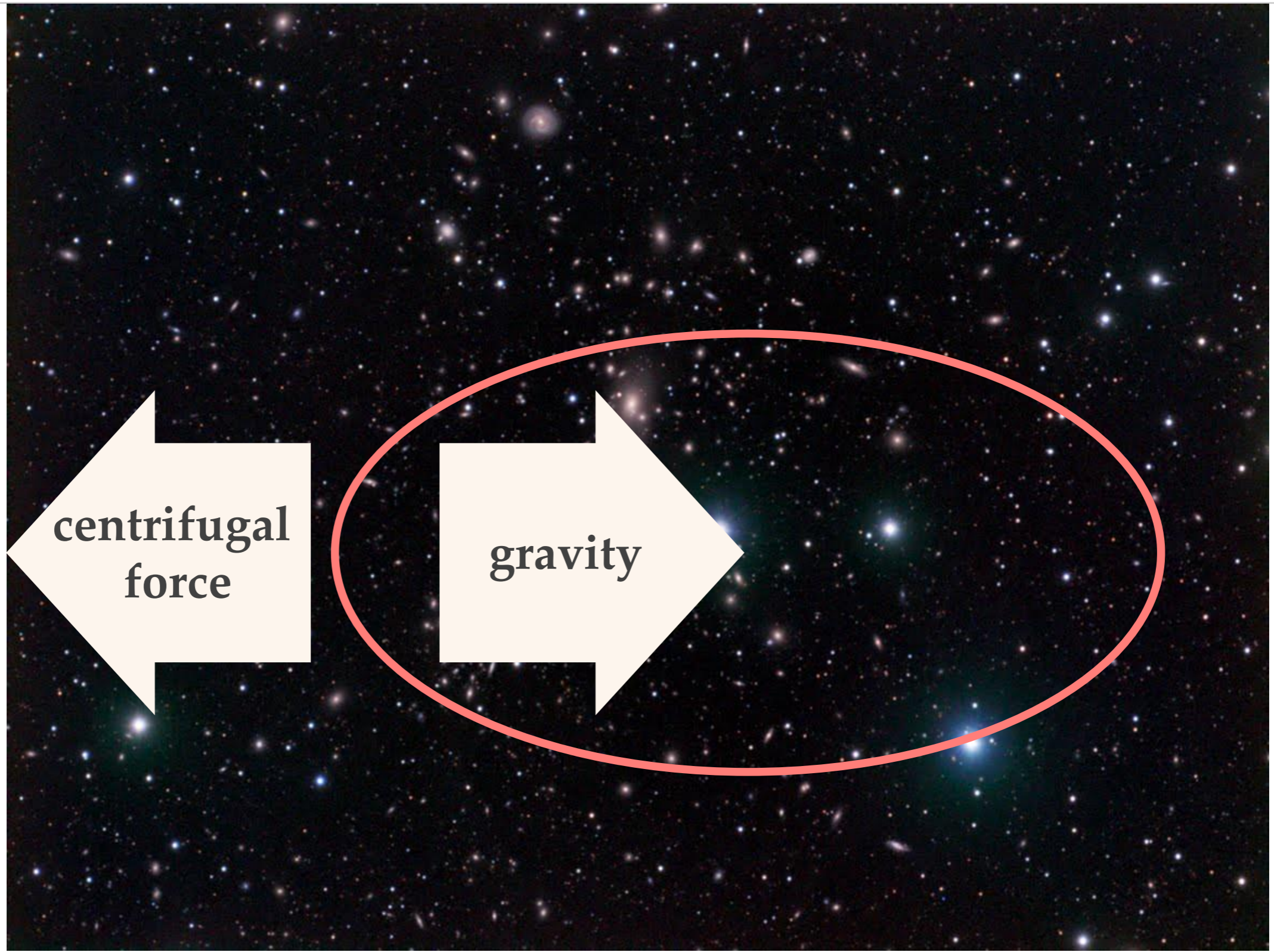
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# Evidence for Dark Matter

# 1) Galaxy Clusters





# 1) Galaxy Clusters

The mass of all the stars and dust is not enough to explain the gravitational force that is needed to keep the cluster together!



The diagram features a dark space background filled with numerous small, bright stars. A large, light-colored oval is drawn around the central part of the image. Inside this oval, a white arrow points to the right, labeled 'gravity'. To the left of the oval, a white arrow points to the left, labeled 'centrifugal force'.

centrifugal  
force

gravity



# 1) Galaxy Clusters

The mass of all the stars and dust is not enough to explain the gravitational force that is needed to keep the cluster together!

centrifugal  
force

- 1) Modified Gravity?
- 2) Or non-luminous matter?



# 1) Galaxy Clusters

The mass of all the stars and dust is not enough to explain the gravitational force that is needed to keep the cluster together!

centrifugal  
force

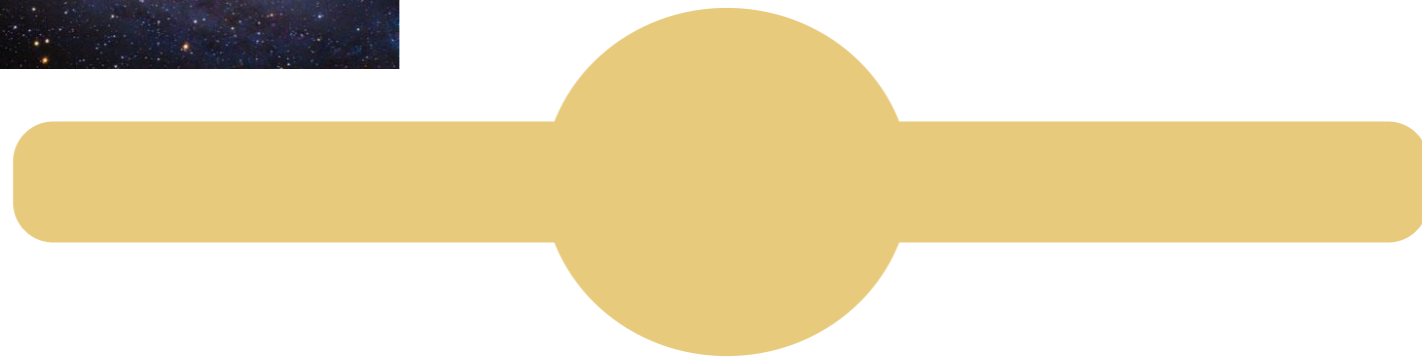
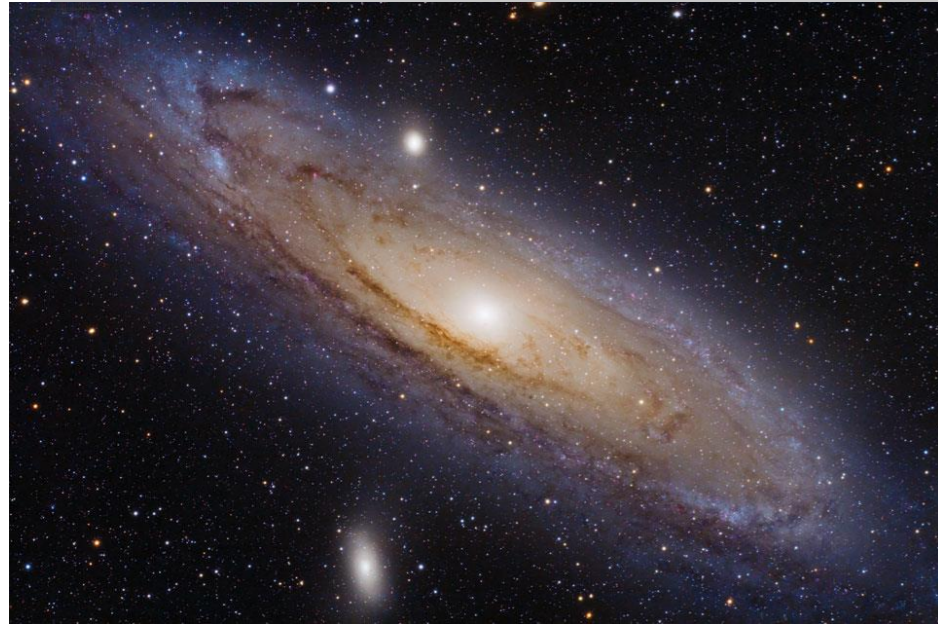
- 1) Modified Gravity?
- 2) Or non-luminous matter?

Both possible.  
And if 2), it could be anything  
that doesn't shine or  
absorb too much

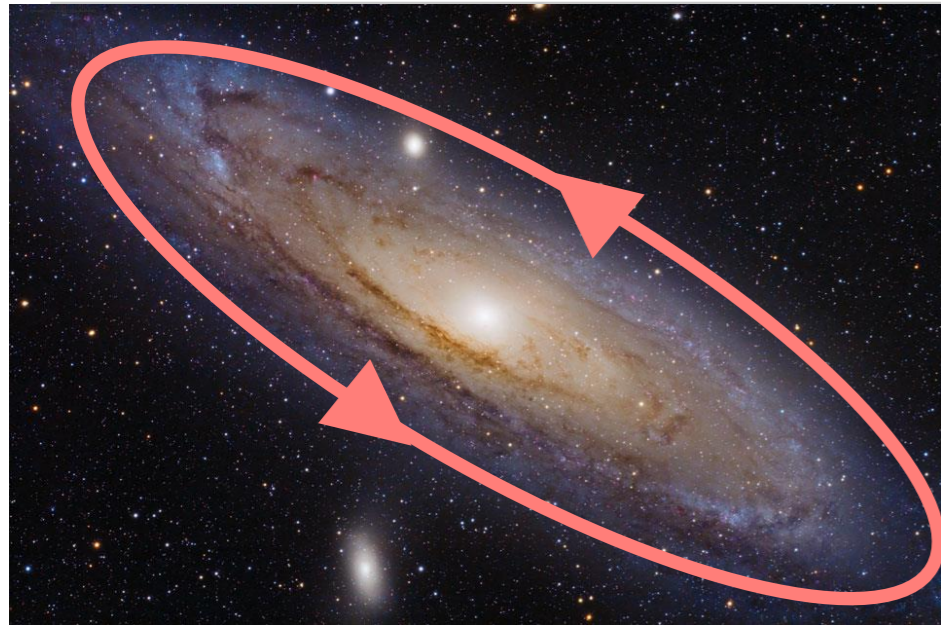
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## 2) Galaxy Rotation Curves

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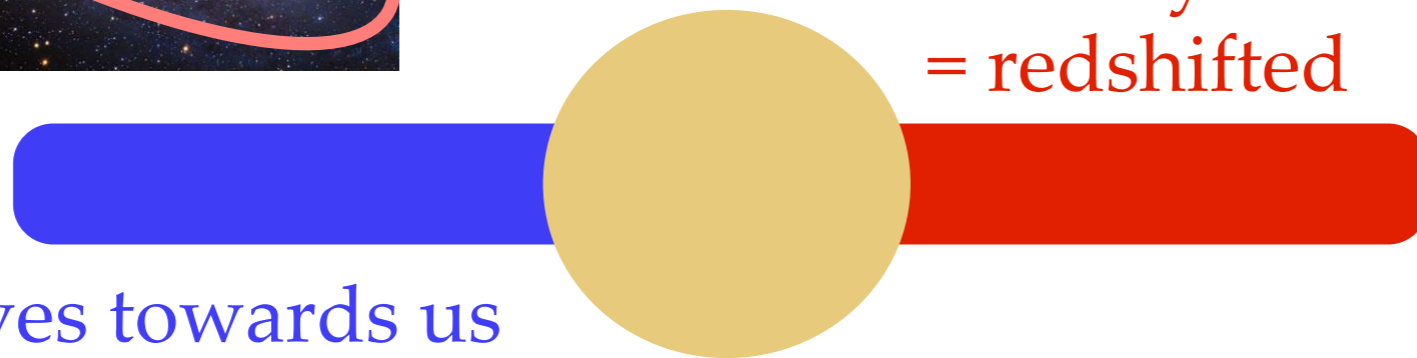


## 2) Galaxy Rotation Curves



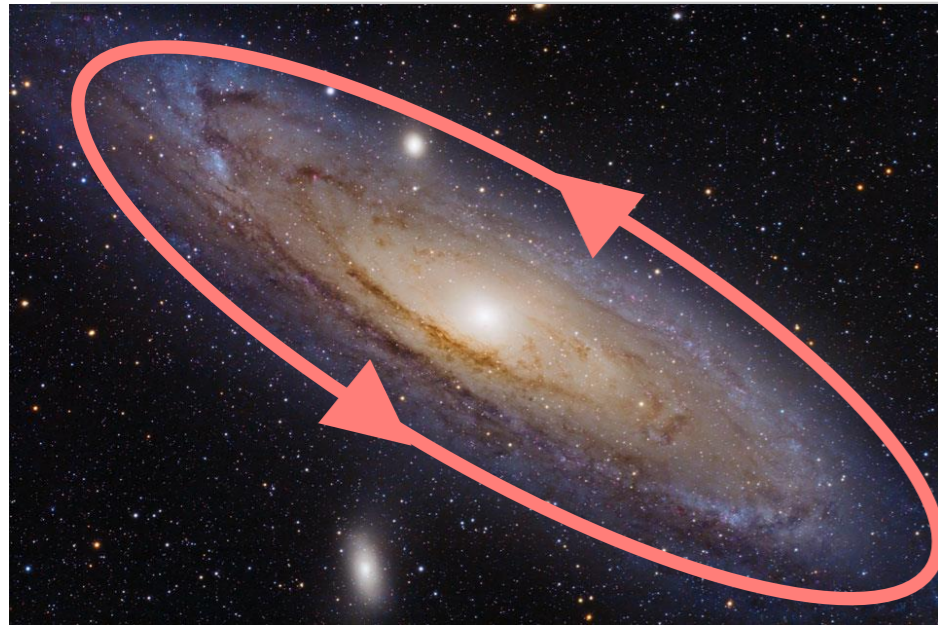
moves away from us  
= redshifted

moves towards us  
= blueshifted





## 2) Galaxy Rotation Curves

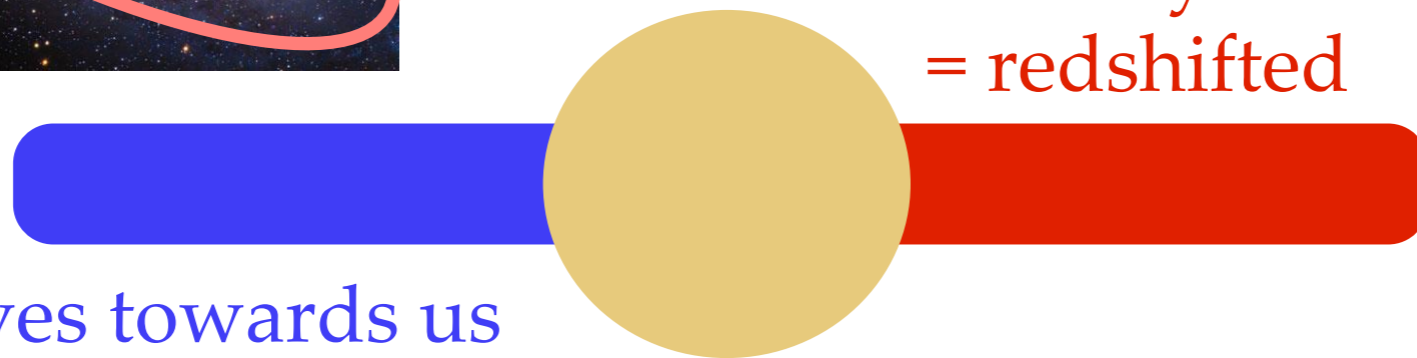


$$\frac{m M(r)}{r^2} G = \frac{m v^2(r)}{r}$$

$$\Rightarrow v^2(r) = \frac{M(r)}{r} G$$

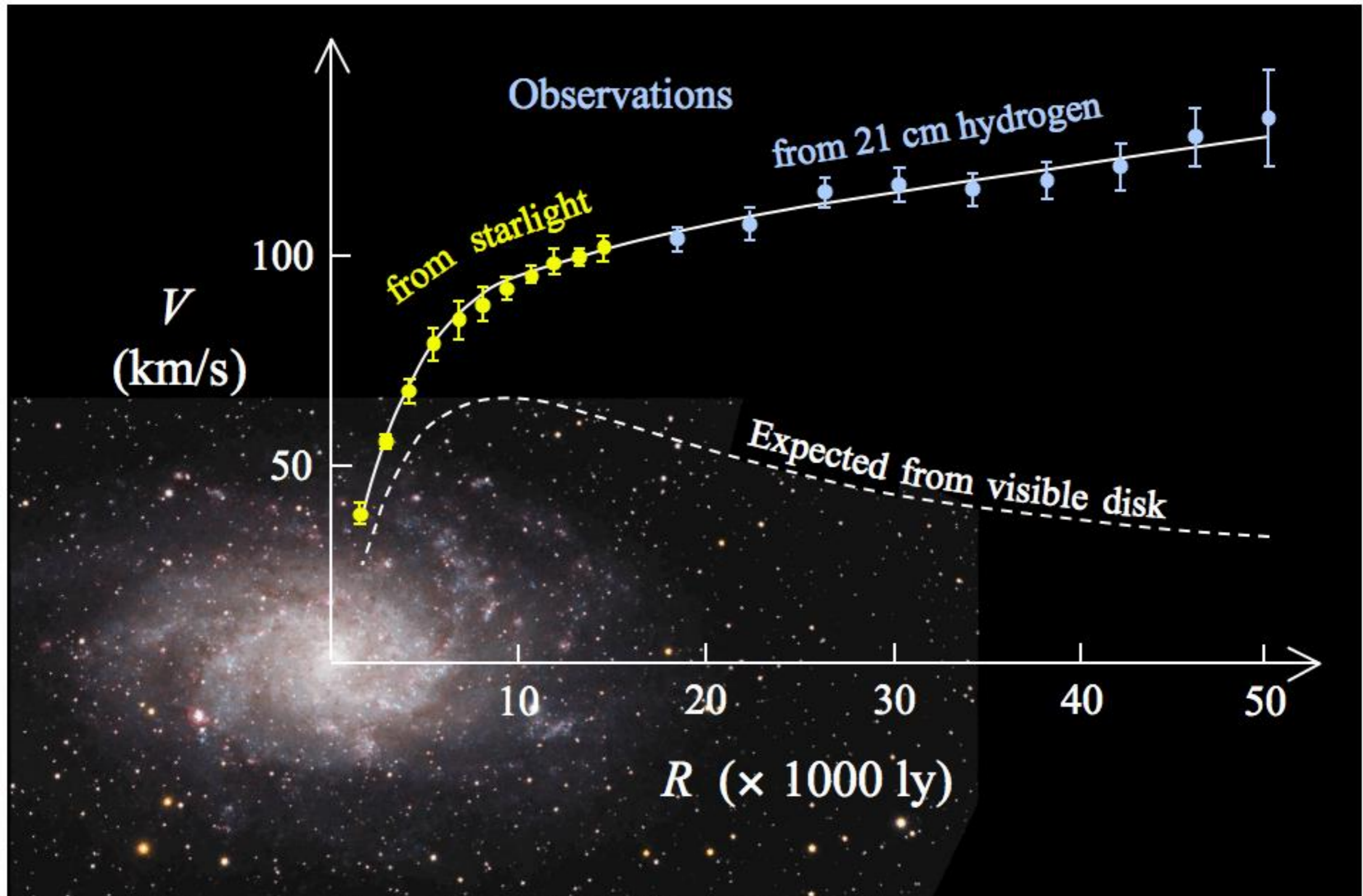
moves away from us  
= redshifted

moves towards us  
= blueshifted

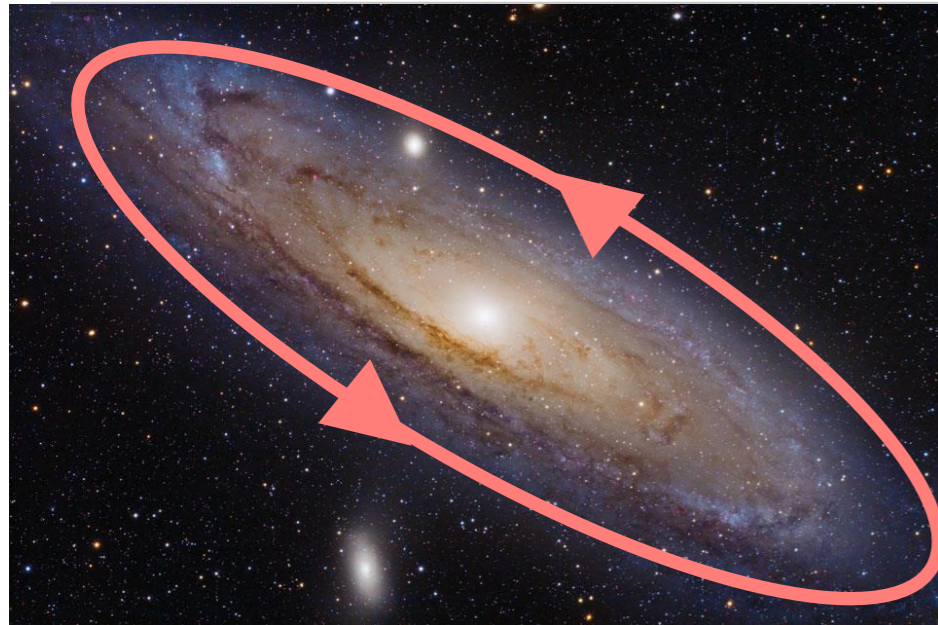




## 2) Galaxy Rotation Curves



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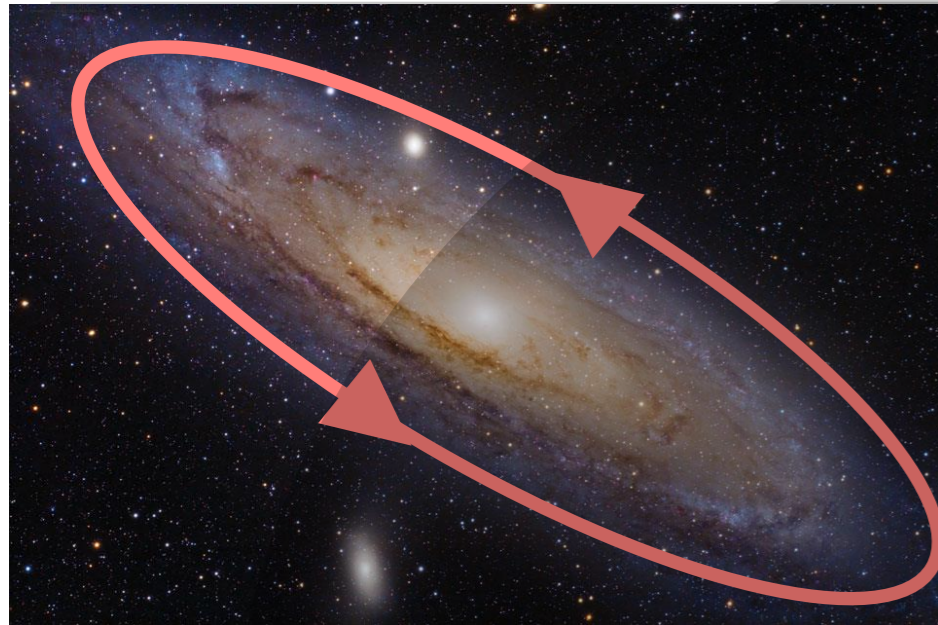
$$\frac{m M(r)}{r^2} G = \frac{m v^2(r)}{r}$$
$$\Rightarrow v^2(r) = \frac{M(r)}{r} G$$

moves away from us  
= redshifted

moves towards us  
= blueshifted

1) Modified Gravity?  
2) Or non-luminous matter?  
e.g. MACHOS (Massive Astrophysical  
Compact Halo Objects)

## 2) Galaxy Rotation Curves



$$\frac{m M(r)}{r^2} G = \frac{m v^2(r)}{r}$$

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- 1) Modified Gravity?
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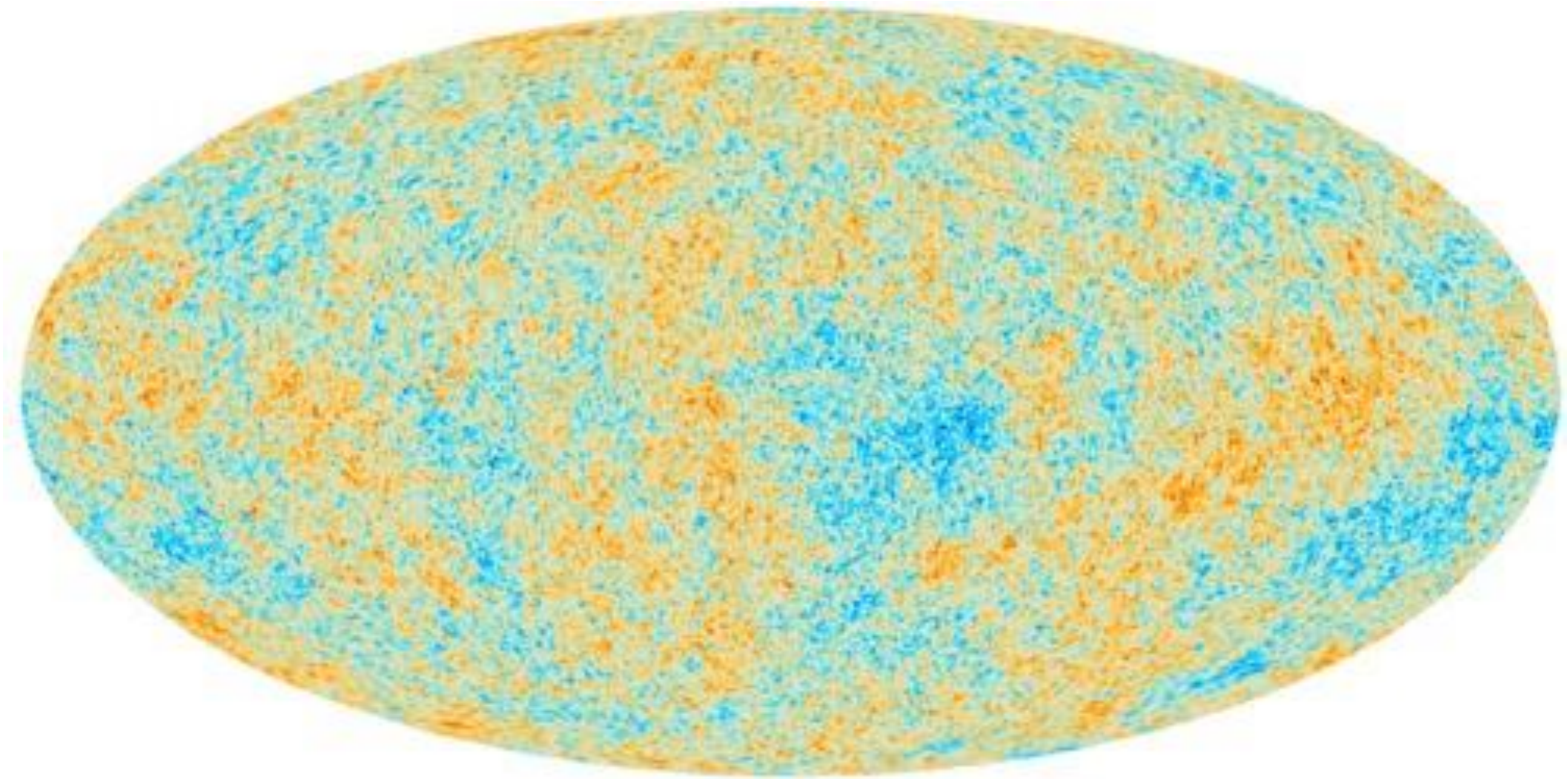
Both possible.  
And if 2), it could be anything  
that doesn't shine or  
absorb too much



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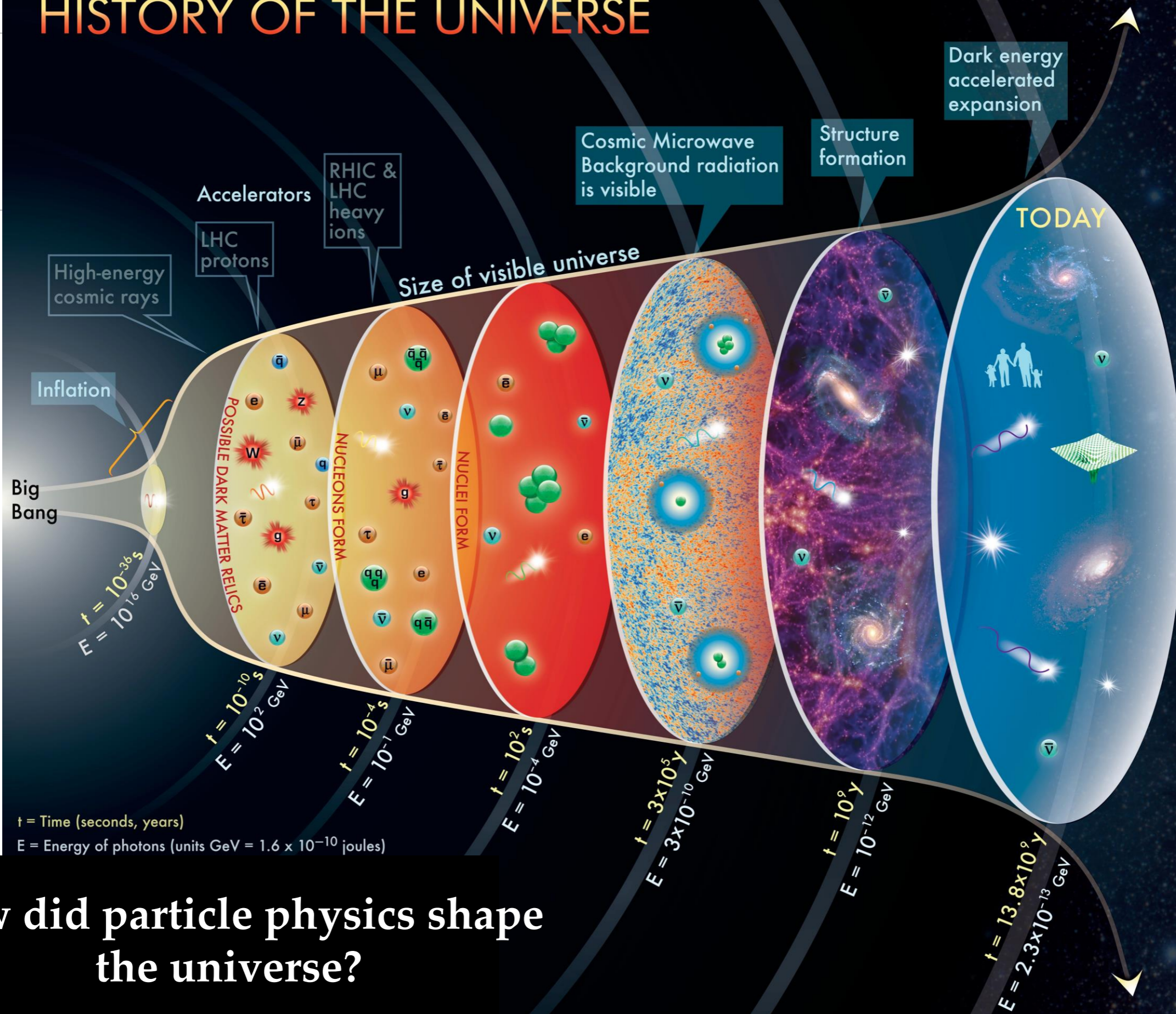
# 3) Cosmic Microwave Background

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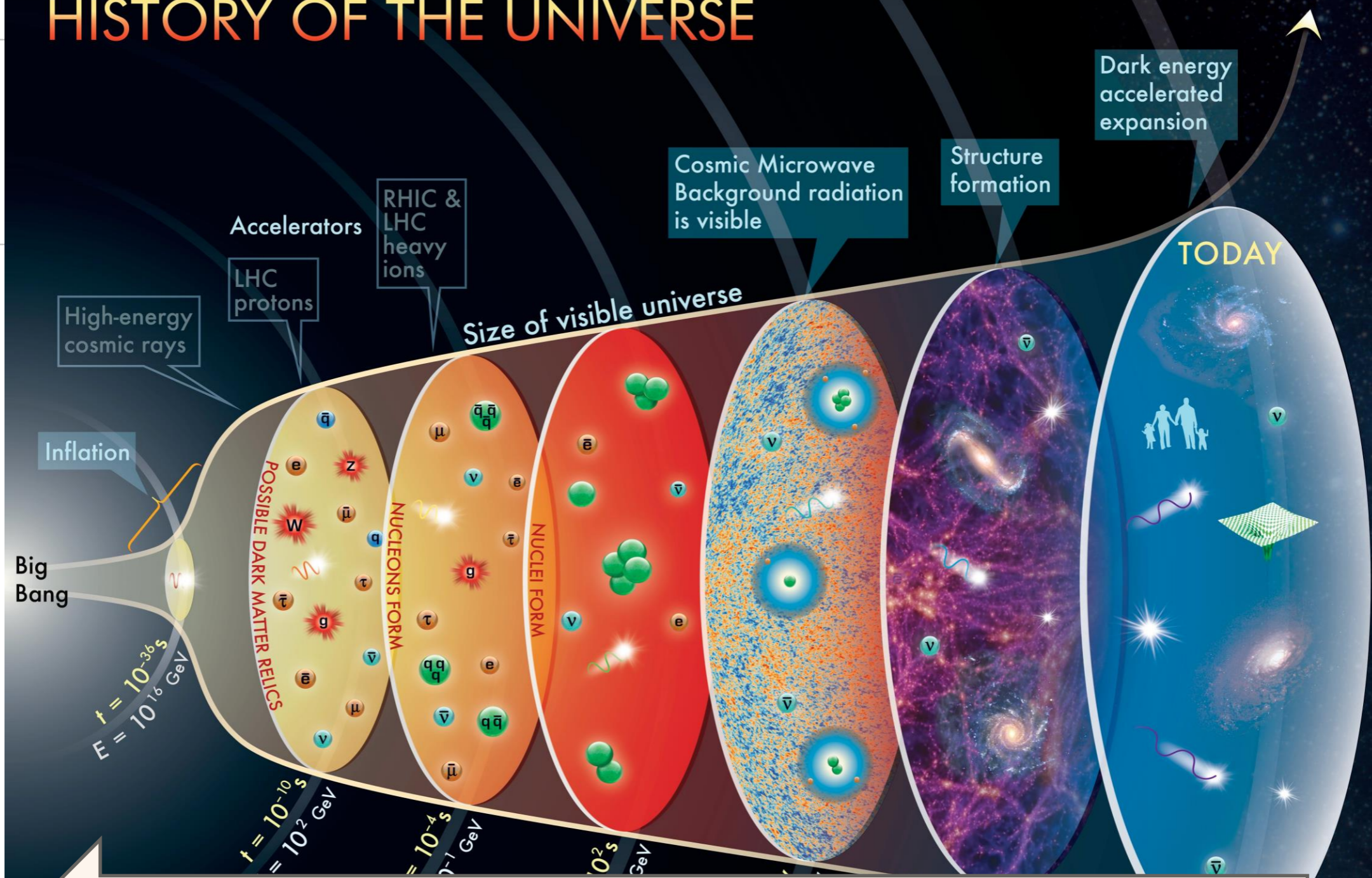
# HISTORY OF THE UNIVERSE



How did particle physics shape the universe?



# HISTORY OF THE UNIVERSE



energy density, temperature

cosmic time

The concept for the above figure originated in a 1986 paper by Michael Turner.

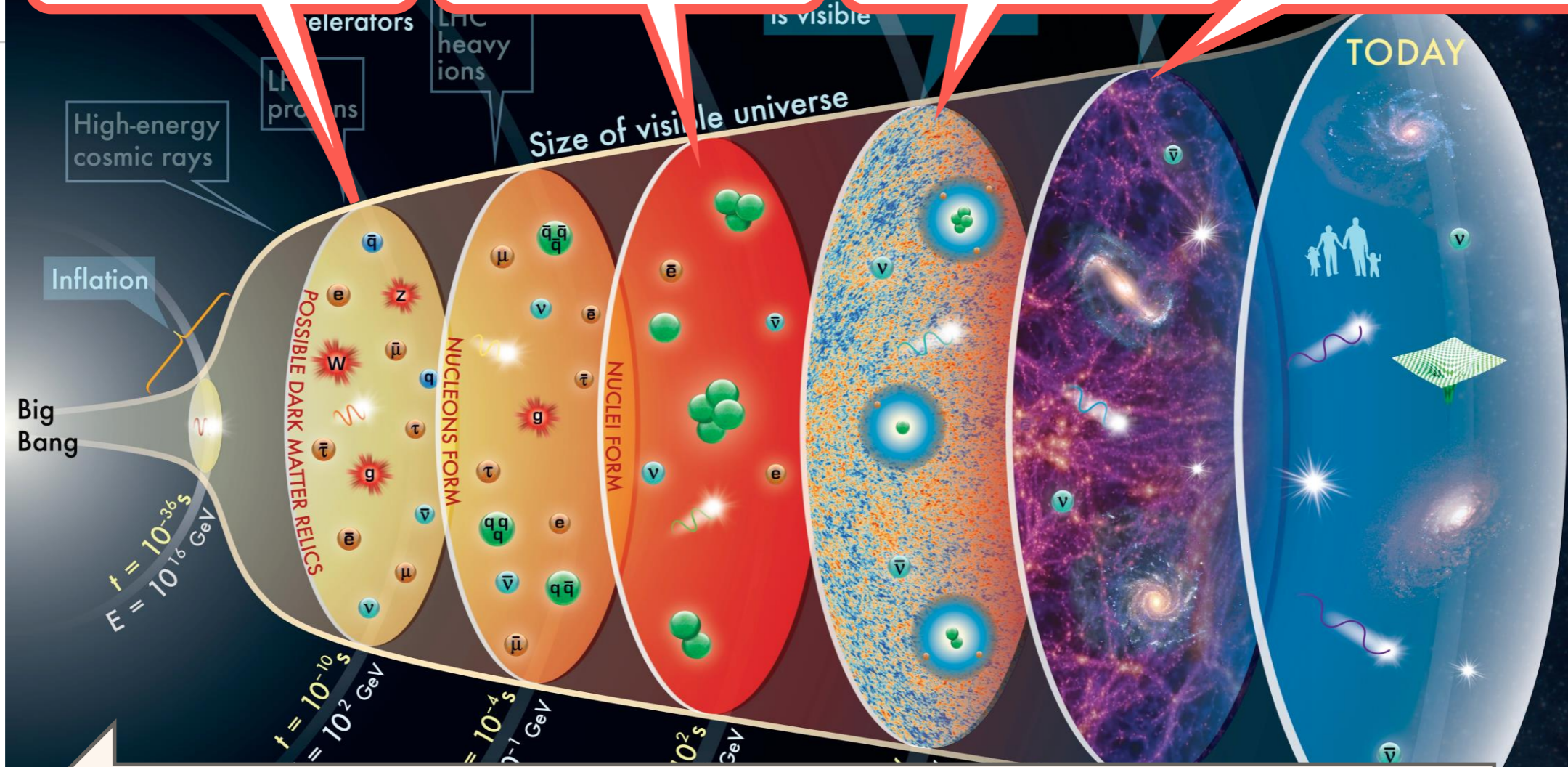


Large Hadron Collider

light element abundances

Cosmic Microwave Background

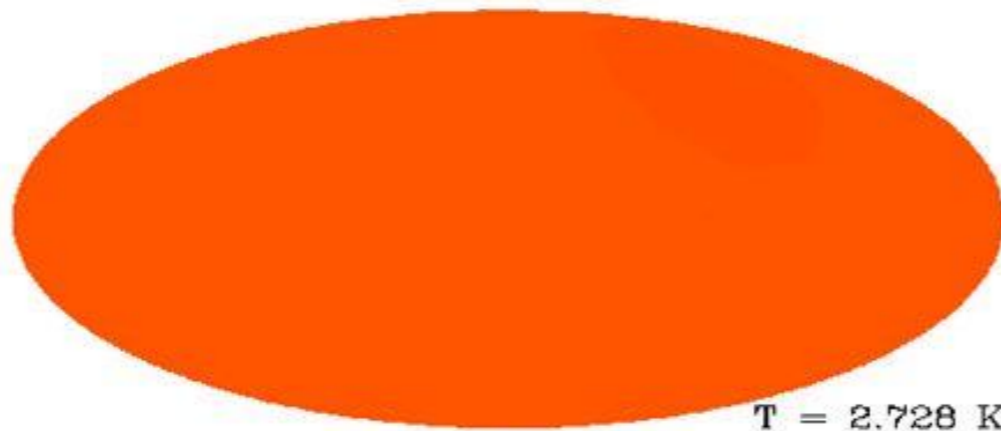
optical astronomy



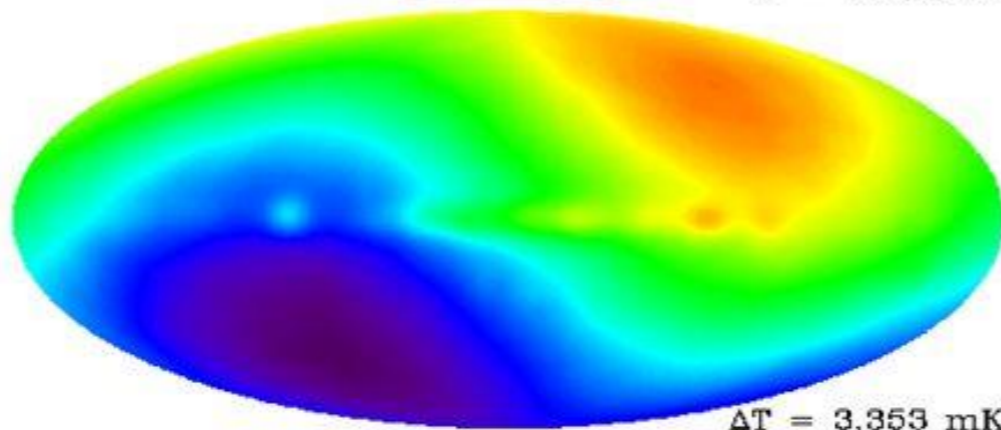
energy density, temperature

cosmic time

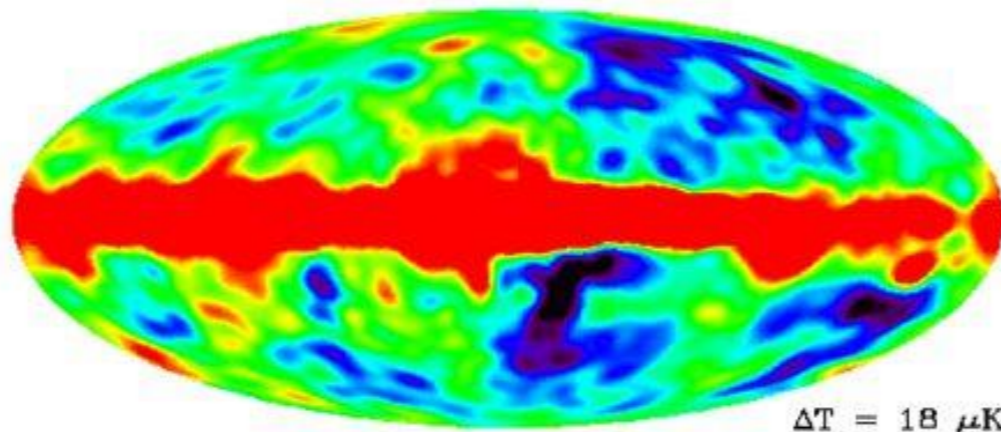
# 3) Cosmic Microwave Background



Isotropic 3K background.  
The most perfect blackbody  
we know



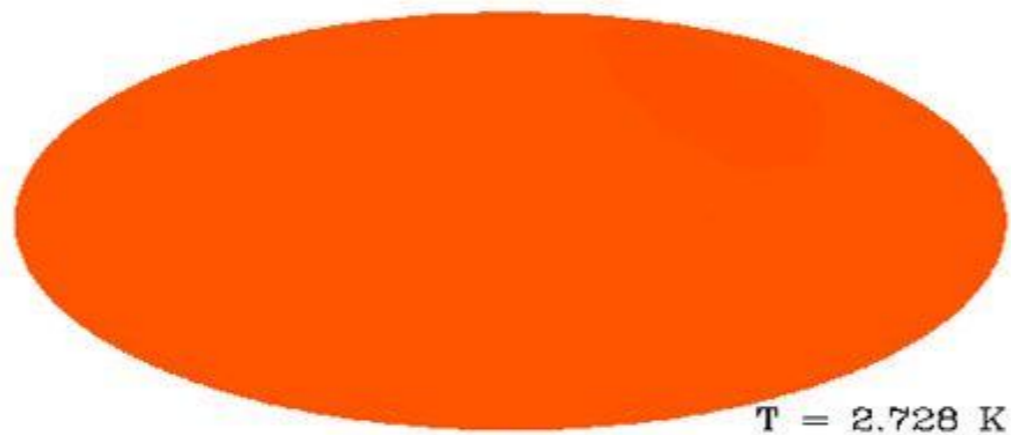
Dipole (3.4 mK).  
Our motion relative to CMB



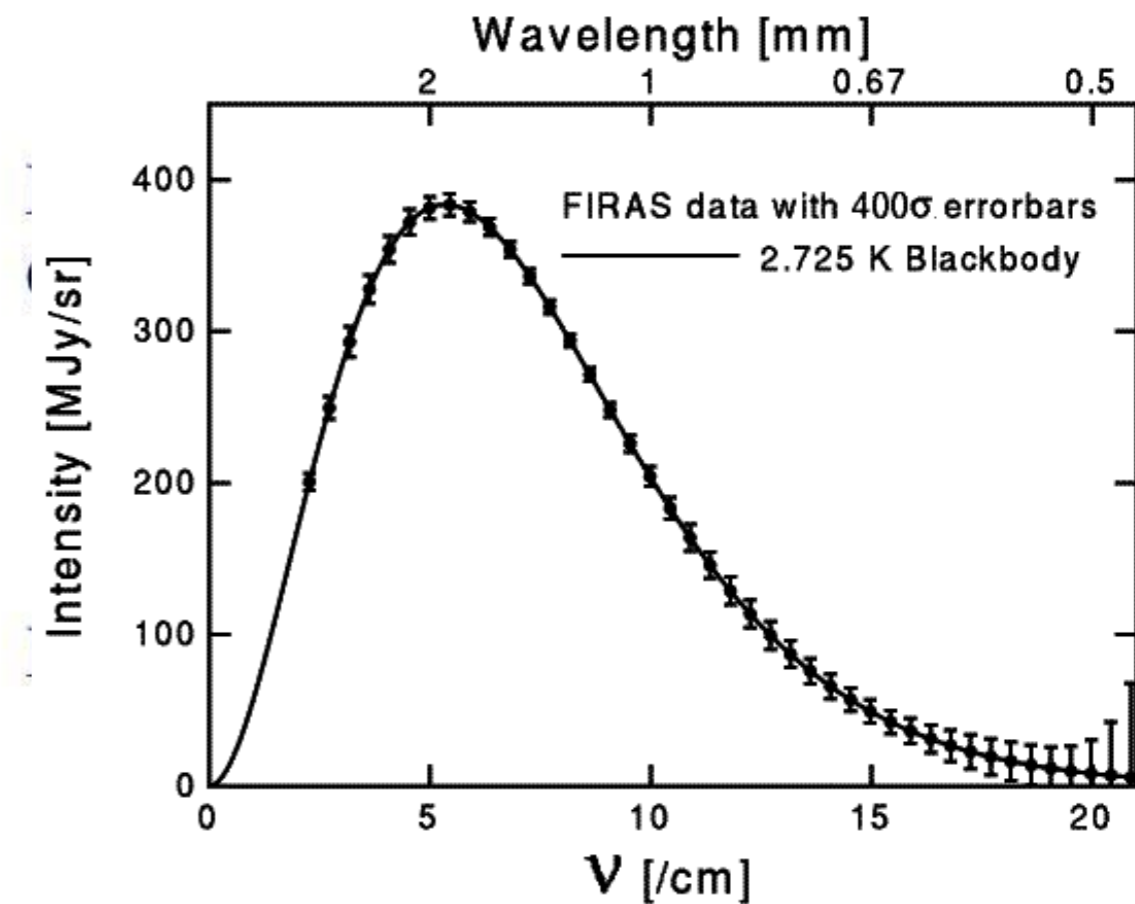
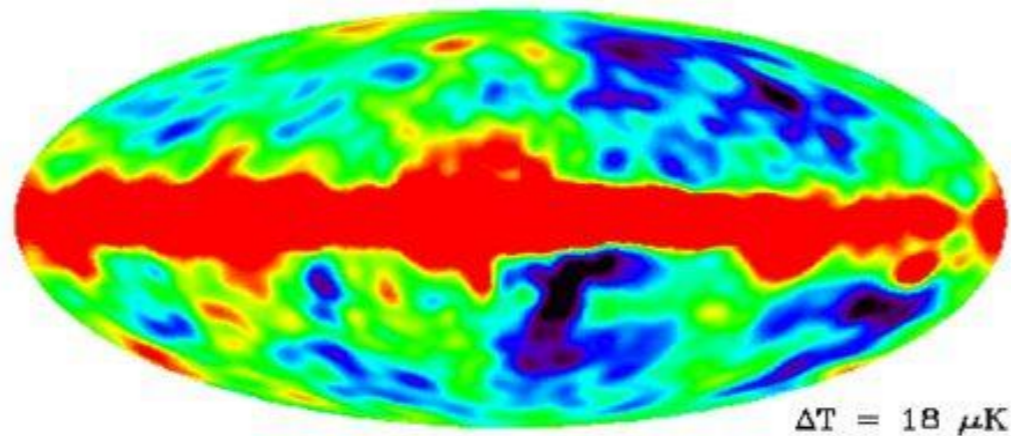
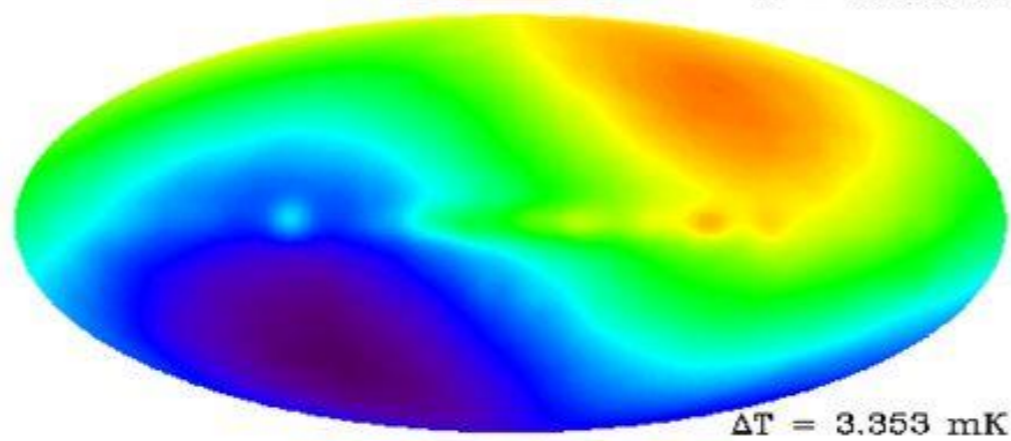
Primordial fluctuations  
 $20 \mu\text{K}$



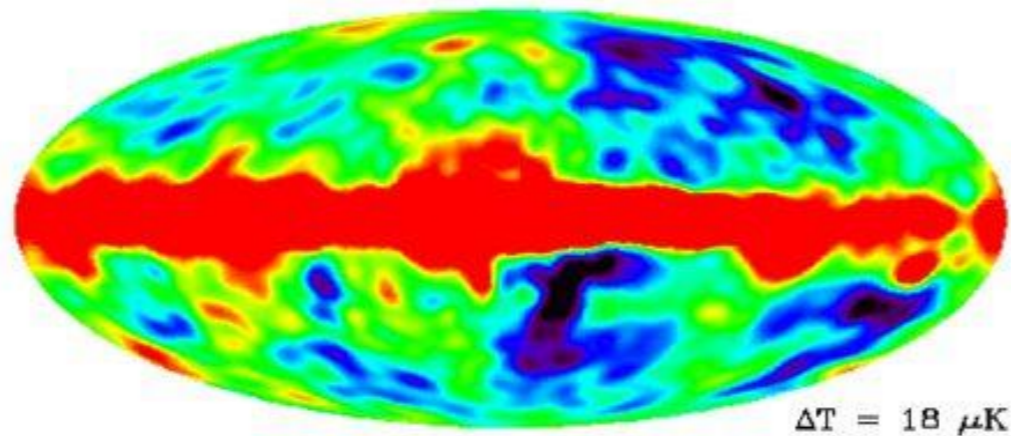
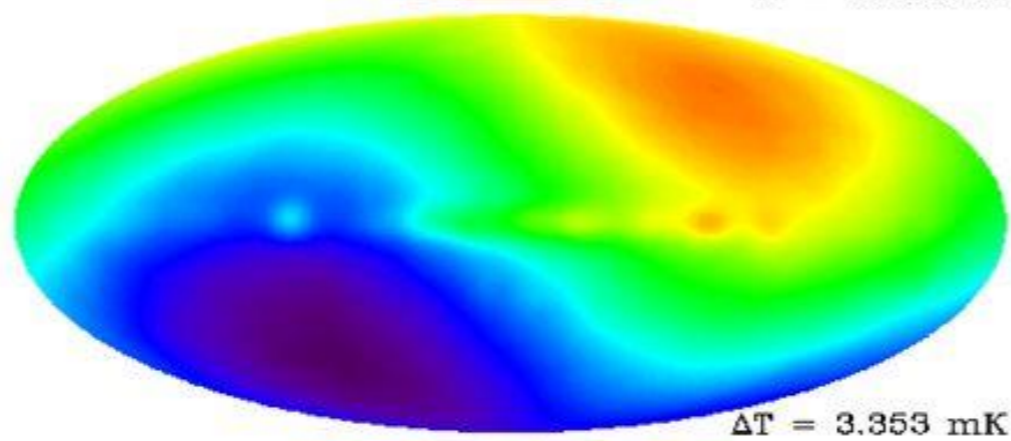
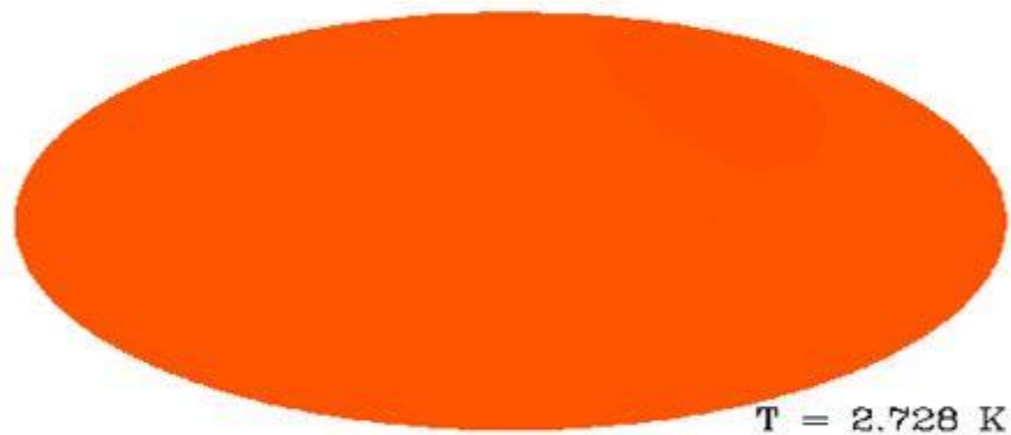
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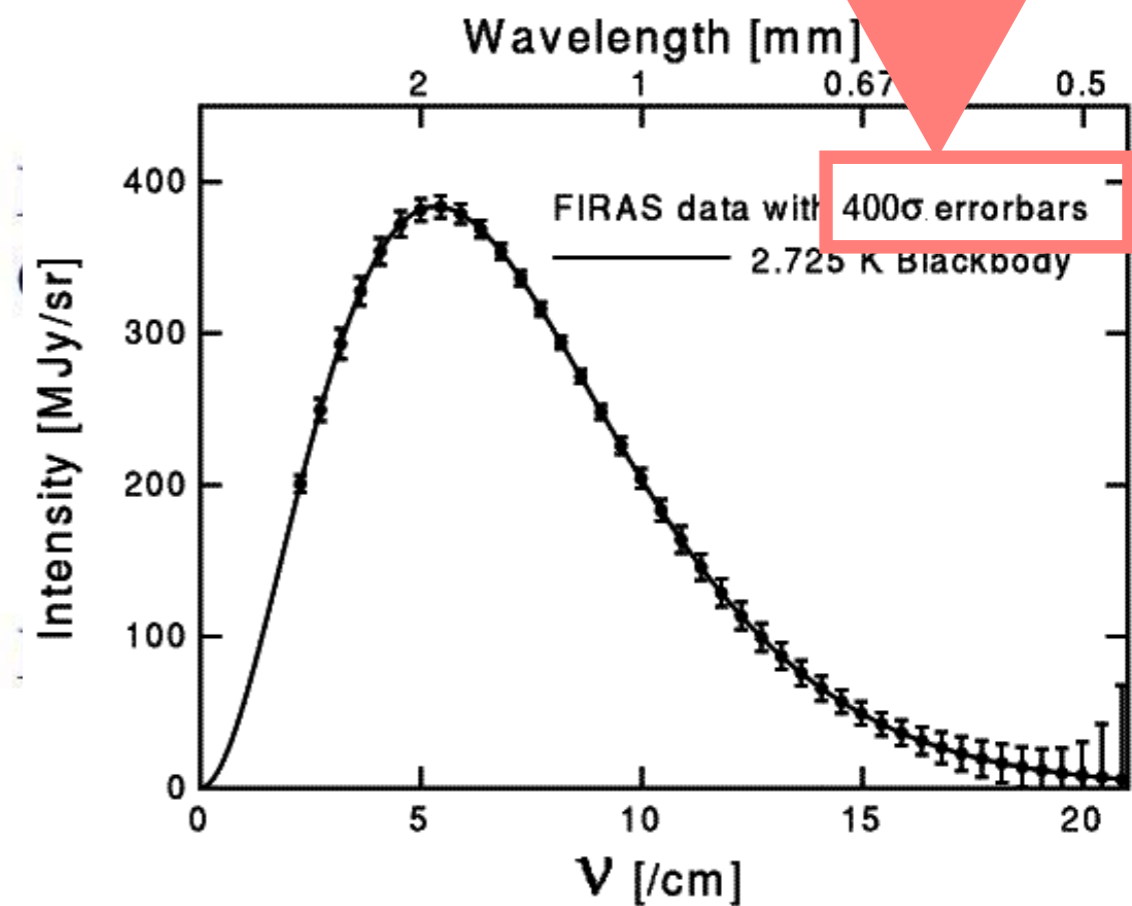
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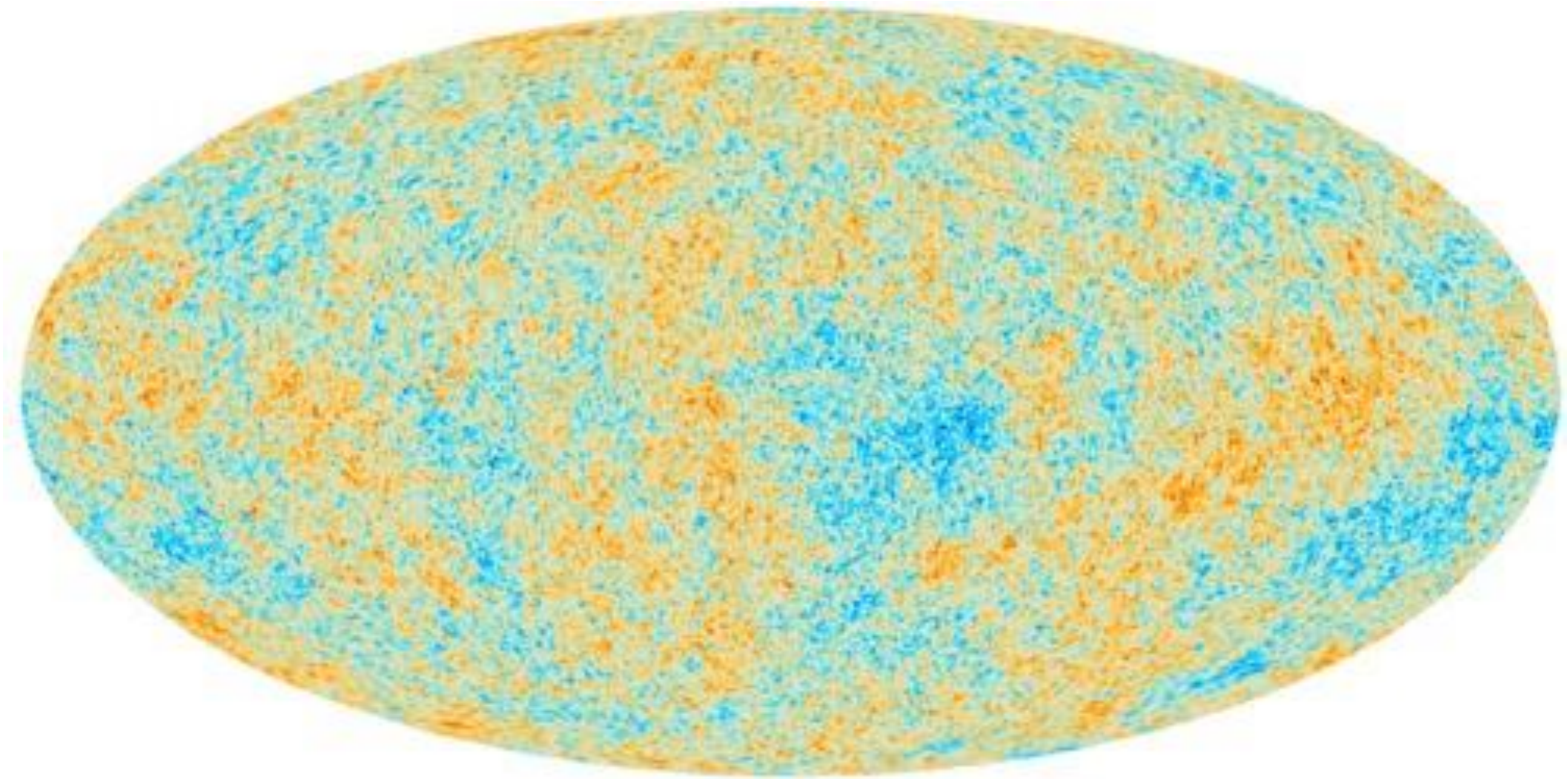




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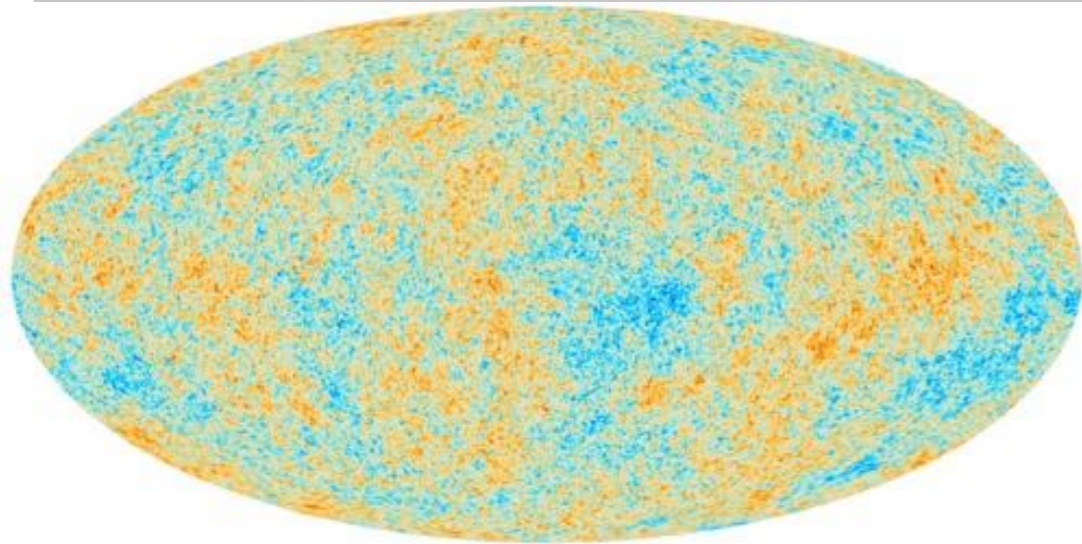
# 3) Cosmic Microwave Background

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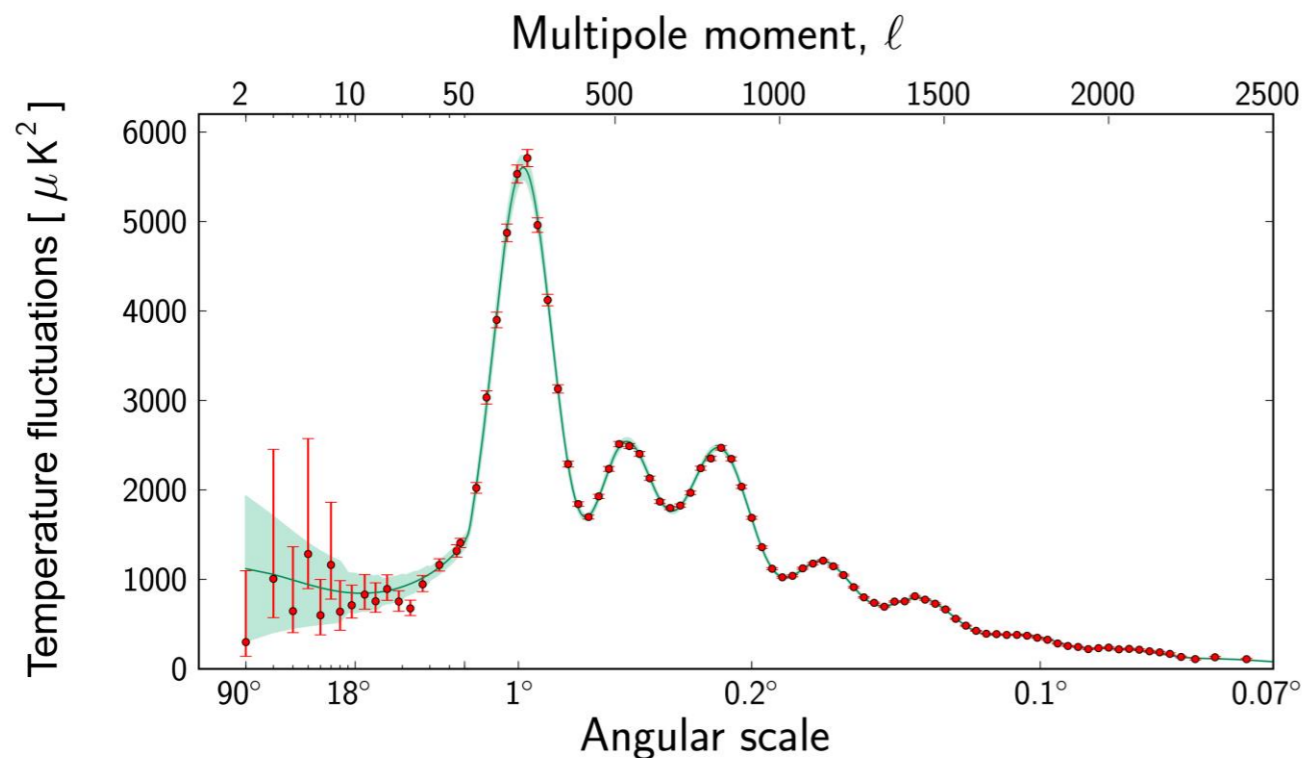




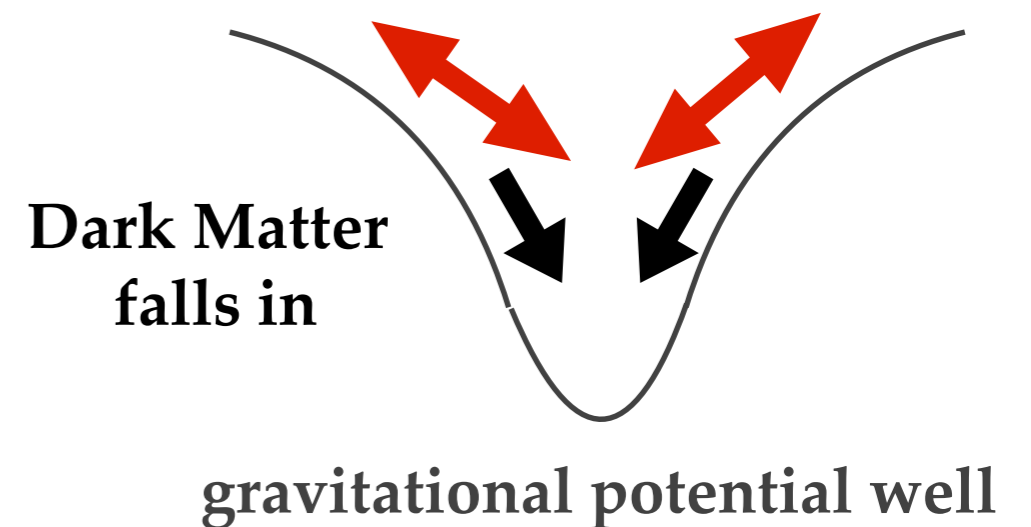
# 3) Cosmic Microwave Background



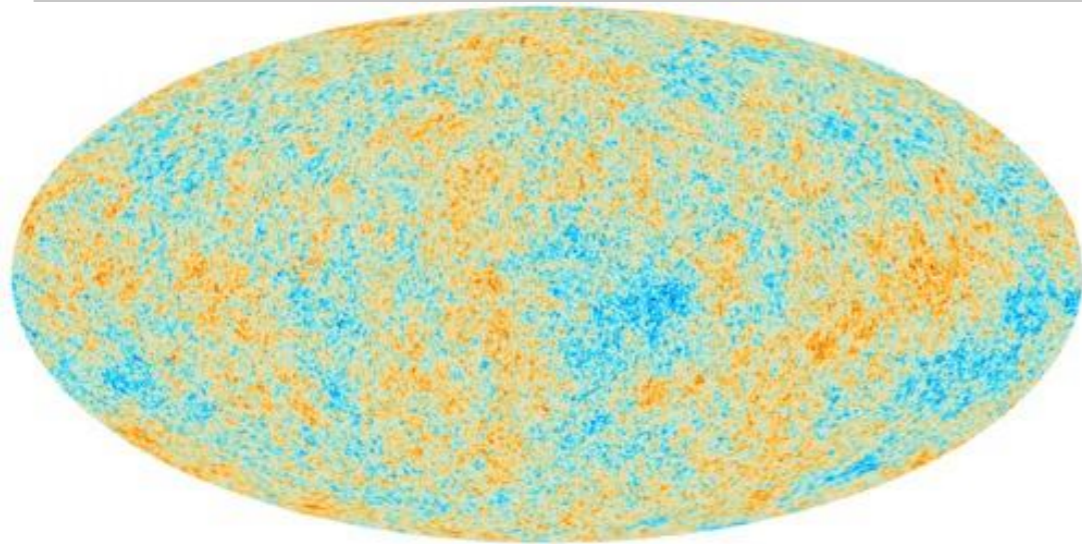
Radiation, baryons and Dark Matter affect the acoustic oscillations in the primordial plasma in different ways...



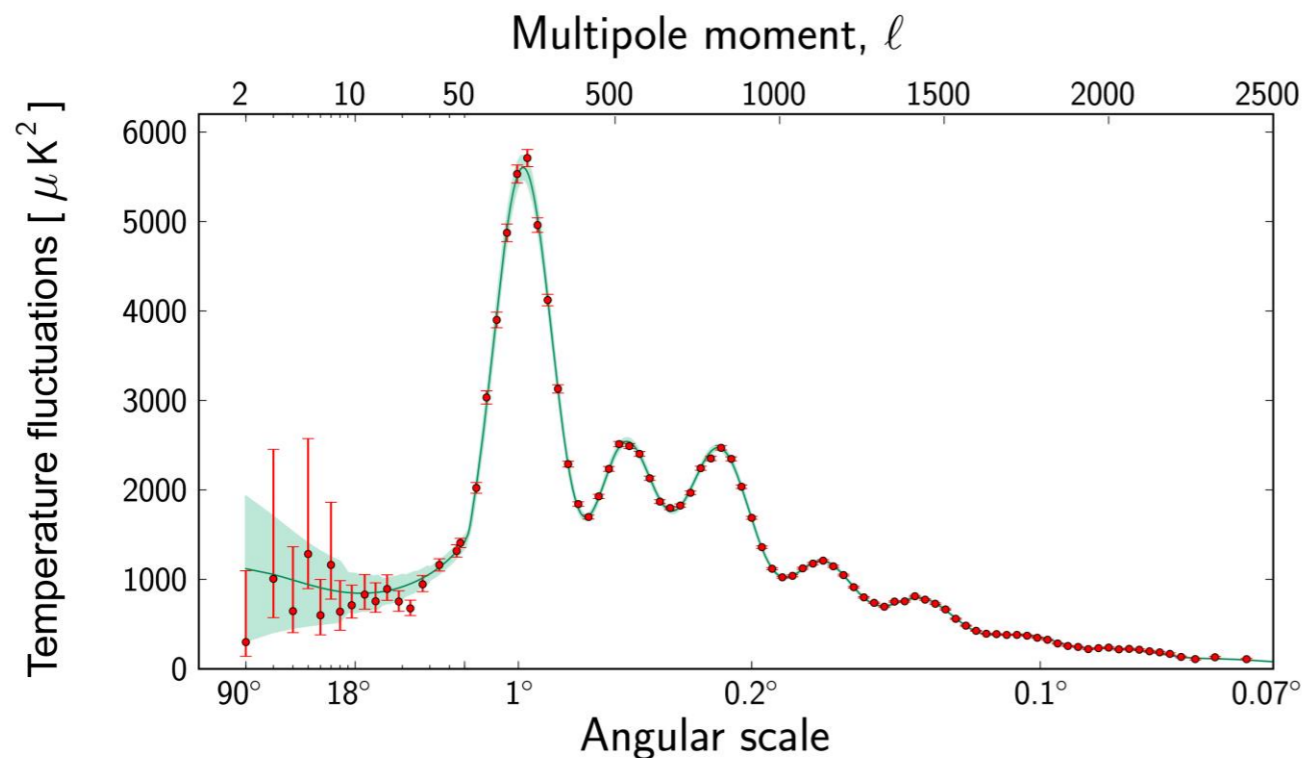
**ordinary matter/radiation oscillates due to radiation pressure**



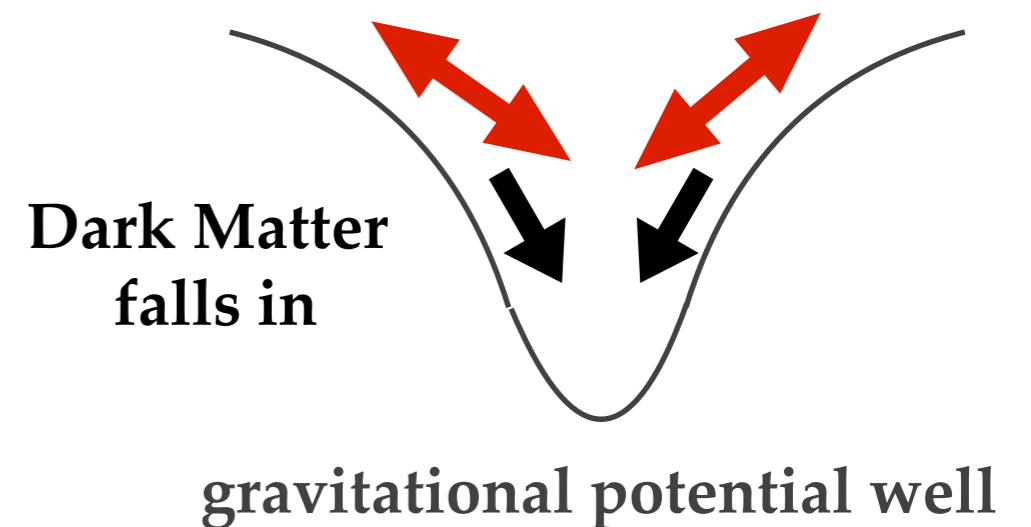
# 3) Cosmic Microwave Background



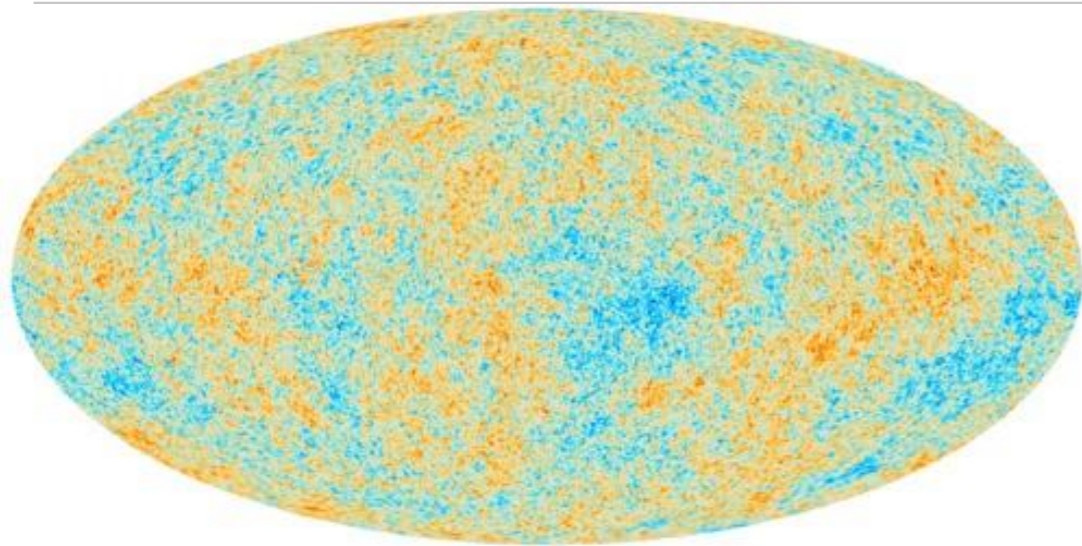
- 1) Modified Gravity?
- 2) Or non-luminous matter?



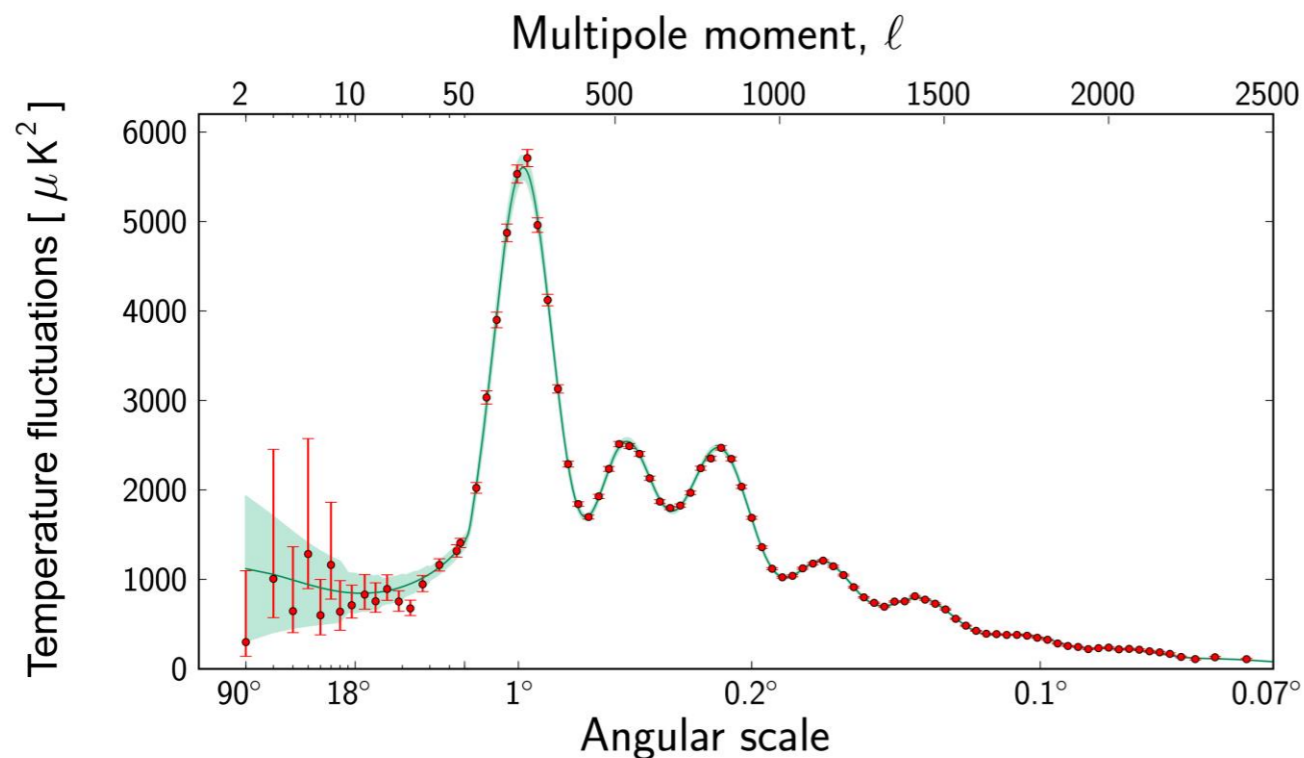
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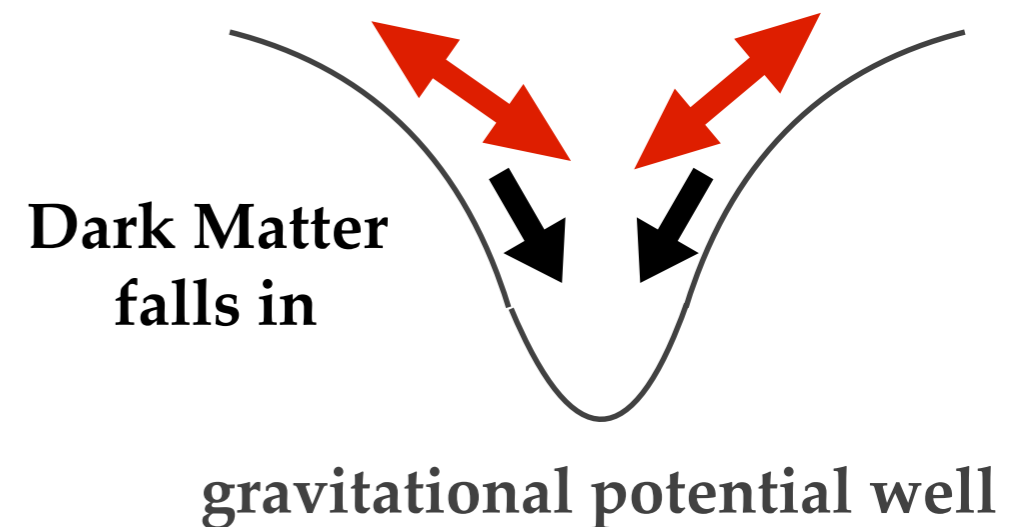
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1) Modified Gravity?  
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Very difficult to explain with modified gravity.  
Shows that DM is not made of lumps of baryons.



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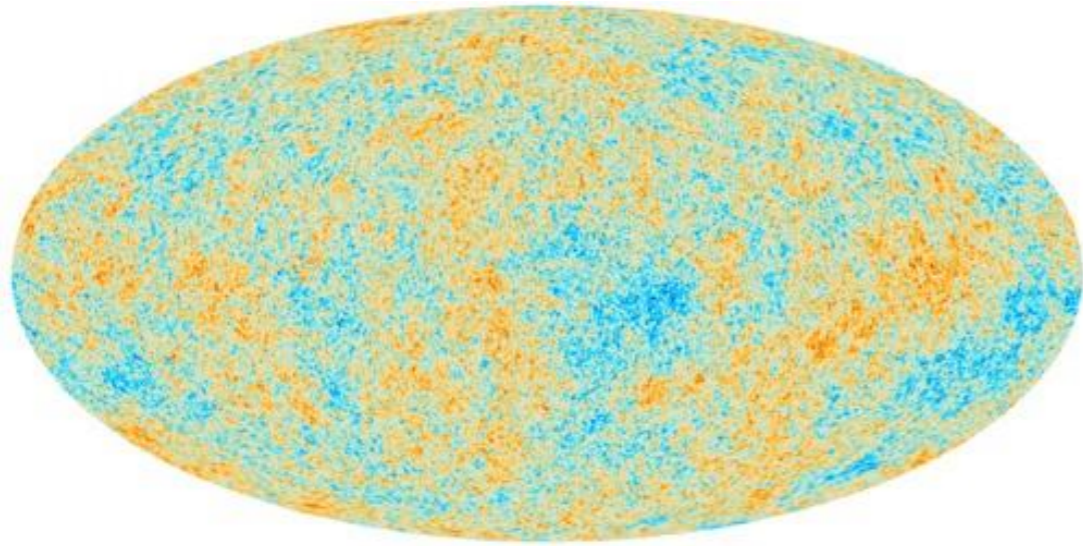




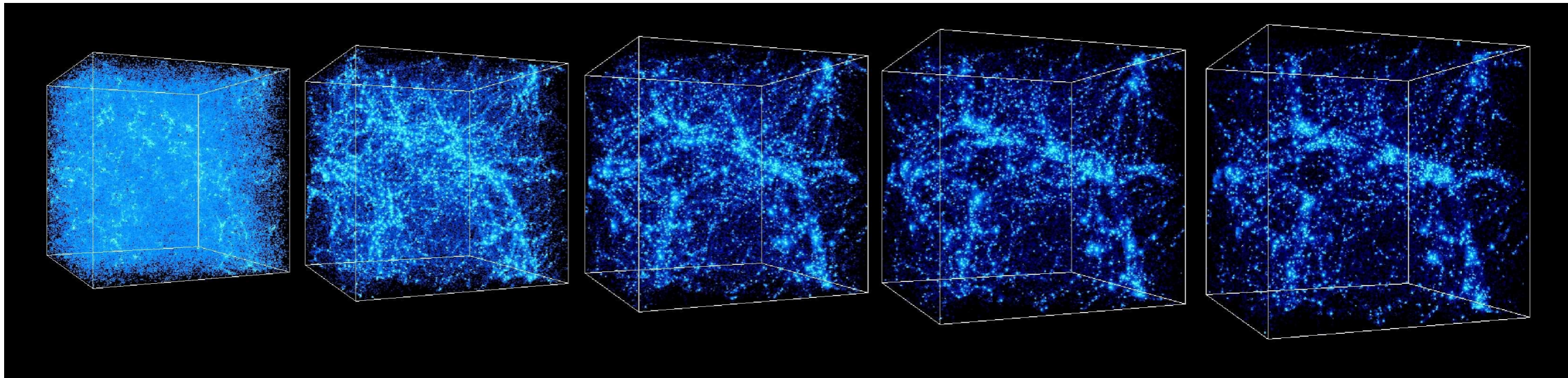
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# 4) Structure Formation

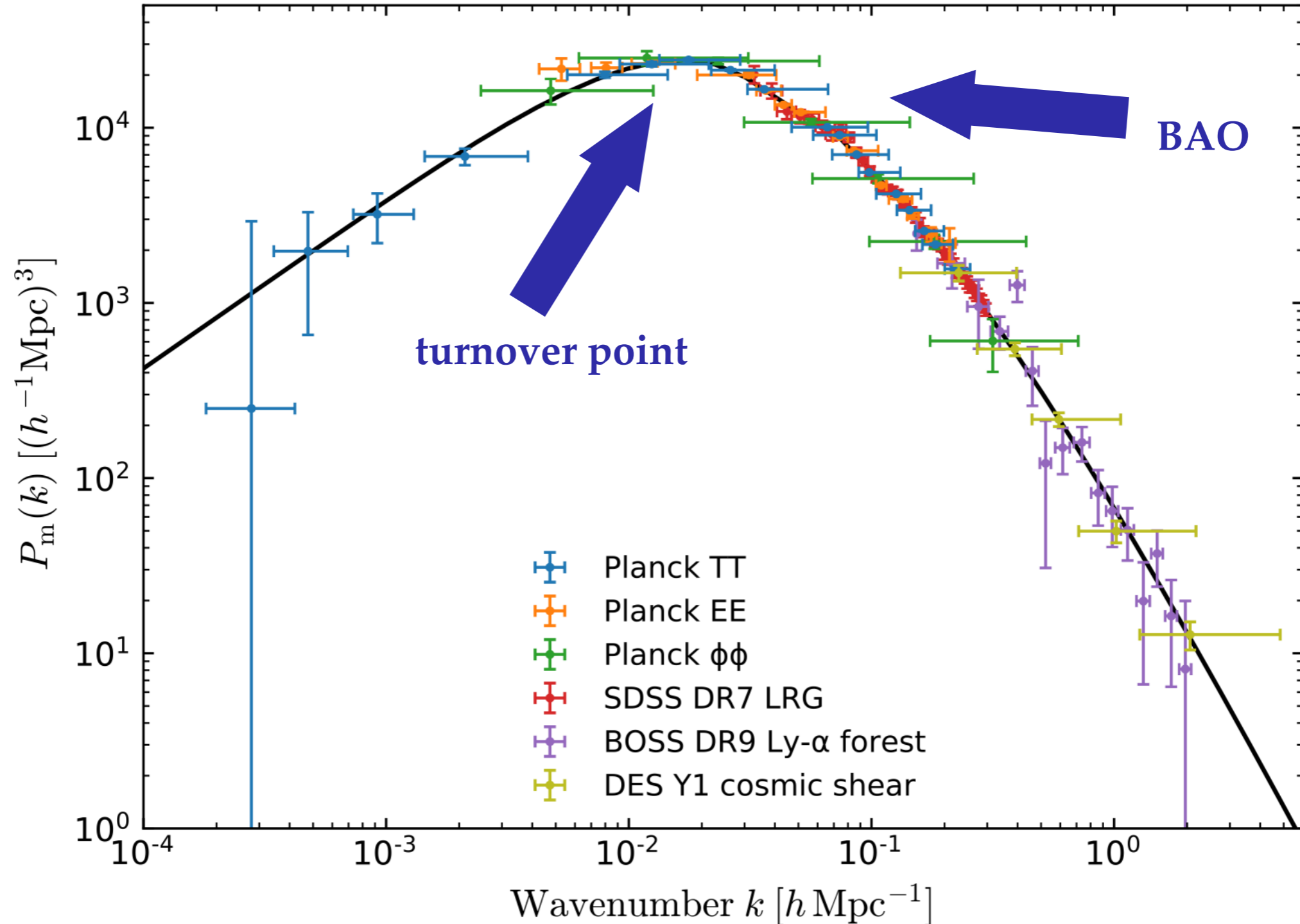
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The early universe was very homogeneous and isotropic...  
... structure formed by gravitational collapse of small inhomogeneities  $\sim 1/100.000$

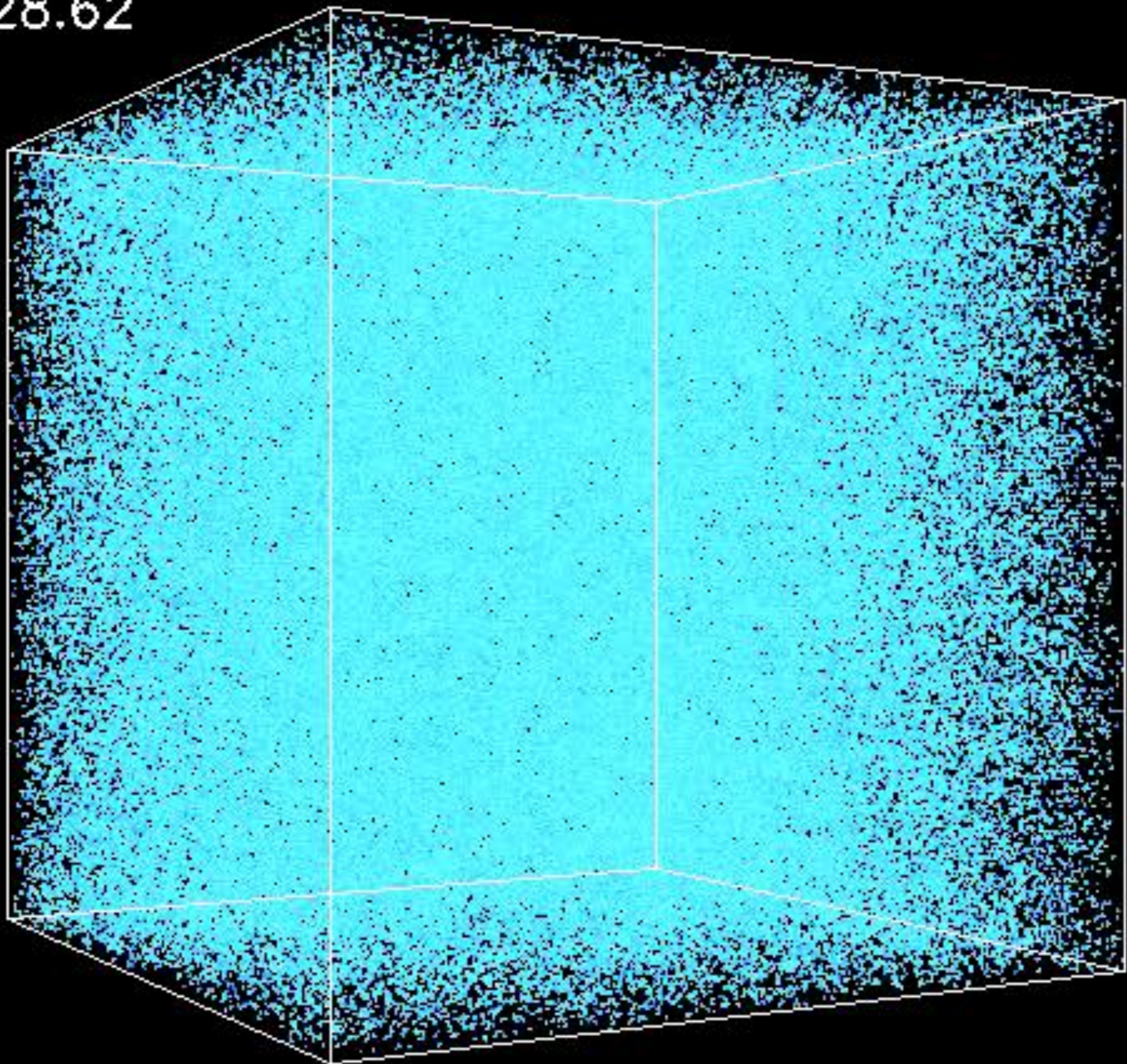


# Matter Power Spectrum



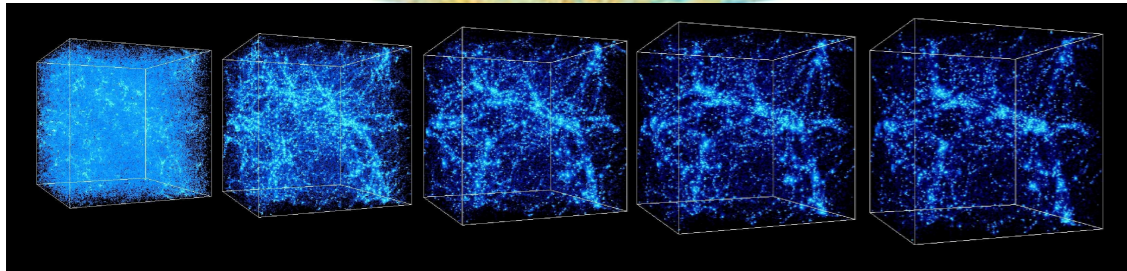
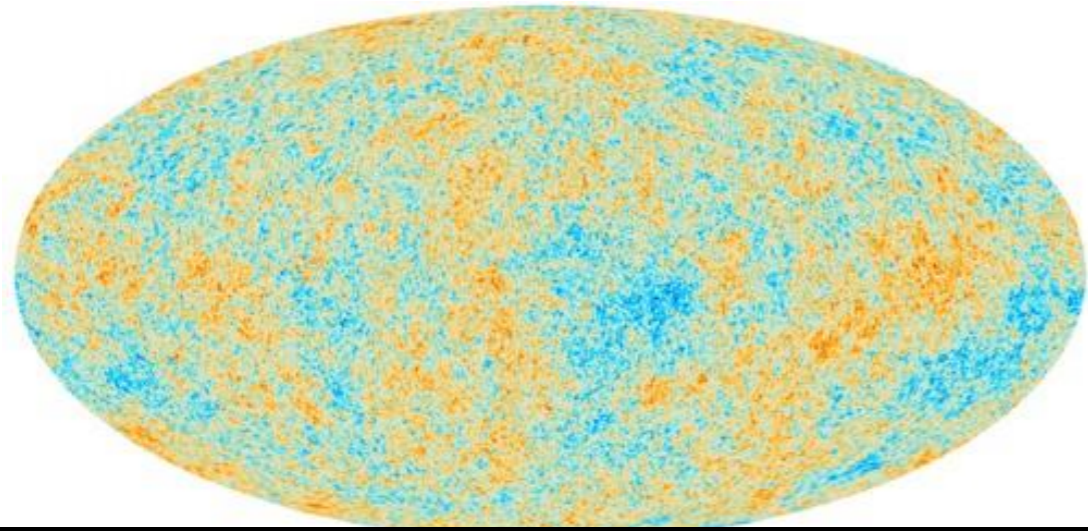


$Z=28.62$



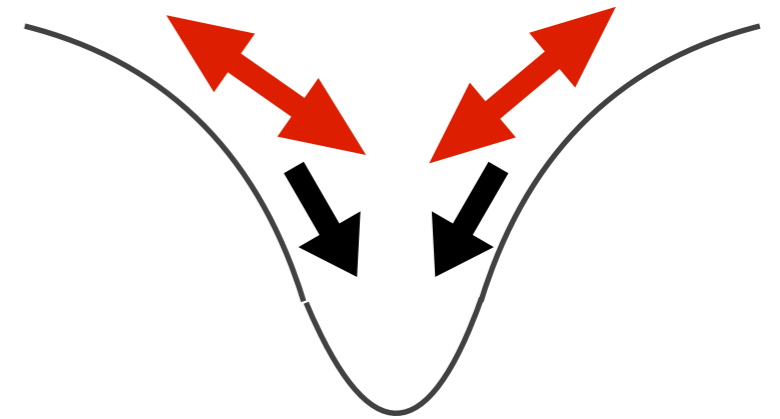


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**ordinary matter/radiation oscillates due to radiation pressure**



gravitational potential well

Simulations only agree with observation if this process starts before the CMB decoupled

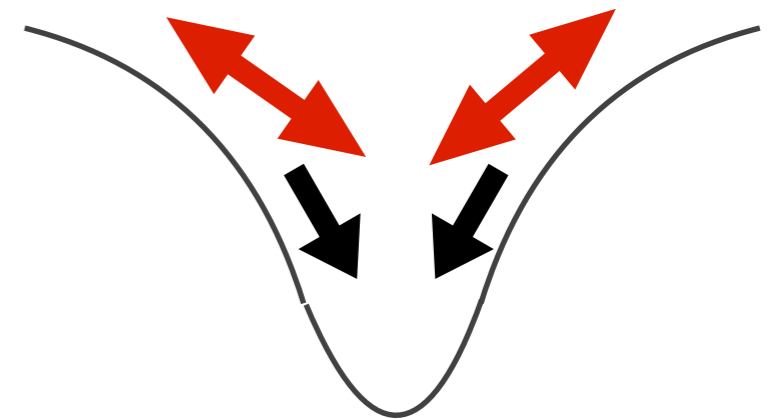
**Must be driven by particles that do not feel the radiation pressure!**

# 4) Structure Formation

- 1) Modified Gravity?
- 2) Or non-luminous matter?

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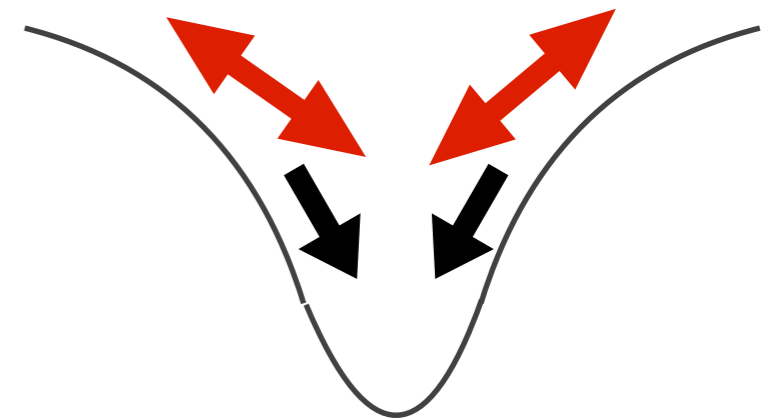
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Very difficult to explain with modified gravity.  
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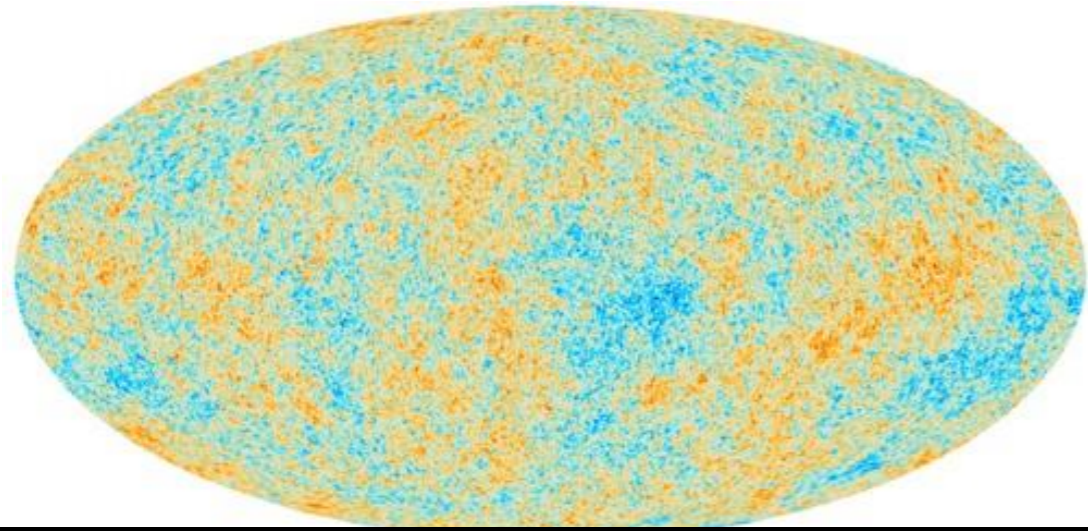
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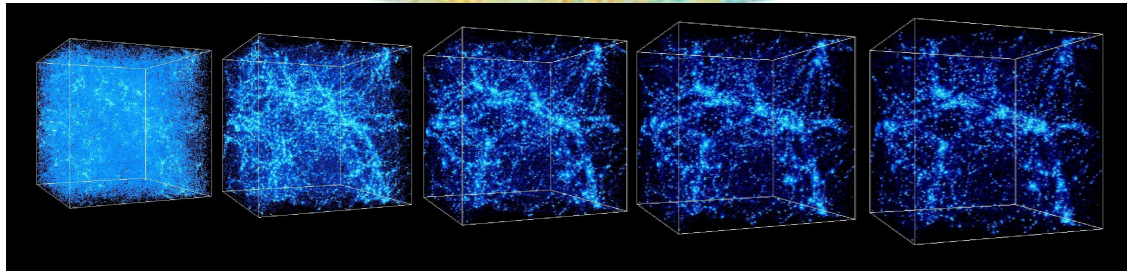


# 4) Structure Formation



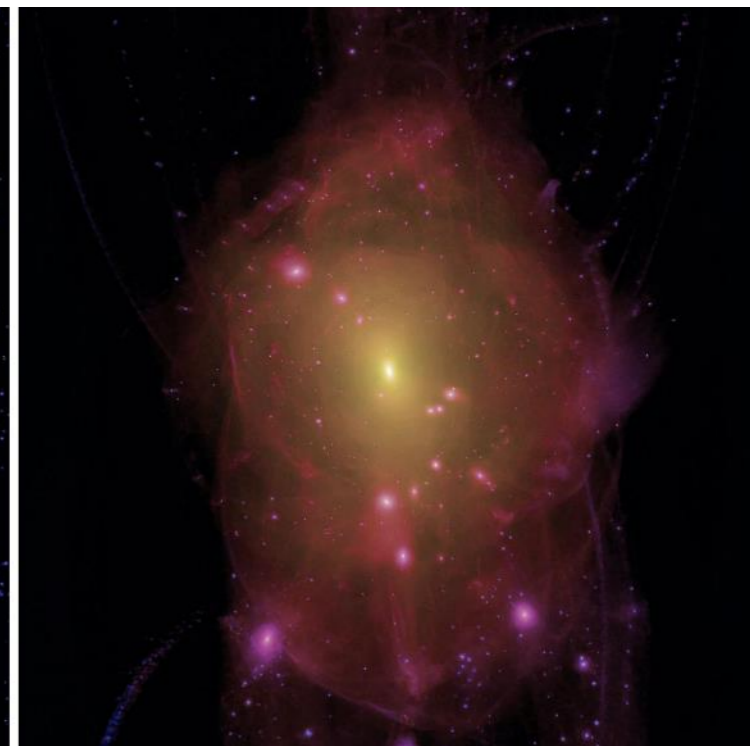
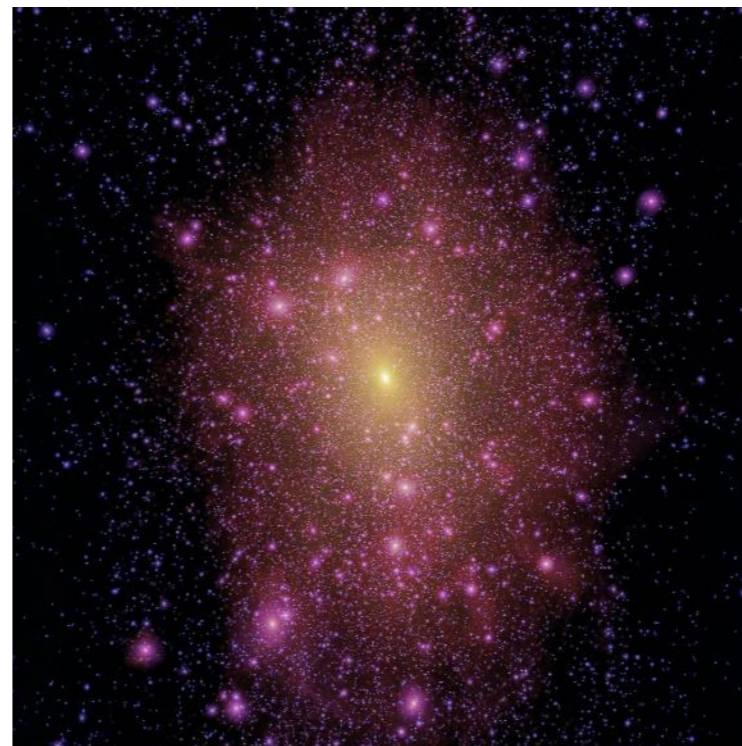
Movement of Dark Matter particles during structure formation “smears out” the inhomogeneities...

... which leads to a suppression of small scale structures in the universe!



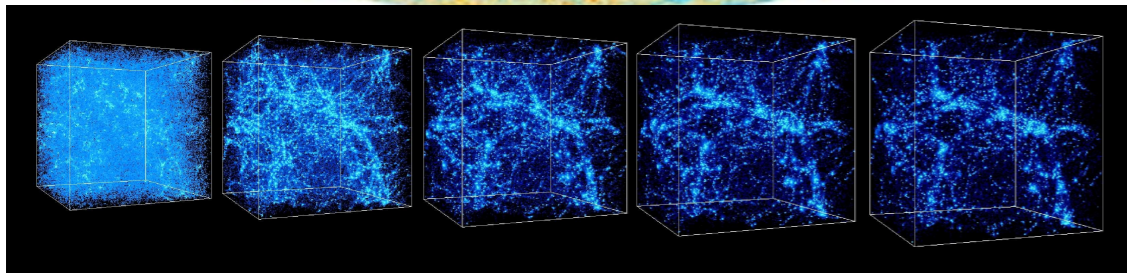
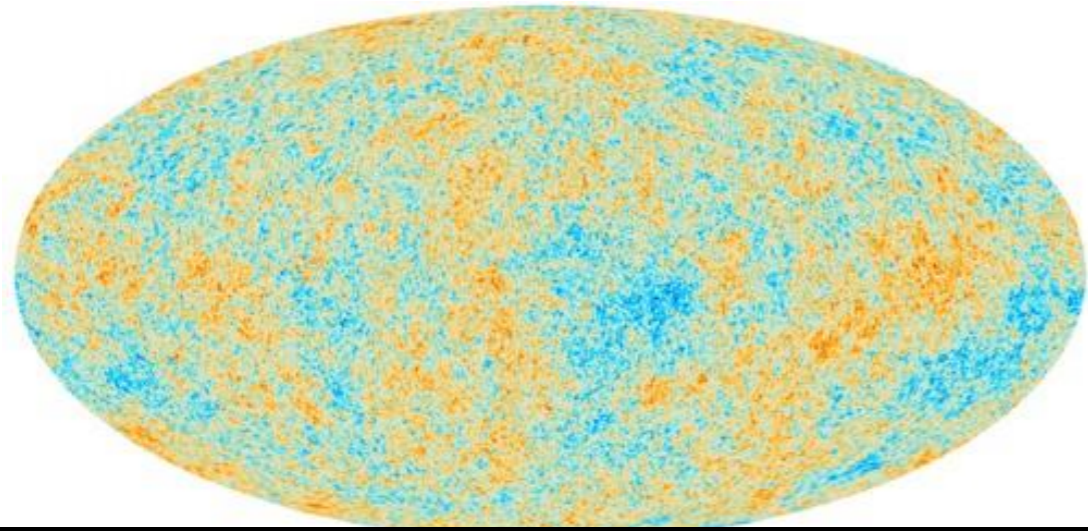
“cold” DM

“warm” DM





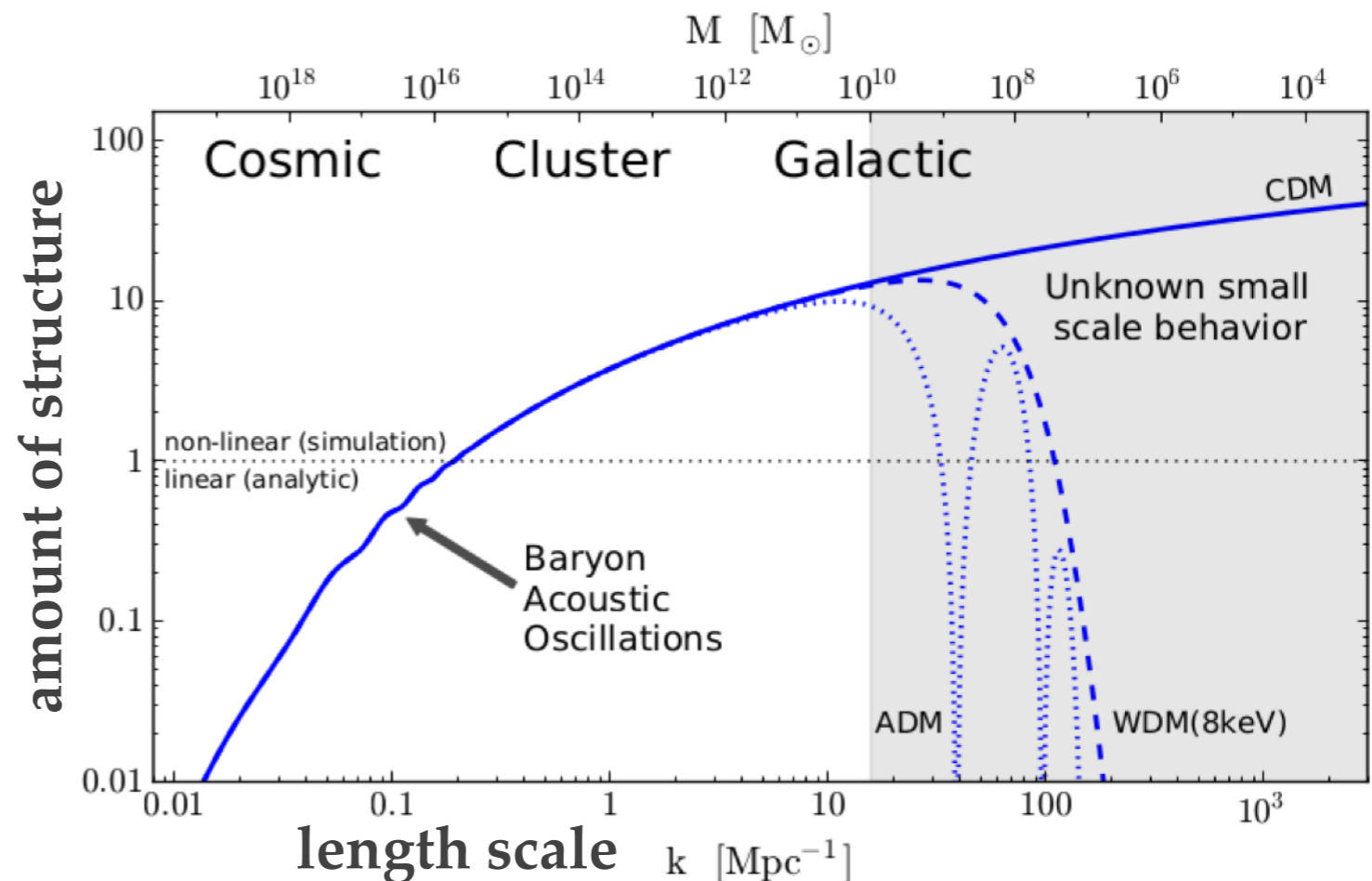
# 4) Structure Formation



This can be quantified in terms of the **matter power spectrum**

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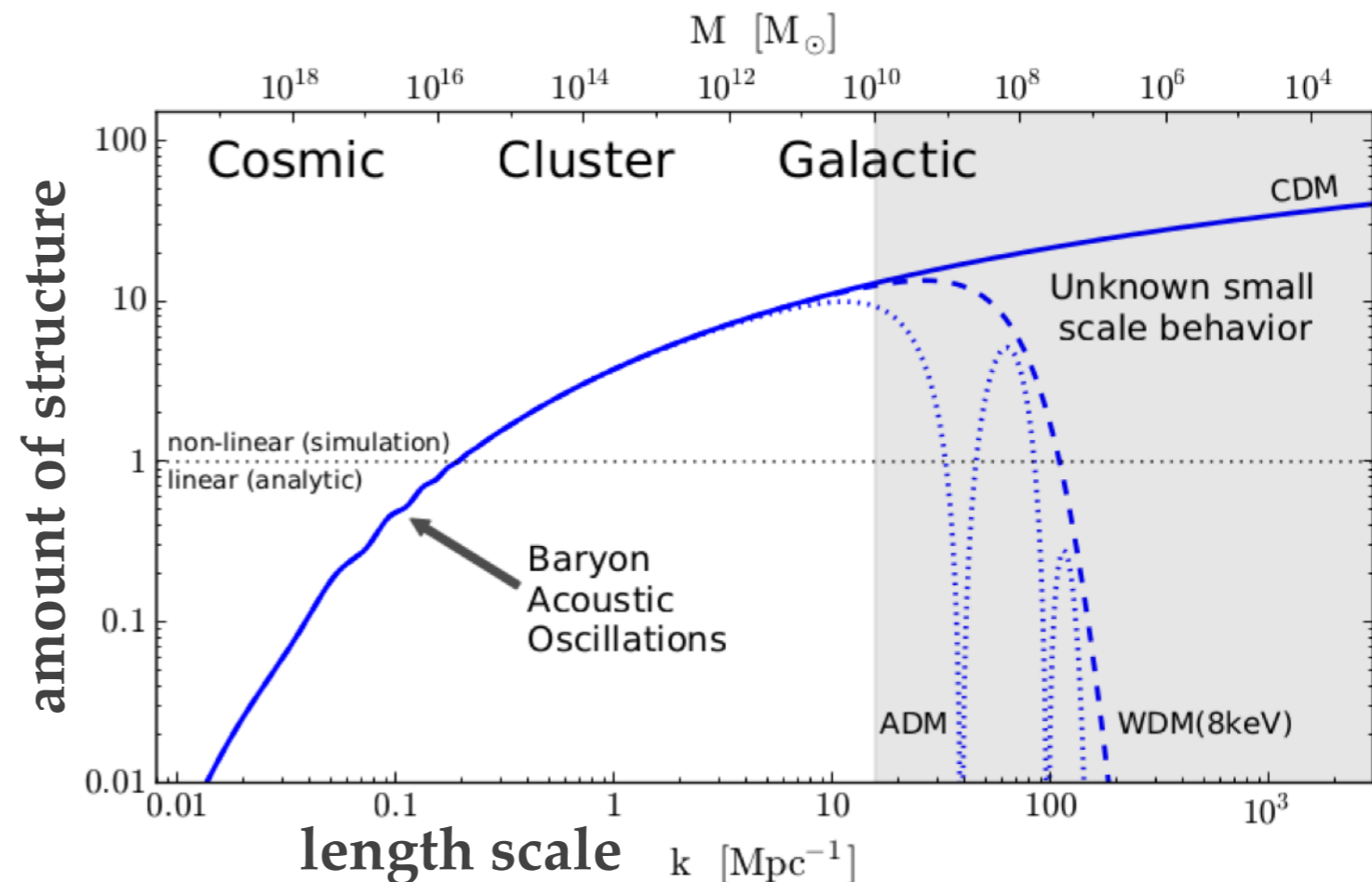
# 4) Structure Formation

Dark Matter must be (relatively) “cold”, i.e., non-relativistic at the time of structure formation

This can be quantified in terms of the **matter power spectrum**

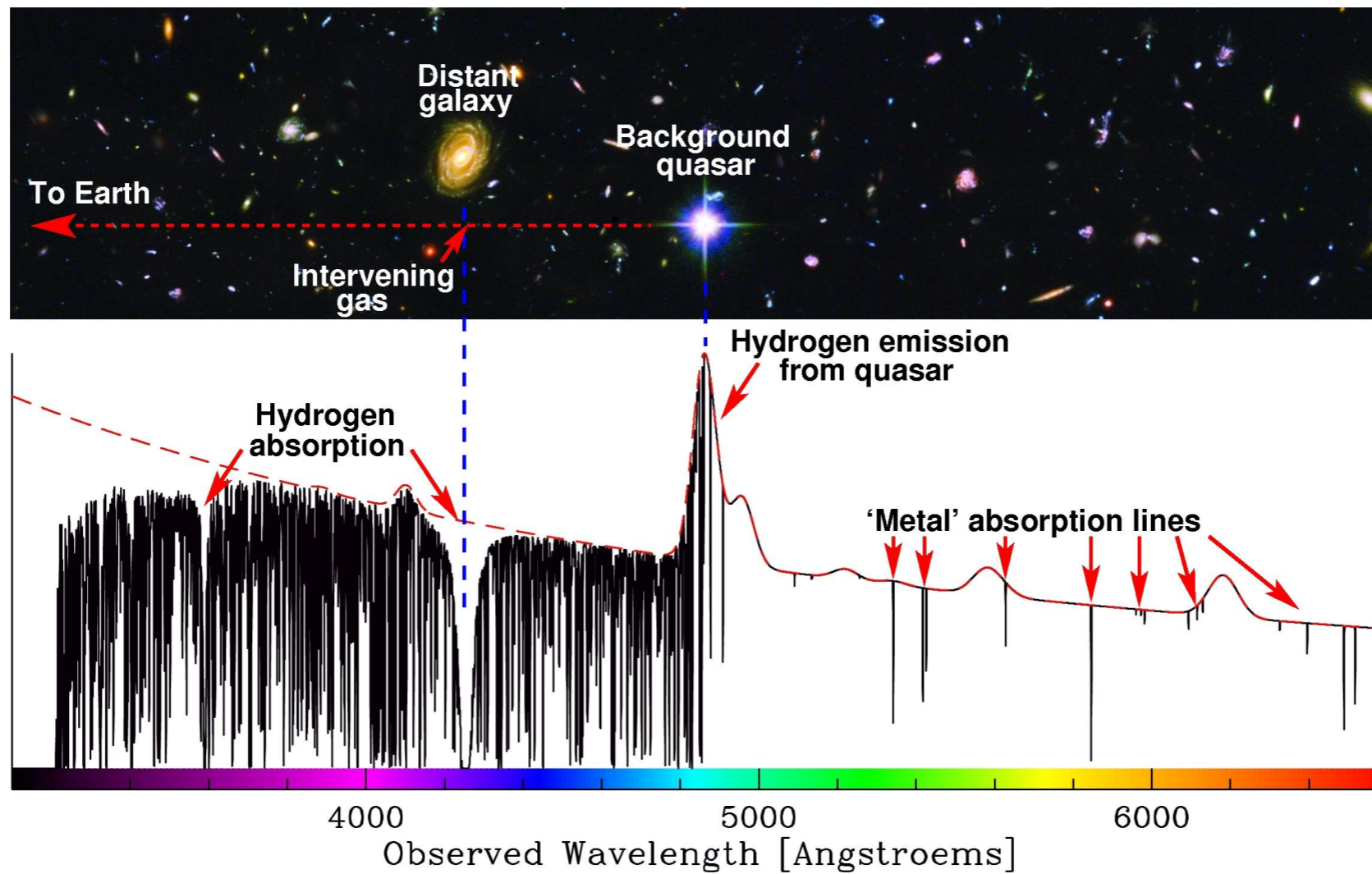
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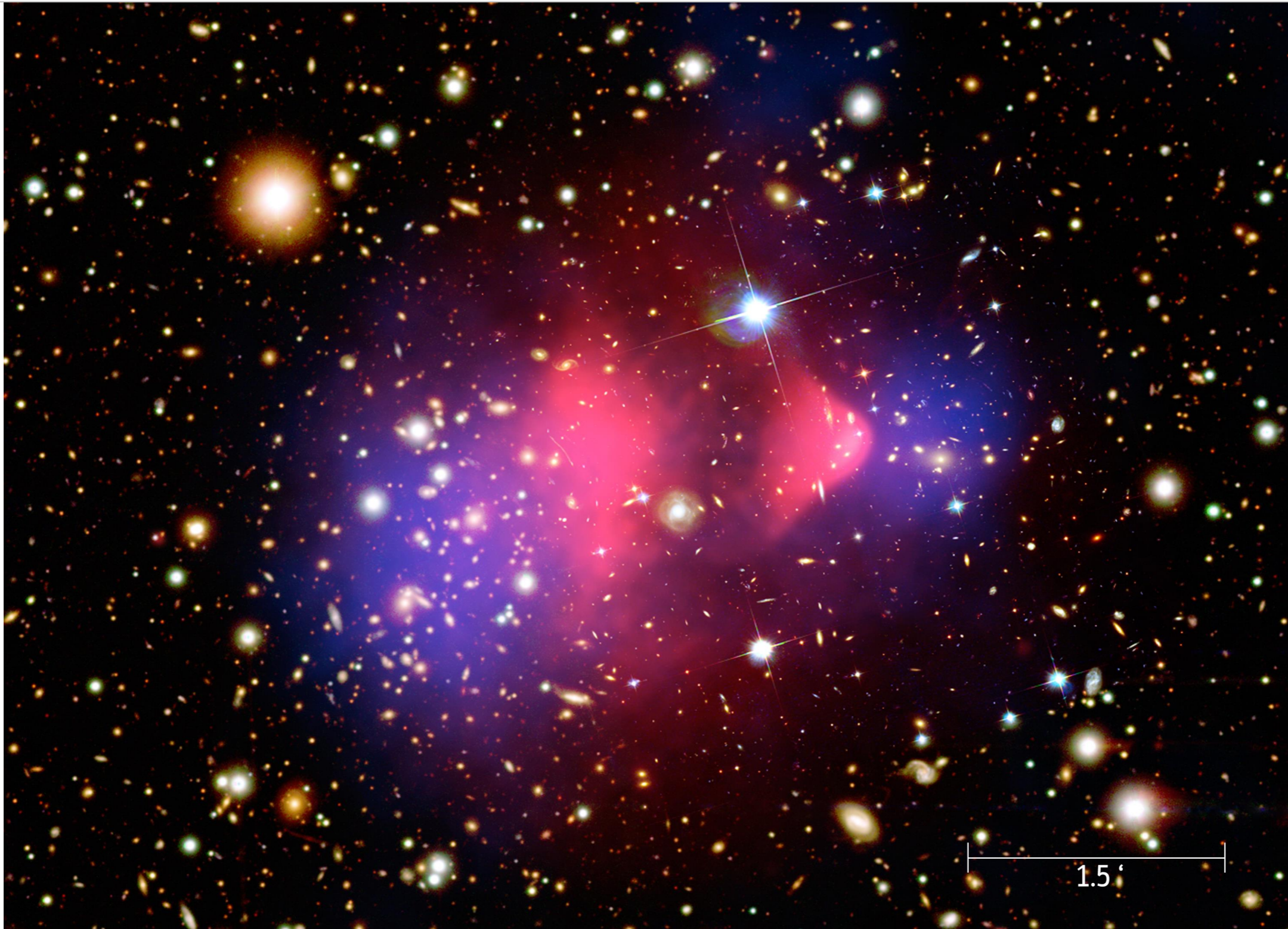


# Lyman Alpha Forest



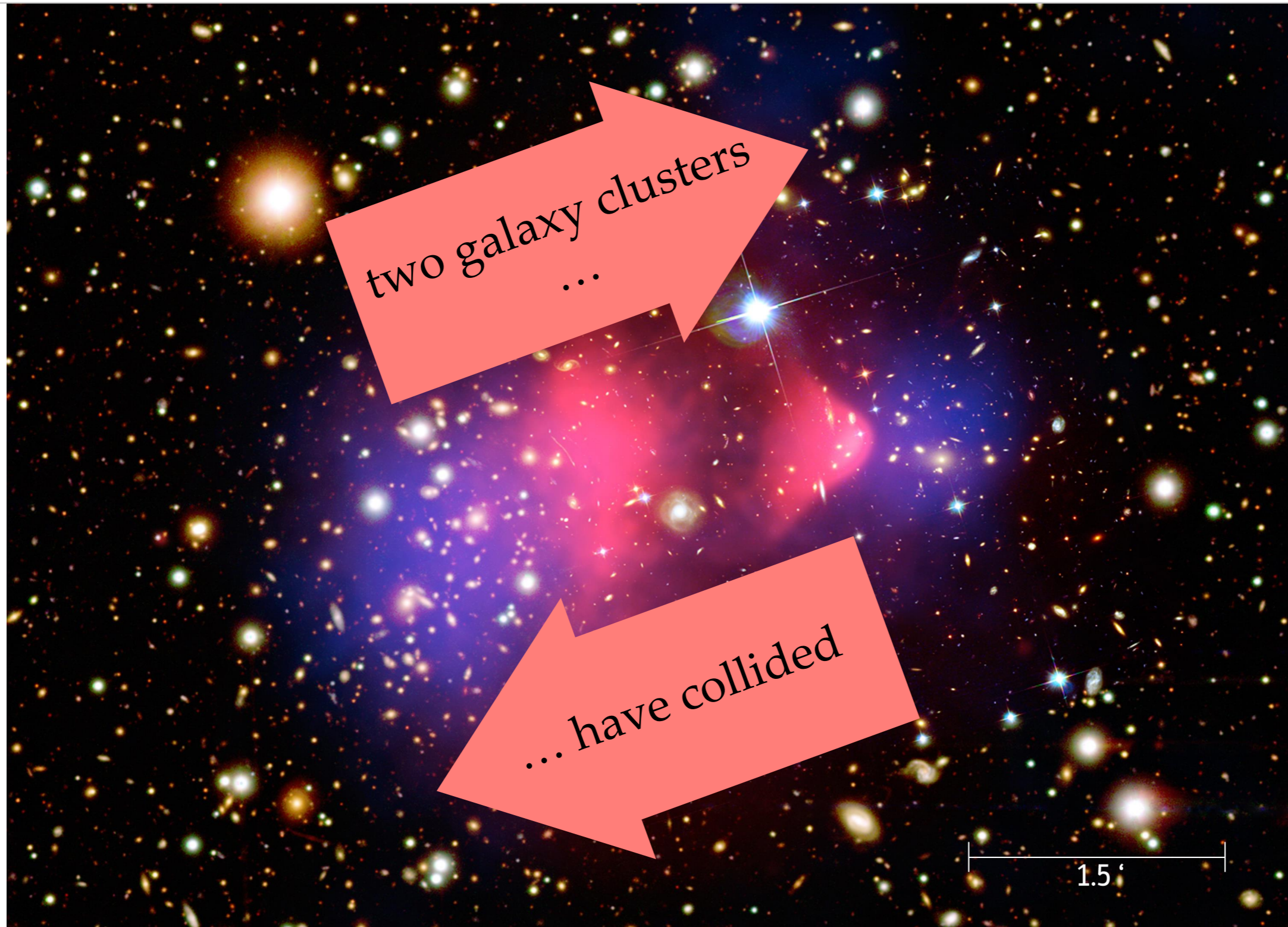


# 5) Bullet Cluster (and friends)



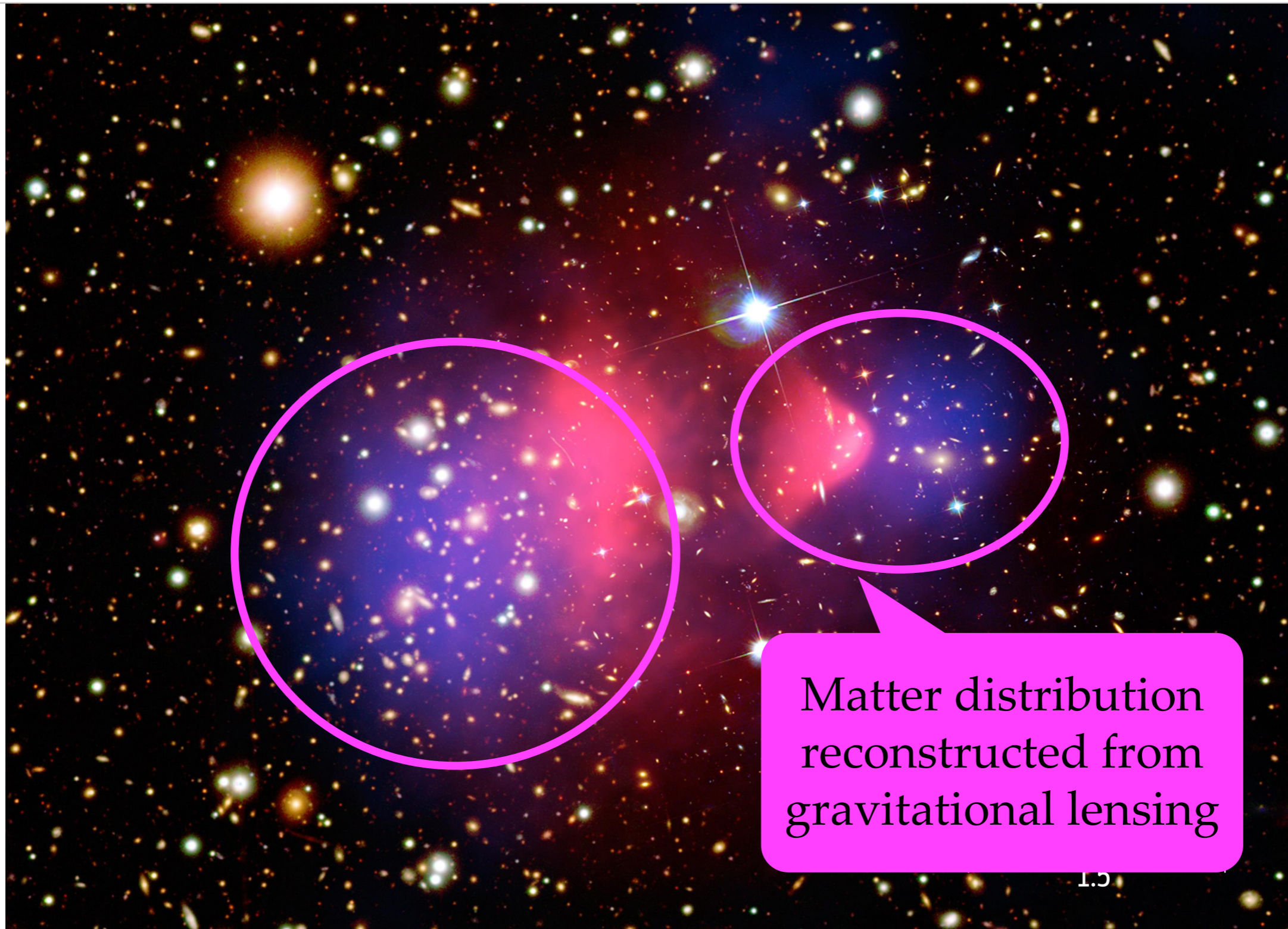


# 5) Bullet Cluster (and friends)





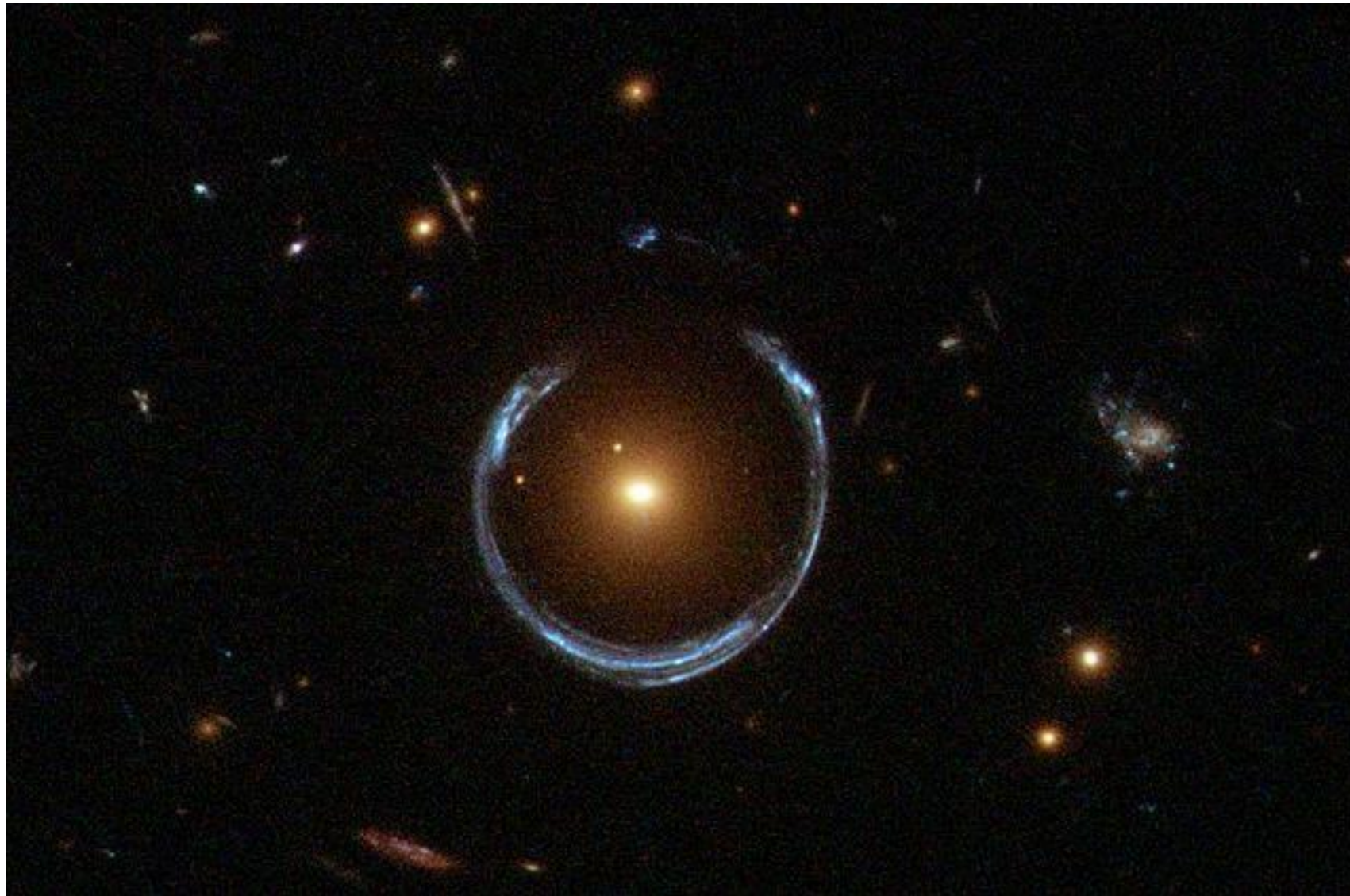
# 5) Bullet Cluster (and friends)



Matter distribution  
reconstructed from  
gravitational lensing



d friends)



Matter distribution  
reconstructed from  
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# 5) Bullet Cluster (and friends)

Matter distribution  
seen in X-ray  
observations

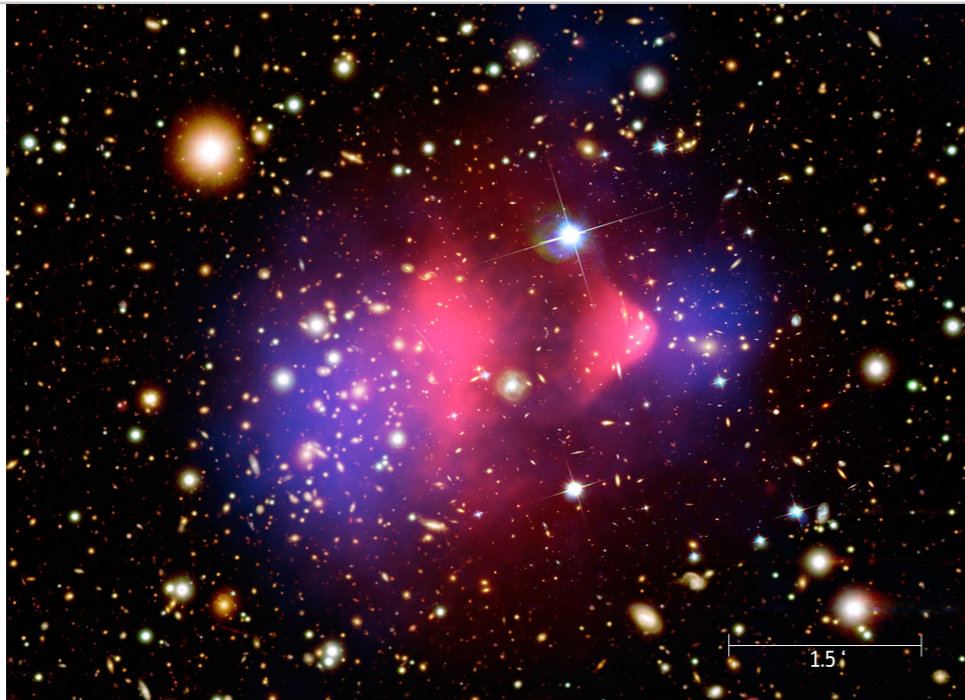
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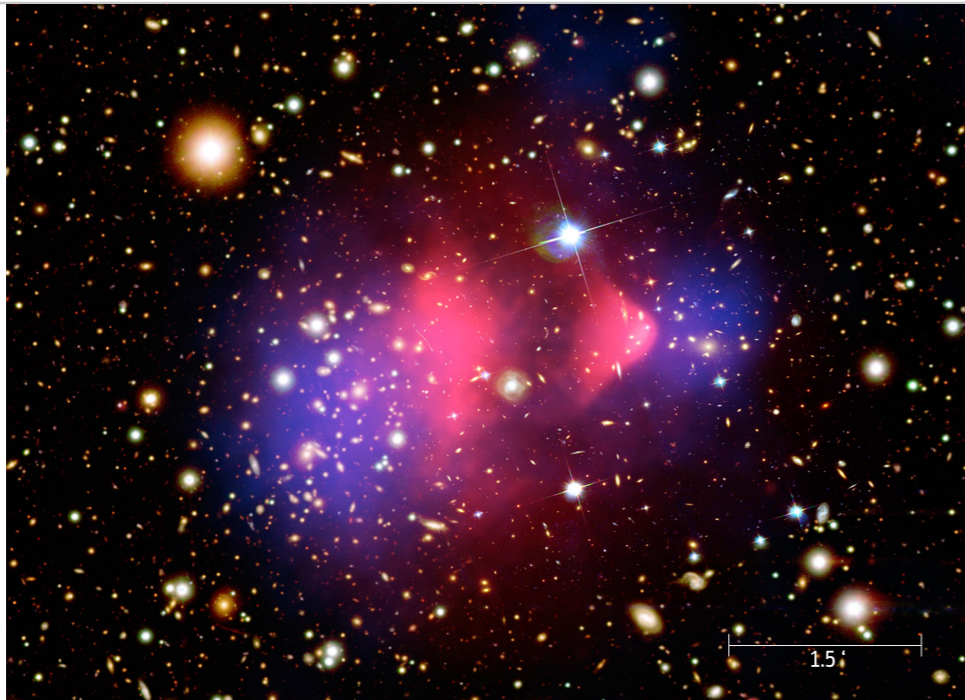


## **Interpretation:**

The visible matter scatters and undergoes a merger.

The Dark Matter is collision free and passes.

# 5) Bullet Cluster (and friends)



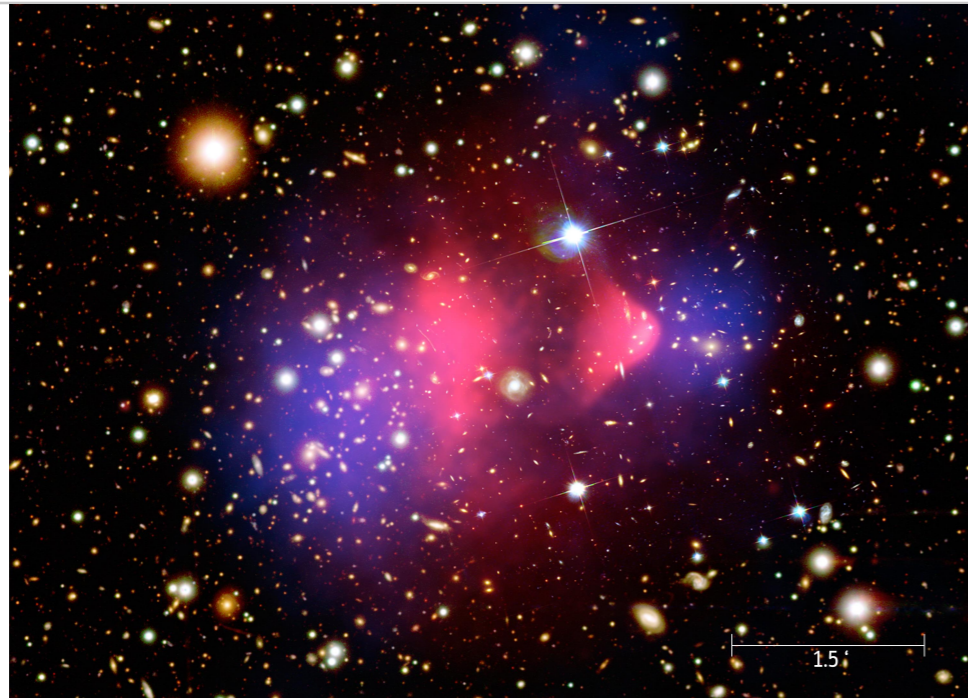
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## **Interpretation:**

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**Very difficult to explain with  
modified gravity.**

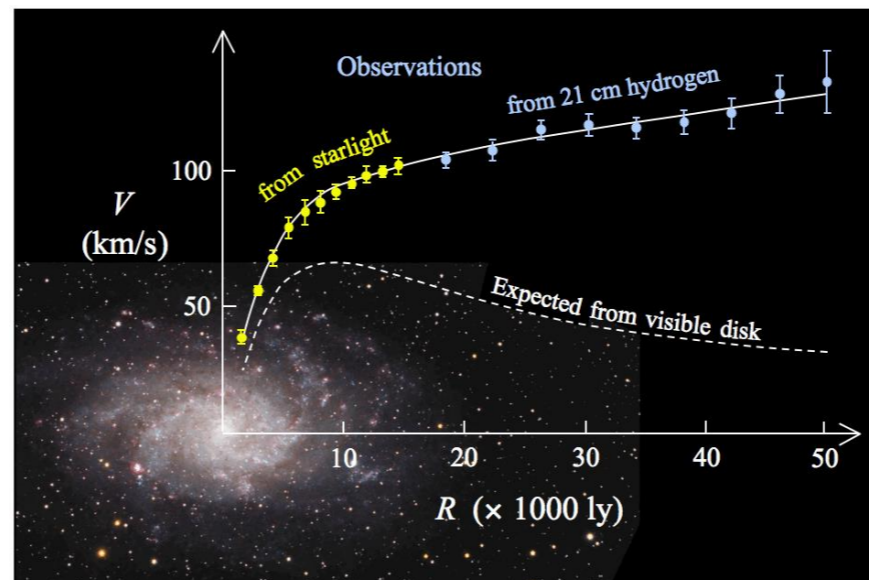
**Shows that DM is collisionless.**



# Summary

There is compelling evidence that ~80% of the mass in the universe is made of particles that are

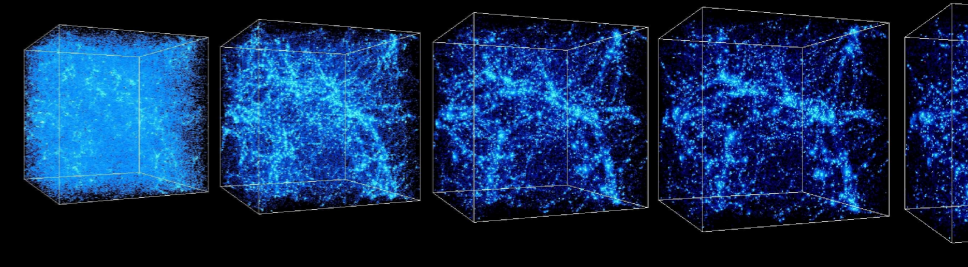
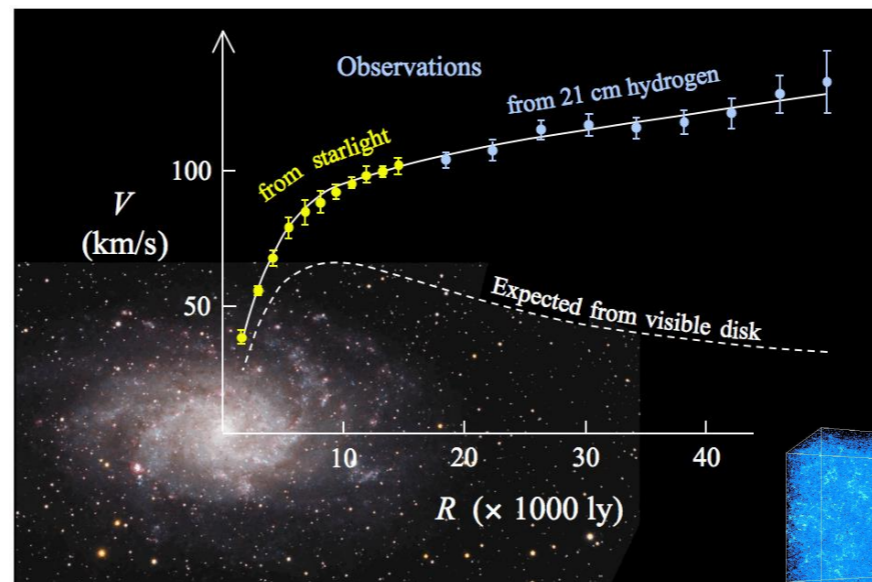
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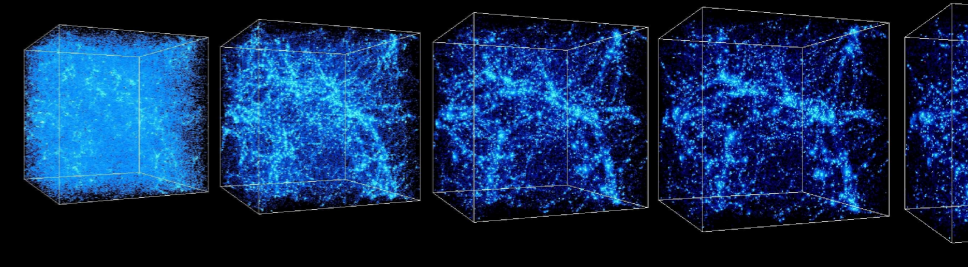
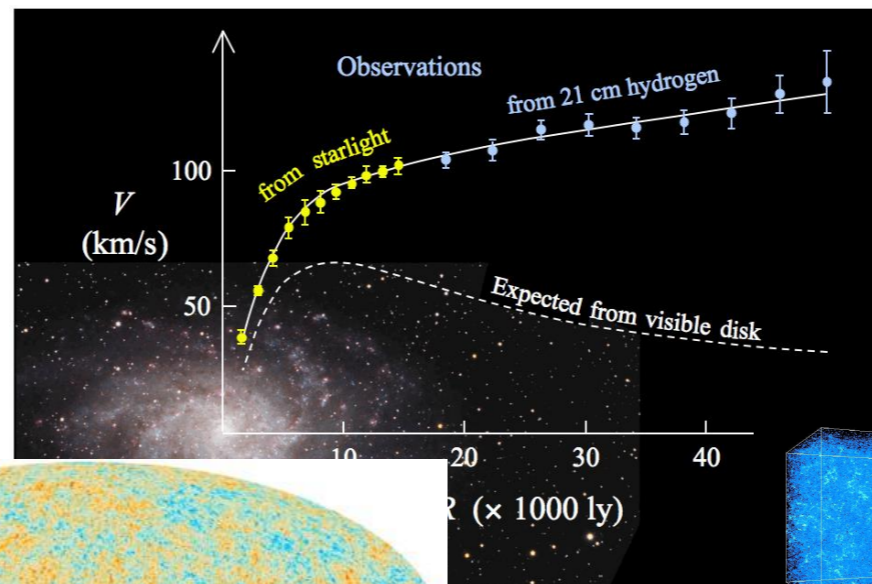
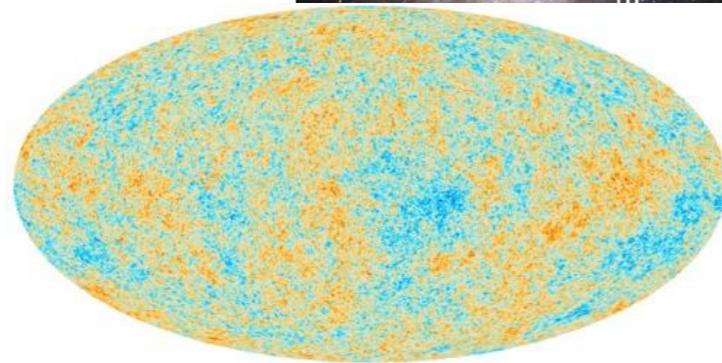
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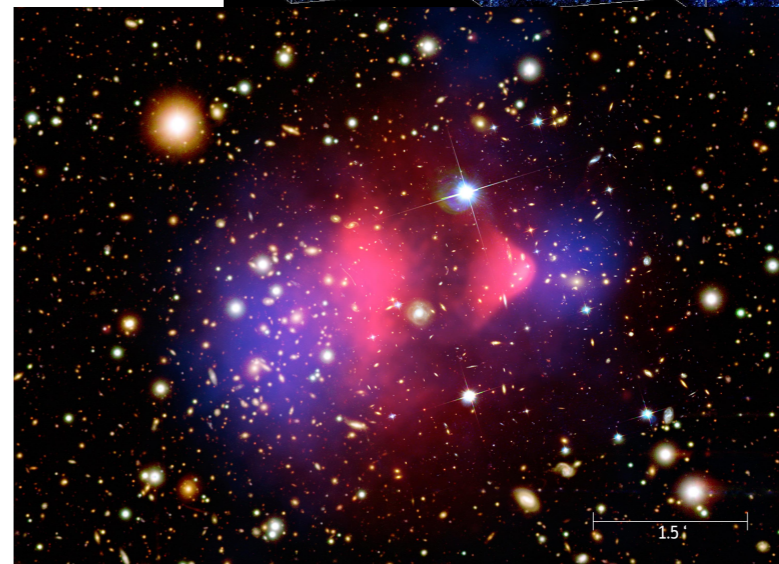
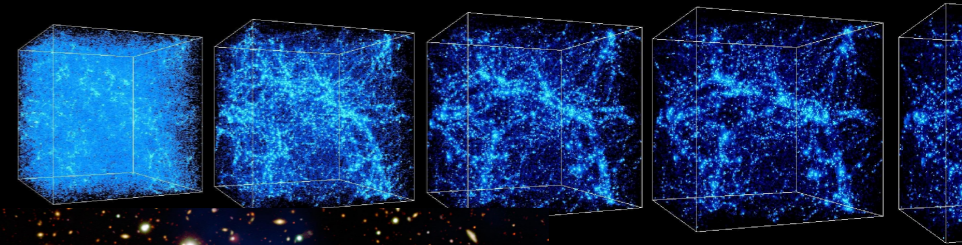
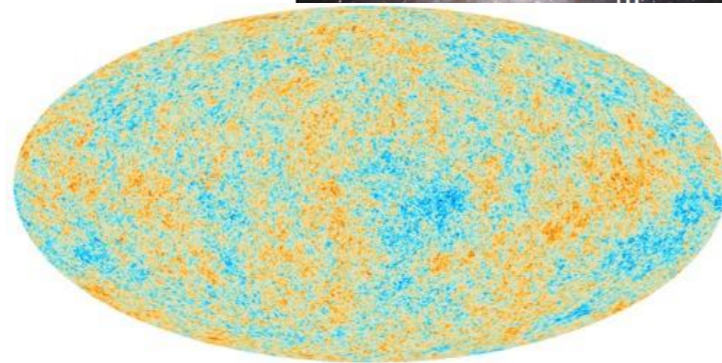
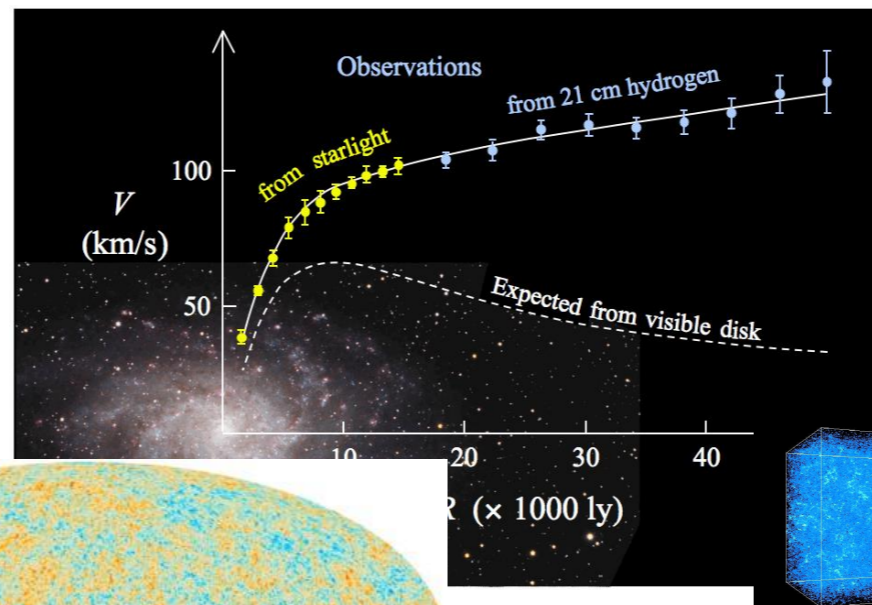




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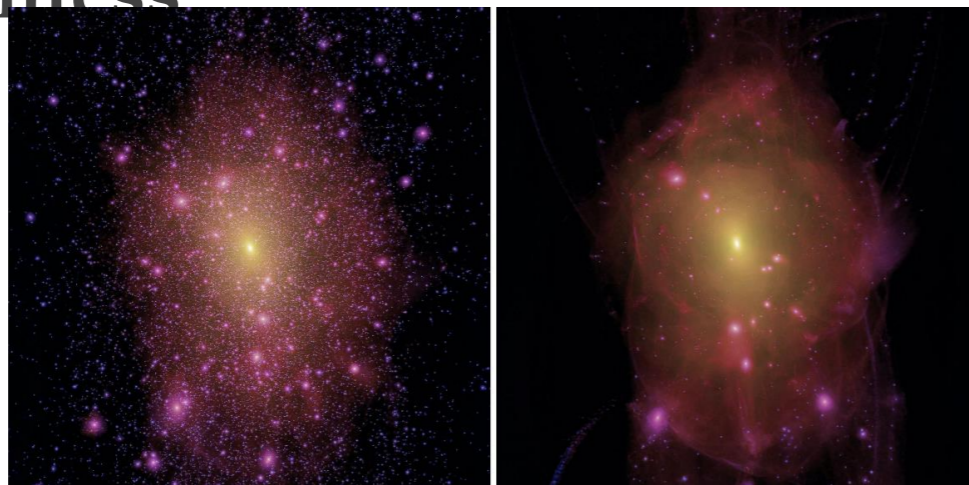
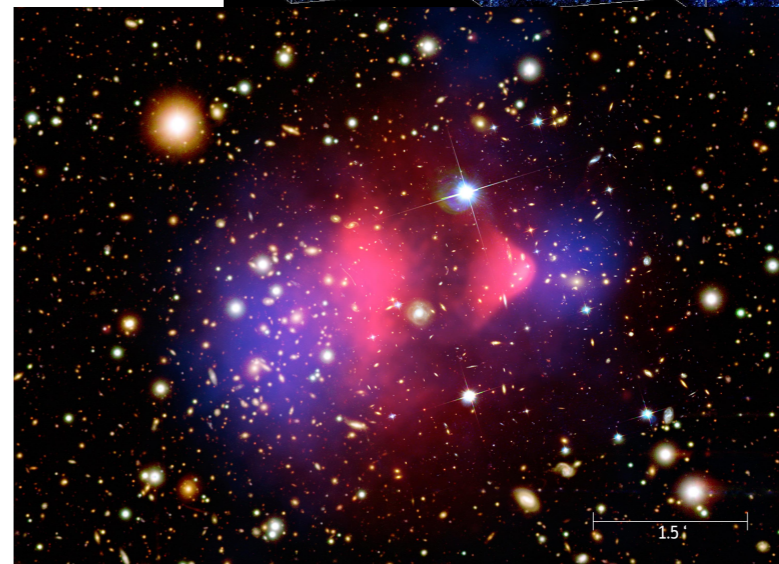
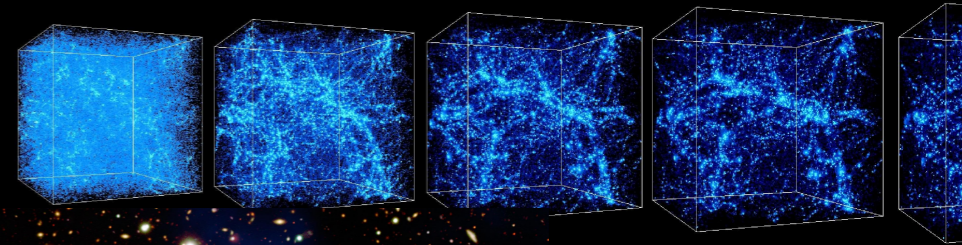
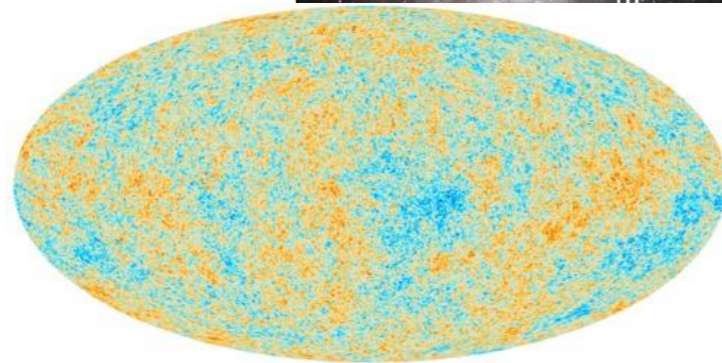
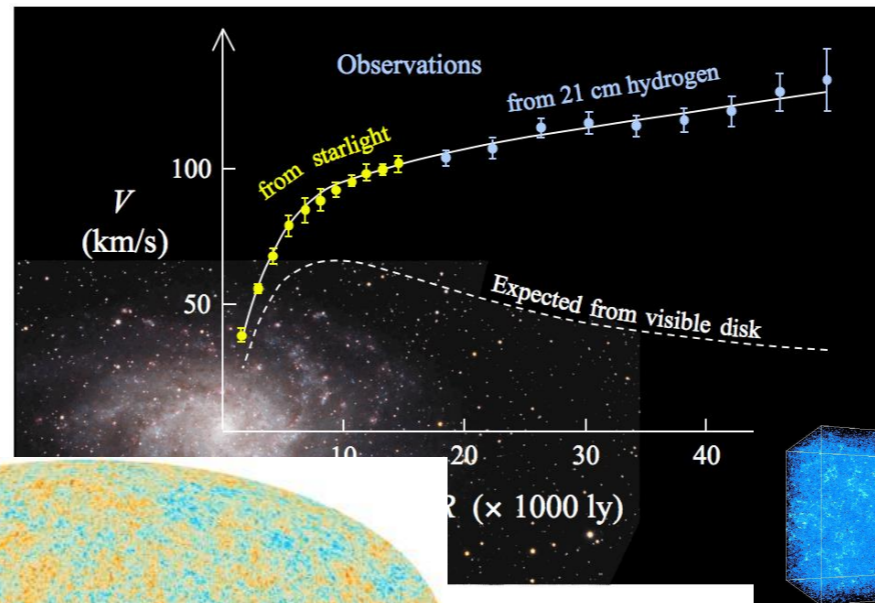




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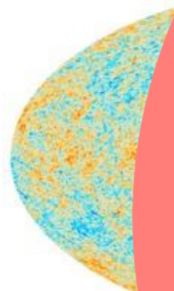
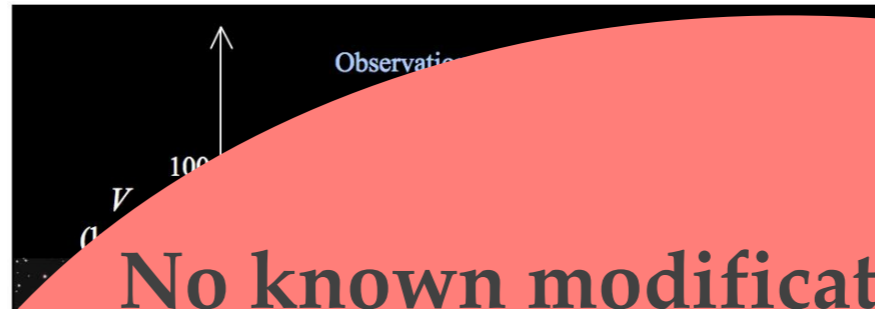
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No known modification of gravity can explain all of this.

The Standard Model cannot, either.

A simple extension by one (or several) new “Dark Matter” particles could do the job.

...but no such “Dark Matter” particle has been seen yet

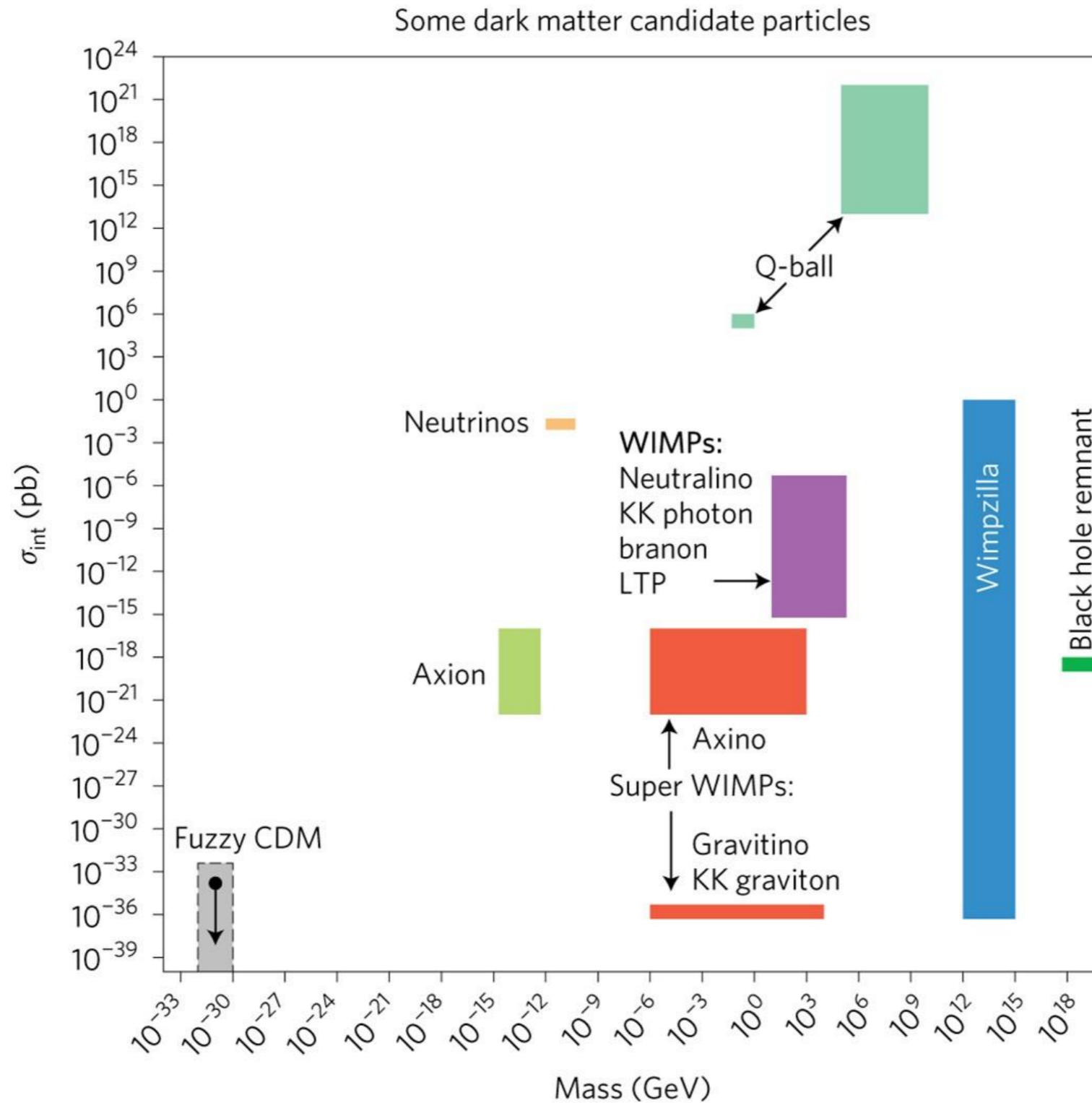




# DM Candidates

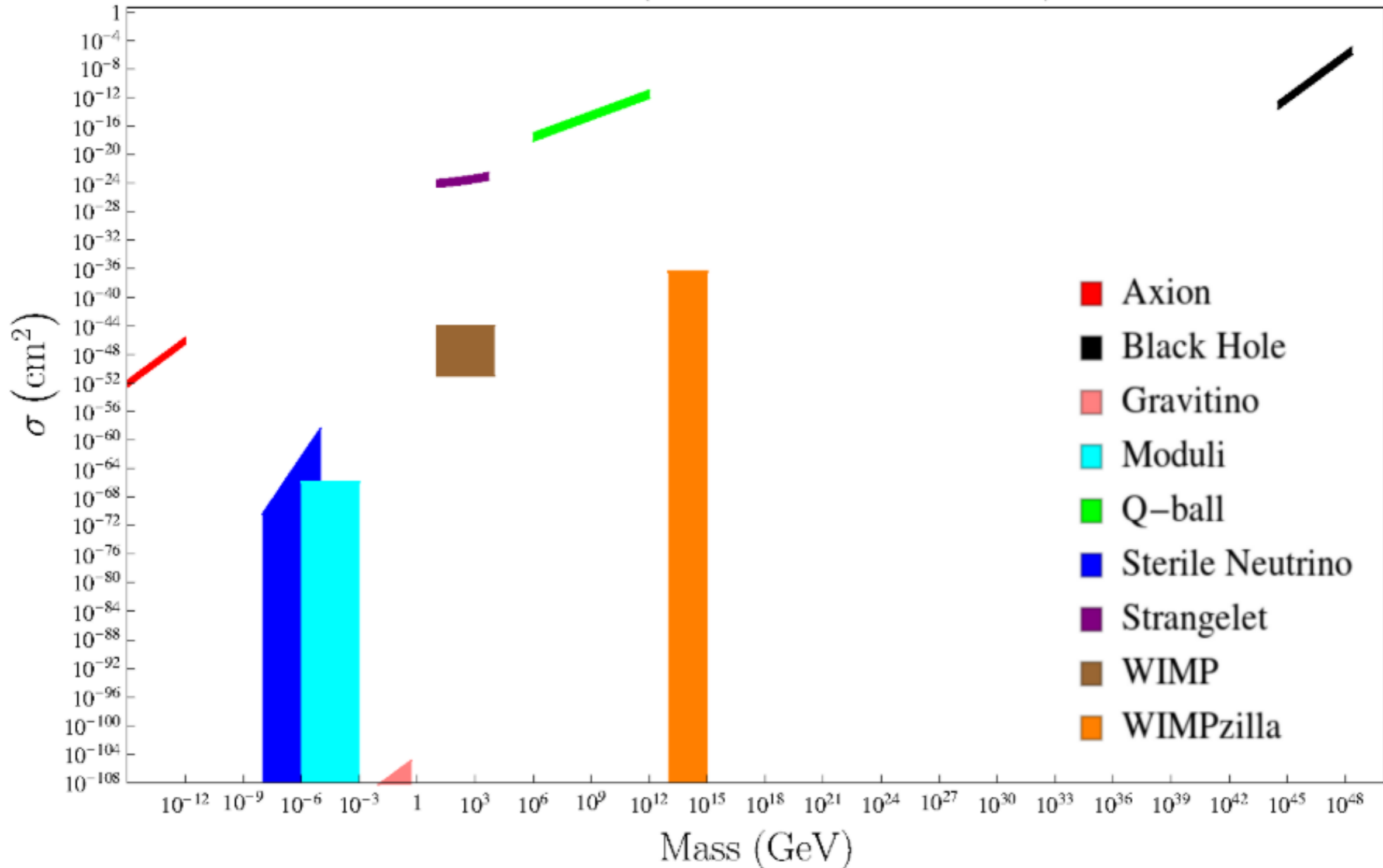


# DM Candidates



# DM Candidates

Cross Section (Xenon for Reference)





# The WIMP Miracle

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# Particle Dark Matter

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**Let's assume that Dark Matter is made of new particles...**

- I) How did it come into existence?**
- II) How can we uncover what it is?**

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# Particle Dark Matter

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Let's assume that Dark Matter is made of new particles...

- I) How did it come into existence?
- II) How can we uncover what it is?

It depends on the kind of particle that we postulate.

Today: **The WIMP.**



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# Weakly Interacting Massive Particles

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**There are two reasons to believe in WIMPs:**

- i) They “naturally” appear in many theories beyond the SM (e.g. supersymmetry)**
- ii) They “naturally” give the correct amount of Dark Matter.**

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# The WIMP Miracle

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**[following Kolb/Turner chapter 5]**

# Useful relations in equilibrium

number density	$n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3p$
energy density	$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3p$
pressure density	$p = \frac{g}{(2\pi)^3} \int \frac{ \vec{p} ^2}{3E} f(\vec{p}) d^3p$

ultra-relativistic limit  
("radiation")

$$\rho = \begin{cases} (\pi^2/30)gT^4 & \text{(BOSE)} \\ (7/8)(\pi^2/30)gT^4 & \text{(FERMI)} \end{cases}$$
$$n = \begin{cases} (\zeta(3)/\pi^2)gT^3 & \text{(BOSE)} \\ (3/4)(\zeta(3)/\pi^2)gT^3 & \text{(FERMI)}, \end{cases}$$
$$p = \rho/3$$

non-relativistic limit  
("matter")

$$n = g \left( \frac{mT}{2\pi} \right)^{3/2} \exp[-(m - \mu)/T]$$
$$\rho = mn$$
$$p = nT \ll \rho.$$

the exponential suppression implies that the number densities are dominated by "radiation"



# Effective number of relativistic degrees of freedom

number density  $\rho = \begin{cases} (\pi^2/30)gT^4 & \text{(BOSE)} \\ (7/8)(\pi^2/30)gT^4 & \text{(FERMI)} \end{cases}$

energy density  $\varepsilon = \begin{cases} (\zeta(3)/\pi^2)gT^3 & \text{(BOSE)} \\ (3/4)(\zeta(3)/\pi^2)gT^3 & \text{(FERMI)}, \end{cases}$

pressure density  $p = \rho/3$

ultra-relativistic limit  
("radiation")

the total radiation and pressure densities can be written as

$$\rho_R = \frac{\pi^2}{30} g_* T^4,$$

$$p_R = \rho_R/3 = \frac{\pi^2}{90} g_* T^4$$

with the effective number of relativistic degrees of freedom

$$g_* = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4$$

analogously we find for the **entropy density**

$$s = \frac{2\pi^2}{45} g_{*s} T^3, \quad \text{with} \quad g_{*s} = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3$$

$$H = \frac{\dot{a}}{a} = \text{HUBBLE PARAMETER}$$

$\Gamma_i$  = RATE AT WHICH PARTICLES OF TYPE  $i$   
INTERACT

$\Gamma_i \gtrsim H \Rightarrow$  PARTICLES ARE "COUPLED"

$\Gamma_i \lesssim H \Rightarrow$  "DECOUPLED"

BOLTZMANN EQ. ABSTRACT:  $f_i(\mathbf{x}, \mathbf{p}) =$  PHASE SPACE  
DISTRIBUTION

ON-STREAM

$$\hat{L}[\{f_i\}] = \mathcal{L}[\{f_i\}]$$

ACTS ON  
 $f_i$  ONE BY  
ONE  
"FREE STREAMING"

COUPLES  
 $f_i$

CLASSICAL NEWTONIAN MECH:

$$\hat{L} = \frac{d}{dt} + \frac{d\vec{x}}{dt} \cdot \vec{\nabla}_x - \frac{d\vec{p}}{dt} \cdot \vec{\nabla}_p$$



$$\hat{L} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \frac{\partial}{\partial p^\mu}$$

$\Gamma^\mu_{\alpha\beta}$  = CHRISTOFFEL SYMBOLS

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\sigma} (\partial_\beta g_{\sigma\alpha} + \partial_\alpha g_{\sigma\beta} - \partial_\sigma g_{\alpha\beta})$$

$g_{\alpha\beta}$  = METRIC

LEMNITZ FRIED MANN - ROBERTSON - WALKER =

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\psi^2 \right]$$

$$\Rightarrow L[\vec{r}_i] = \omega_p \frac{d\vec{r}_i}{dt} - \underbrace{\frac{\dot{a}}{a}}_H |\vec{p}|^2 \frac{\partial L}{\partial \omega_p}$$

DM MOM. DISTR.  $f(x, p)$  IS IMPORTANT  
FOR STRUCTURE FORMATION

↳ RELEVANT FOR SOME DM CANDIDATE

• BUT FOR WIMP  $\vec{p}$  IS MEASURABLE WHEN  
MATTER DOMINATION STARTS ( $T \sim 0.8 \text{ eV}$ )

$\Rightarrow$  ONLY  $n(t) = g \int \frac{d^3 \vec{p}}{(2\pi)^3} f$  MATTERS.

⇒ DIVIDE BY  $\omega$

$$\dot{f}_i - H \frac{\vec{p}^2}{\omega} \frac{\partial f_i}{\partial \omega} = \frac{1}{\omega} C \left[ \begin{matrix} \vec{p} \\ f_i \end{matrix} \right]$$

$$\int u'v = uv - \int u'v$$

INTEGRATE

$$\dot{f}_i - gH \int \frac{d^3 p}{(2\pi)^3} \underbrace{\frac{\vec{p}^2}{\omega} \frac{\partial f_i}{\partial \omega}} = g \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\omega} C \left[ \begin{matrix} \vec{p} \\ f_i \end{matrix} \right]$$

$$\frac{\vec{p}^2}{\omega} \frac{\partial f}{\partial p} \frac{\partial f}{\partial \omega} = \frac{\vec{p}^2}{\omega} \frac{\omega}{p} \frac{\partial f}{\partial p} = p \frac{\partial f}{\partial p}$$

$$\text{now } g \int \frac{d^3 p}{(2\pi)^3} p \frac{\partial f}{\partial p} = \int d\ell d(\cos \theta) dp p^3 \frac{\partial f}{\partial p} = -g3 \int d\ell d(\cos \theta) dp p^2 f = 3n$$



$$\dot{n} + 3 H n = \frac{g}{(2\pi)^3} \int \frac{d^3 p}{w} C[\mathbf{p}]$$

NOW WE NEED AN EXPRESSION FOR  $C_0$

ASSUME:

- PARTICLES PROPAGATE FREELY BETWEEN ISOLATED SCATTERINGS
- SCATTERINGS DESCRIBED BY S-MATRIX ELEMENTS FROM VACUUM QFT

## NOT CAPTURED

- QUANTUM EFFECTS IN PROPAGATION  
(E.G. COHERENT OSC.)
- SCREENING BY PLASMA
- THERMAL CORRECTIONS TO  
SCATTERING RATES

PROCESS  $abc \dots \rightarrow ij k$

$$\frac{g}{(2\pi)^3} \int C(A) \frac{d^3p}{w} = \int \Pi_a \Pi_b \dots \Pi_i \Pi_j \dots \times (2\pi)^4 \delta(p_a + p_b \dots - p_i - p_j \dots)$$
$$\times \left[ |M_{ab \dots \rightarrow ij \dots}|^2 L_a f_b \dots (1 \pm f_i) (1 \pm f_j) \dots \right. \\ \left. - |M_{ij \dots \rightarrow ab \dots}|^2 L_i f_j \dots (1 \pm f_a) (1 \pm f_b) \dots \right]$$

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- QUANTUM EFFECTS IN PROPAGATION  
(E.G. COHERENT OSC.)
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PROCESS  $abc... \rightarrow ij k$

$$\frac{g}{(2\pi)^3} \int C(A) \frac{d^3p}{w} = \int \Pi_a \Pi_b - \Pi_i \Pi_j \dots \times (2\pi)^4 \delta(p_a + p_b - p_i - p_j \dots)$$
$$\times \left[ |M_{ab \rightarrow ij}|^2 f_a f_b \dots (1 \pm f_i) (1 \pm f_j) \dots \right. \\ \left. - |M_{ij \rightarrow ab}|^2 f_i f_j \dots (1 \pm f_a) (1 \pm f_b) \dots \right]$$



with

$$d\pi_i = g_i \frac{1}{(2\pi)^3} \frac{d^3 p_i}{2\omega_{p_i}}$$

ASSUMPTIONS =

$$1) |M_{d_i \rightarrow \bar{d}_j}|^2 = |M_{\bar{d}_i \rightarrow d_j}|^2 \equiv |M|^2$$

$$2) \text{BOLTZMANN} = 1 \pm f \approx 1, \quad f \approx \exp(-(\omega_i - \mu_i)/T)$$

$$\dot{n}_d + \underbrace{3H n_d}_{\text{DILUTION}} = - \int d\pi_i \dots d\pi_a (2\pi)^4 |M|^2 \delta^{(4)}(p_0 + p_a \dots - p_i - p_j \dots) \\ \times [f_0 f_b \dots - f_i f_j \dots]$$

INTRODUCE

$$Y = \frac{n}{s}$$

$$S = \text{ENTROPY DENSITY} = \frac{2\pi^2}{45} g_{*s} T^3$$

ADIABATIC EXPANSION:  $sa^3 = \text{CONST}$

$$\Rightarrow \frac{d}{dt}(sa^3) = 0 = 3a^2 \dot{a}s + a^3 \dot{s}$$

$$\Rightarrow \dot{s} = -3 \frac{a^2 \dot{a}}{a^3} s = -3Hs$$

NOW

$$\dot{n} + 3Hn = \frac{d}{dt}(Ys) + 3HYs$$

$$= \dot{Y}s + \dot{s}Y + 3HYs = \dot{Y}s - 3HsY + 3HYs$$

$$= \dot{Y}s$$

$$t = \frac{1}{2} \sqrt{\frac{45}{4\pi^3 g_s}} \frac{m_{pl}}{T^2} = \frac{1}{2} \sqrt{\frac{45}{4\pi^3 g_s}} \frac{m_{pl}}{m^2} X^2 = \frac{1}{2H}$$

$$\left( \text{with } H = \frac{1}{m_{pl}} \sqrt{\frac{4\pi^3 g_s}{45}} \right)$$

$$\frac{d}{dt} = \frac{\partial X}{\partial t} \frac{d}{\partial X} = \left[ \frac{\partial t}{\partial X} \right]^{-1} \frac{d}{\partial X} = \left[ \frac{\partial}{\partial X} \frac{1}{2H} \right]^{-1} \frac{d}{\partial X}$$

WE CAN DEFINE

$$\mathcal{H} = \frac{1}{X} \left( \frac{\partial}{\partial X} \frac{1}{2H} \right)^{-1} = \frac{1}{X} \left( \frac{\partial}{\partial X} \frac{1}{2} \sqrt{\frac{45}{4\pi g_s}} \frac{m_{pl}}{m^2} X^2 \right)$$

$$\frac{d}{dt} = \mathcal{H} X \frac{d}{\partial X}$$

$$\Rightarrow \frac{d}{dt} Y = \frac{1}{5} \int d\pi_1 \dots d\pi_n \dots M^2 \delta^4(p_0 - p_1) \dots [f_0 - f_1 \dots]$$



TAKE STABLE WIMP  $\Psi$  WITH



X W EQUILIBRIUM  
NO CHM. POT  $\rightarrow f_X = f_{\bar{X}} = \exp(-\omega_X/T)$

$\delta$ -FUNCTION,

$$f_X f_{\bar{X}} = \exp(-(\omega_X + \omega_{\bar{X}})/T) = \exp(-(\omega_\Psi + \omega_{\bar{\Psi}})/T) = f_\Psi^{EQ} f_{\bar{\Psi}}^{EQ}$$

$$\rightarrow [f_\Psi f_{\bar{\Psi}} - f_X f_{\bar{X}}] = [f_\Psi f_{\bar{\Psi}} - f_\Psi^{EQ} f_{\bar{\Psi}}^{EQ}]$$

$f_{\psi}$  IN GENERAL IS NOT IN EQUILIBRIUM  
HOWEVER, WE CAN ASSUME  
"KINETIC EQUILIBRIUM"

$$f_{\psi} \propto f_{\psi}^{EQ} \Rightarrow f_{\psi} = C_{\psi} f_{\psi}^{EQ}$$

$$\Rightarrow n_{\psi} = C_{\psi} n_{\psi}^{EQ}$$

THEN (ASSUMING  $g_x = g_\psi = 1$ )

$$= \frac{-1}{5} \int \frac{d^3 p_x}{(2\pi)^3} \frac{d^3 p_{\bar{x}}}{(2\pi)^3} \frac{d^3 p_\psi}{(2\pi)^3} \frac{d^3 p_{\bar{\psi}}}{(2\pi)^3} |M|^2 \delta(p_\psi + p_{\bar{\psi}} - p_x - p_{\bar{x}})$$

$$\times \left[ f_\psi f_{\bar{\psi}} - f_\psi^{EG} f_{\bar{\psi}}^{EG} \right]$$

$$f_\psi f_{\bar{\psi}}^{EG} [c_\psi c_{\bar{\psi}} - 1]$$

$$= \underbrace{-\frac{[c_\psi c_{\bar{\psi}} - 1]}{5}}_{\text{bracket}} \int \frac{d^3 p_x d^3 p_{\bar{x}} d^3 p_\psi d^3 p_{\bar{\psi}}}{(2\pi)^{12}} |M|^2 \delta(p_\psi + p_{\bar{\psi}} - p_x - p_{\bar{x}}) f_\psi^{EG} f_{\bar{\psi}}^{EG}$$

$$= \frac{1}{5} [n_\psi n_{\bar{\psi}} - n_\psi^{EG} n_{\bar{\psi}}^{EG}] \frac{1}{n_\psi^{EG} n_{\bar{\psi}}^{EG}}$$



$$\Rightarrow H_X \frac{dY}{dX} = -\frac{S}{\hbar} (\cancel{Y_\psi} \cancel{Y_{\bar{\psi}}} - Y_{\psi}^{EQ} Y_{\bar{\psi}}^{EQ}) \langle \sigma | \sigma \rangle$$

$$\langle \sigma | \sigma \rangle = \frac{1}{n_{\psi}^{EQ} n_{\bar{\psi}}^{EQ}} \int \frac{d^3 p_\psi d^3 p_{\bar{\psi}} d^3 p_\psi d^3 p_{\bar{\psi}}}{(2\pi)^4} |M|^2 \delta(p_\psi + p_{\bar{\psi}} - p_\psi - p_{\bar{\psi}}) \dots$$

ASSUMING  $Y_\psi = Y_{\bar{\psi}}$  (SYMM. DA)

$$H_X \frac{dY}{dX} = -S (Y_\psi^2 - Y_{\psi}^{EQ^2}) \langle \sigma | \sigma \rangle \quad \Bigg| \quad \frac{dY}{dX} = -\frac{\langle \sigma \sigma \rangle S}{H_X} (Y_\psi^2 - Y_{\psi}^{EQ^2})$$

OR

---

The Boltzmann equation

$$\frac{x}{Y_{\text{EQ}}} \frac{dY}{dx} = -\frac{\Gamma_A}{H} \left[ \left( \frac{Y}{Y_{\text{EQ}}} \right)^2 - 1 \right]$$

---

in terms of abundances  $Y \equiv \frac{n_\psi}{s}$

and the annihilation rate  $\Gamma_A \equiv n_{\text{EQ}} \langle \sigma_A |v| \rangle$

which is given by the thermally averaged cross section

$$\langle \sigma_{\psi\bar{\psi} \rightarrow X\bar{X}} |v| \rangle \equiv \left( n_\psi^{\text{EQ}} \right)^{-2} \int d\Pi_\psi d\Pi_{\bar{\psi}} d\Pi_X d\Pi_{\bar{X}} (2\pi)^4 \\ \times \delta^4(p_\psi + p_{\bar{\psi}} - p_X - p_{\bar{X}}) |\mathcal{M}|^2 \exp(-E_\psi/T) \exp(-E_{\bar{\psi}}/T),$$

with the convenient integration measure  $d\Pi \equiv g \frac{1}{(2\pi)^3} \frac{d^3p}{2E}$

and the dimensionless time variable  $x \equiv m/T,$

---

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particle "freezes out" when the interaction rate  $\Gamma$  is much smaller than the Hubble rate

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in terms of abundances

$$Y \equiv \frac{n_\psi}{s}$$

particle "freezes out" when the interaction rate  $\Gamma$  is much smaller than the Hubble rate

and the annihilation rate

$$\Gamma_A \equiv n_{\text{EQ}} \langle \sigma_A |v| \rangle$$

$\Gamma$  decreases with temperature and is exponentially surpassed for  $T < M$ , i.e.,  $x > 1$

which is given by the thermally averaged cross section

$$\langle \sigma_{\psi\bar{\psi} \rightarrow X\bar{X}} |v| \rangle \equiv (n_\psi^{\text{EQ}})^{-2} \int d\Pi_\psi d\Pi_{\bar{\psi}} d\Pi_X d\Pi_{\bar{X}} (2\pi)^4 \times \delta^4(p_\psi + p_{\bar{\psi}} - p_X - p_{\bar{X}}) |\mathcal{M}|^2 \exp(-E_\psi/T) \exp(-E_{\bar{\psi}}/T)$$

with the convenient integration measure

$$d\Pi \equiv g \frac{1}{(2\pi)^3} \frac{d^3p}{2E}$$

and the dimensionless time variable

$$x \equiv m/T$$

# Hot Relics

If a particle freezes out while being relativistic ( $x_f < 3$ ), then it simply has an equilibrium distribution

$$Y_\infty = Y_{\text{EQ}}(x_f) = 0.278 g_{\text{eff}} / g_{*S}(x_f) \quad (x_f \lesssim 3)$$

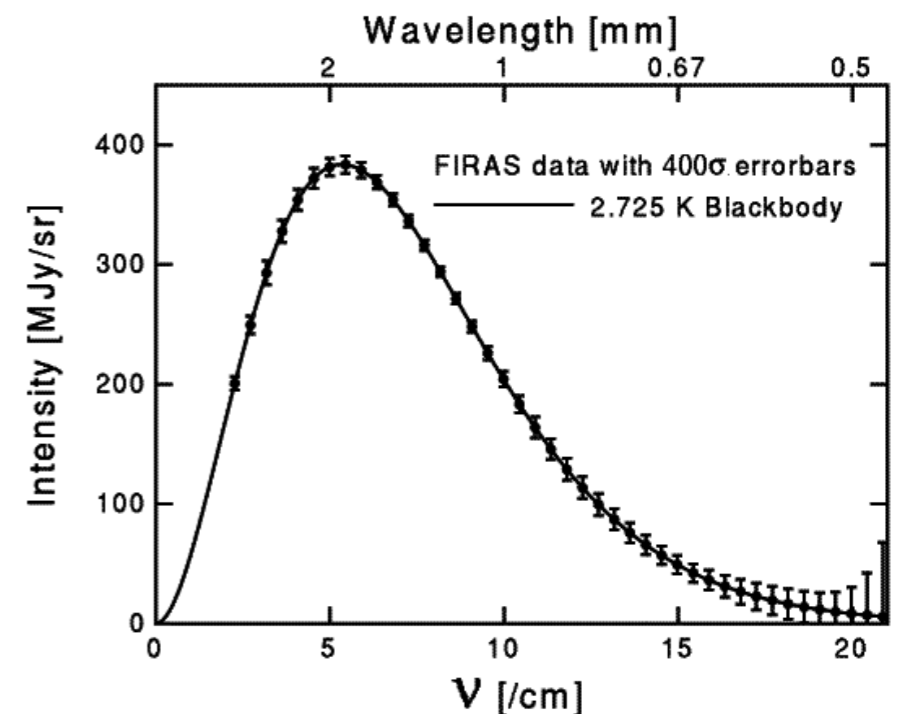
$$\begin{aligned} n_{\psi 0} &= s_0 Y_\infty = 2970 Y_\infty \text{ cm}^{-3} \\ &= 825 [g_{\text{eff}} / g_*(x_f)] \text{ cm}^{-3}. \end{aligned}$$

Example I: photons.

- The CMB photons have a perfect black body spectrum even though they are not “in equilibrium” because they do not interact.
- The shape of the spectrum is invariant because  $g_{*S} a^3 T^3$  is constant for adiabatic evolution, i.e., for a massless particle the energy and temperature in the distribution function redshift in the same way

$$T \propto g_{*S}^{-1/3} a^{-1}$$

$$E = p \propto a^{-1}$$



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# Hot Relics

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Example II: neutrinos.

- Neutrinos are relativistic when they freeze out, but become non-relativistic afterwards. Their energy density is now

$$\rho_{\psi 0} = s_0 Y_\infty m = 2.97 \times 10^3 Y_\infty (m/\text{eV}) \text{ eV cm}^{-3}$$

$$\Omega_\psi h^2 = 7.83 \times 10^{-2} [g_{\text{eff}} / g_{*S}(x_f)] (m/\text{eV}).$$

- note that  $g$  has changed after neutrino decoupling due to electron-positron annihilation,

$$\Omega_\nu = \frac{\rho_\nu^0}{\rho_{\text{crit}}^0} = \frac{\sum m_\nu}{93.14 h^2 \text{ eV}}$$



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# Cold Relic

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We can parameterise the averaged cross section as

$$\langle \sigma_A |v| \rangle \equiv \sigma_0 (T/m)^n = \sigma_0 x^{-n} \quad (\text{for } x \gtrsim 3)$$

and hence write the Boltzmann equation as

$$dY/dx = -\lambda x^{-n-2} (Y^2 - Y_{\text{EQ}}^2)$$

where we have further introduced

$$\lambda = \left[ \frac{x \langle \sigma_A |v| \rangle s}{H(m)} \right]_{x=1} = 0.264 (g_{*s}/g_*^{1/2}) m_{\text{Pl}} m \sigma_0,$$

$$Y_{\text{EQ}} = 0.145 (g/g_{*s}) x^{3/2} e^{-x}.$$

It is more convenient to track the deviation from equilibrium  $\Delta \equiv Y - Y_{\text{EQ}}$

$$\Delta' = -Y'_{\text{EQ}} - \lambda x^{-n-2} \Delta (2Y_{\text{EQ}} + \Delta)$$

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# Cold Relic

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$$\Delta' = -Y'_{\text{EQ}} - \lambda x^{-n-2} \Delta (2Y_{\text{EQ}} + \Delta)$$

At early times we can neglect  $\Delta'$  and solve the equation algebraically:

$$\begin{aligned} \Delta &\simeq -\lambda^{-1} x^{n+2} Y'_{\text{EQ}} / (2Y_{\text{EQ}} + \Delta) \\ &\simeq x^{n+2} / 2\lambda. \end{aligned}$$

At late times the equilibrium abundance is negligible and we can approximate

$$\Delta' = -\lambda x^{-n-2} \Delta^2, \quad \text{and hence} \quad Y_{\infty} = \Delta_{\infty} = \frac{n+1}{\lambda} x_f^{n+1}.$$

Matching the two solutions requires determination of the freeze-out point  $x_f$

We define  $x_f$  as the point when  $\Delta(x_f) = c Y_{\text{EQ}}(x_f)$

and use the early time solution  $\Delta(x_f) \simeq x_f^{n+2} / \lambda(2+c)$  to find

$$x_f \cong \ln [(2+c)\lambda ac] - \left(n + \frac{1}{2}\right) \ln \{\ln [(2+c)\lambda ac]\}$$

with  $a = 0.145(g/g_{*s})$ .

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# Cold Relic

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The analytic estimate is relatively insensitive to the choice of  $c$ , best agreement with numerical results is obtained for  $c(c + 2) = n + 1$ , which yields

$$x_f = \ln[0.038(n + 1)(g/g_*^{1/2})m_{Pl} m \sigma_0] \\ - \left(n + \frac{1}{2}\right) \ln \left\{ \ln [0.038(n + 1)(g/g_*^{1/2})m_{Pl} m \sigma_0] \right\}$$

$$\text{and } Y_\infty = \frac{3.79(n + 1)x_f^{n+1}}{(g_*s/g_*^{1/2})m_{Pl} m \sigma_0} \quad \text{or} \quad \Omega_\psi h^2 = 1.07 \times 10^9 \frac{(n + 1)x_f^{n+1} \text{ GeV}^{-1}}{(g_*s/g_*^{1/2})m_{Pl}\sigma_0}.$$

A similar result could have been obtained if one determined  $x_f$  by solving  $H = \Gamma$  for  $x$ .

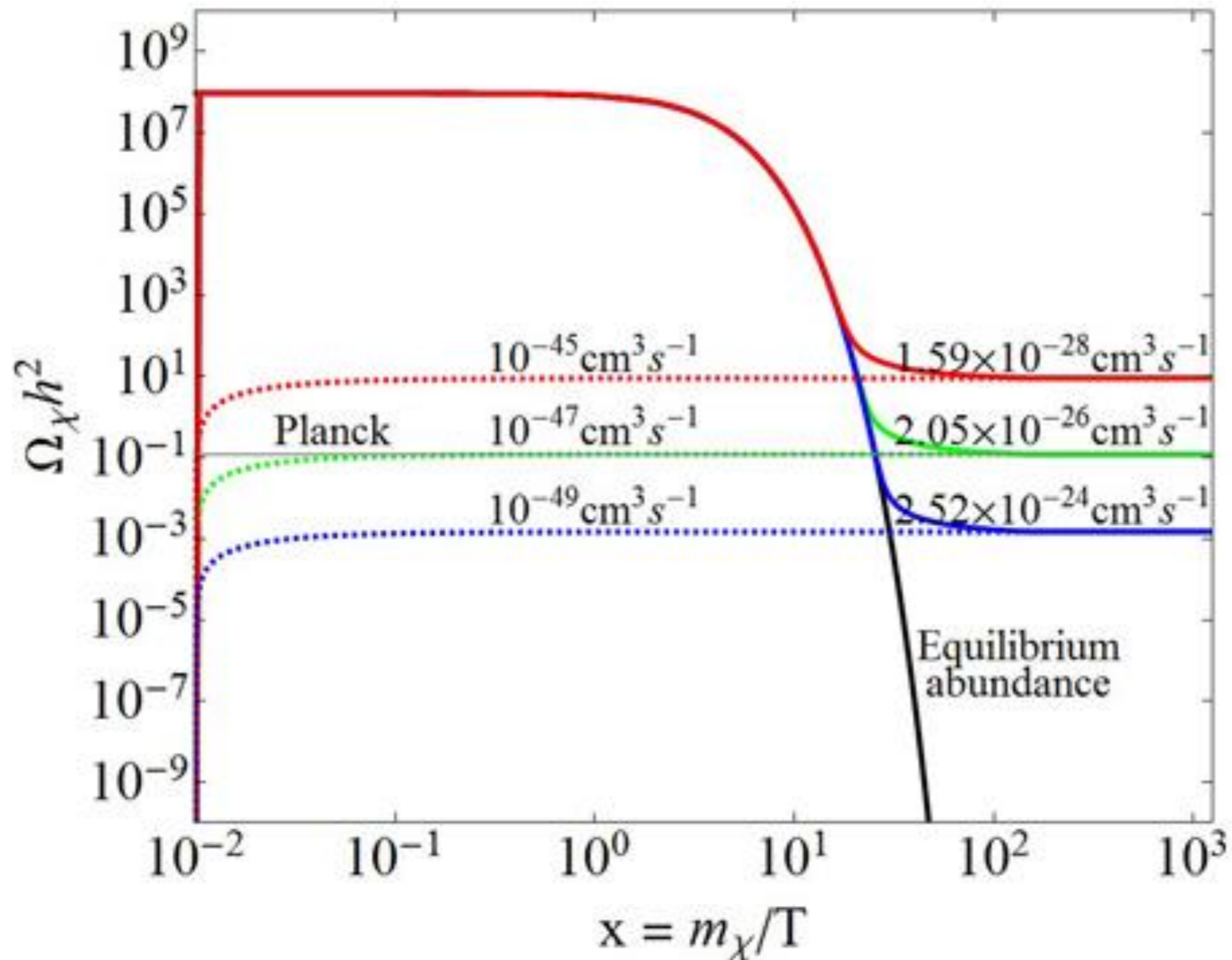
It is worthwhile noting that the final abundance is inversely proportional to the averaged cross section and the DM mass:

$$Y_\infty = \frac{3.79(n + 1)(g_*^{1/2}/g_*s)x_f}{m m_{Pl} \langle \sigma_A |v| \rangle}$$

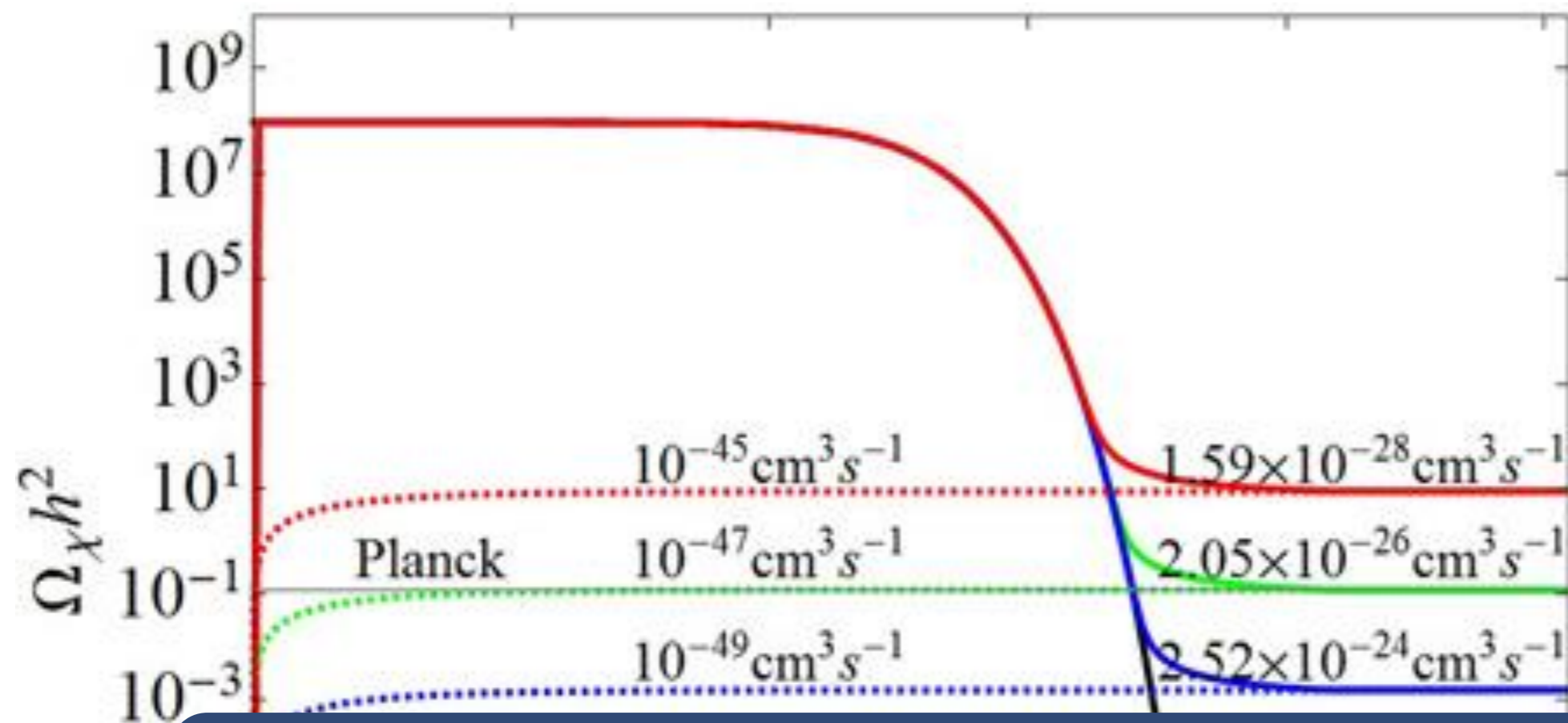
**WIMP Miracle:** If one inserts a mass and cross section of the EW order, one roughly gets the observed DM density! (not really, but good enough to call it a miracle and dominate the field for decades...)



# Freeze-out and freeze-in



# Freeze-out and freeze-in



Freeze-in and freeze-out are both thermal production mechanisms because the DM is made in thermal scatterings in the plasma. There are also non-thermal mechanisms, such as

- decay of heavy particles (e.g. sterile neutrino DM)
- formation of a coherent condensate (e.g. axion DM)

$$x = m_\chi/T$$

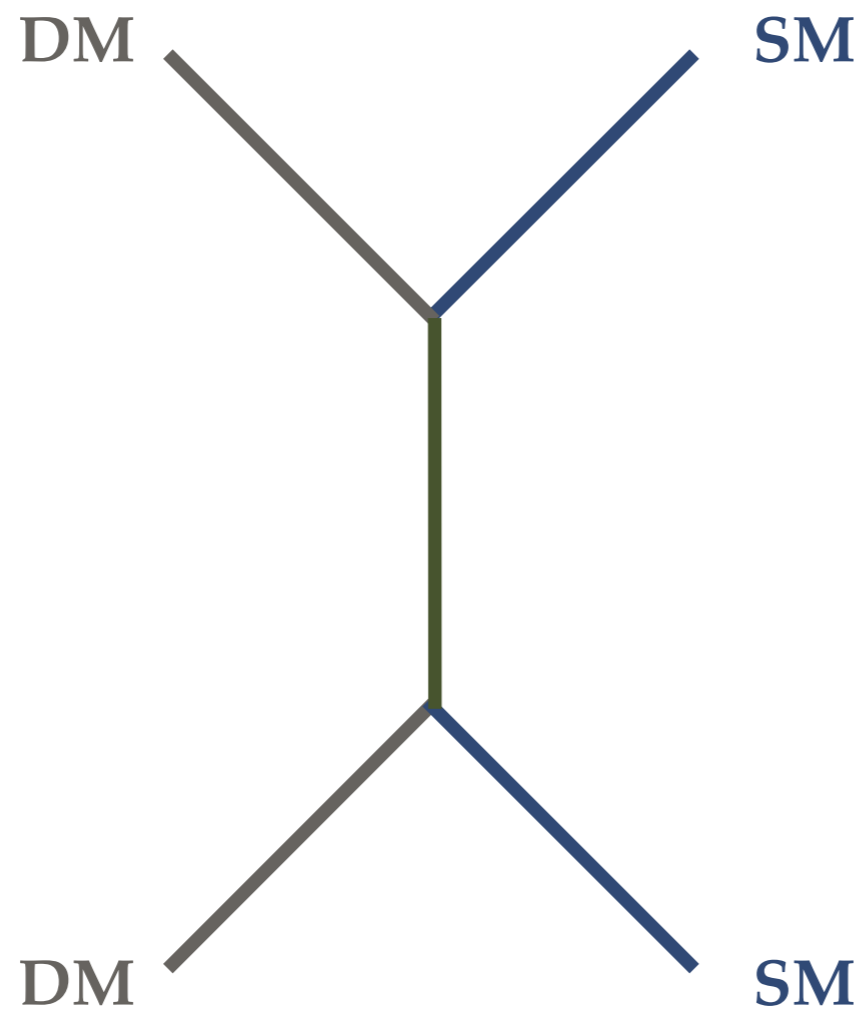
# WIMP DM Searches



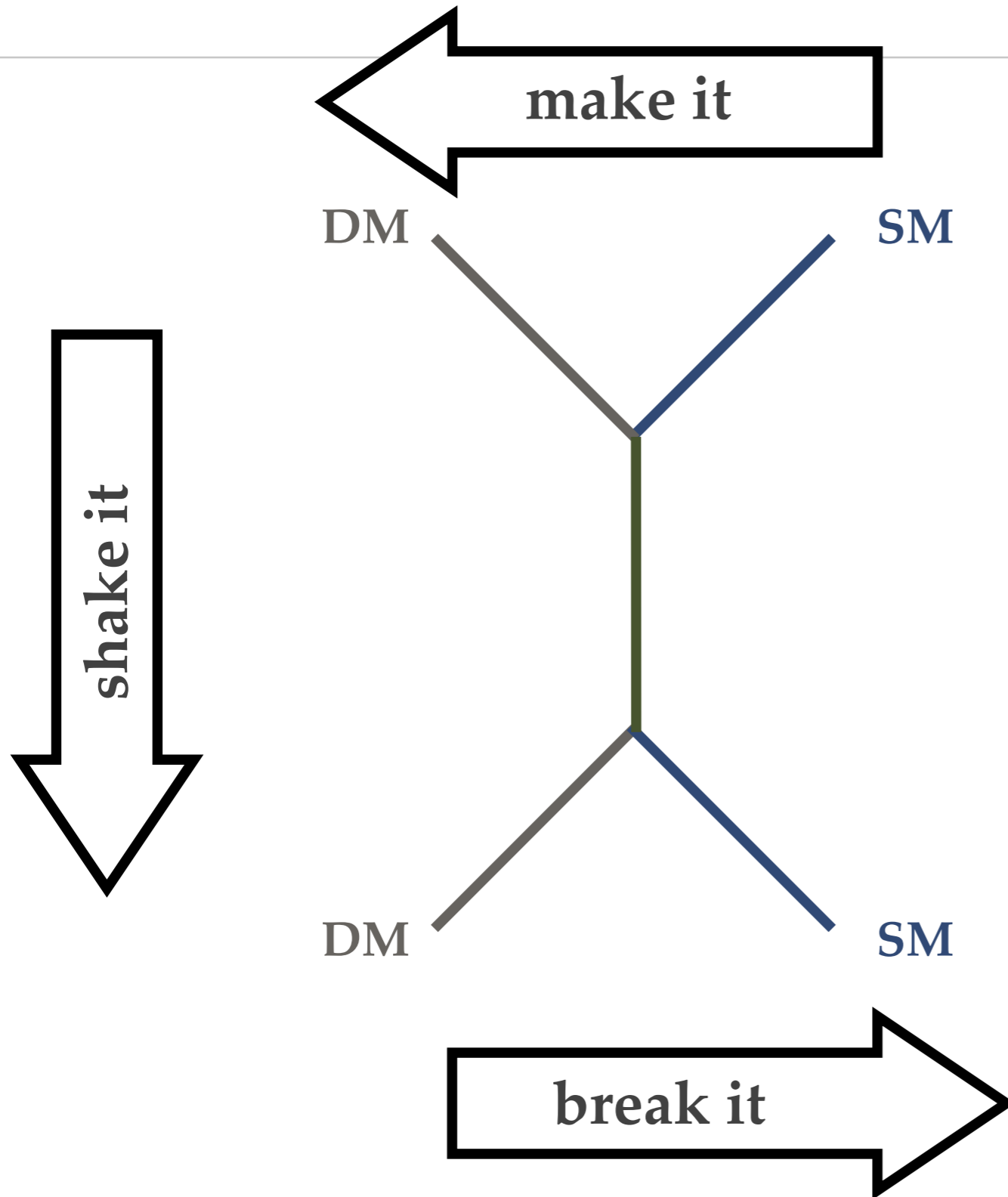
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# How to detect Dark Matter?

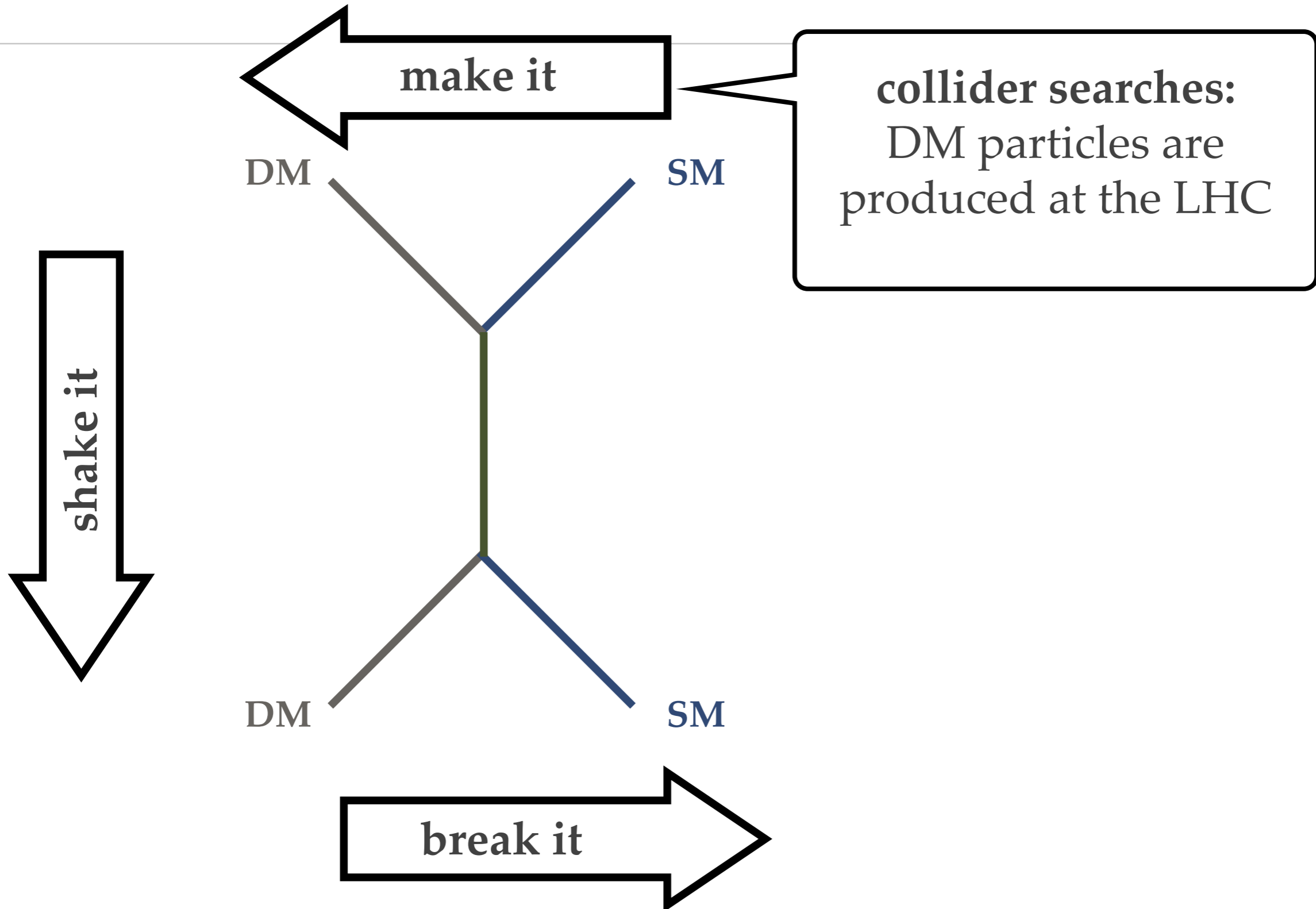
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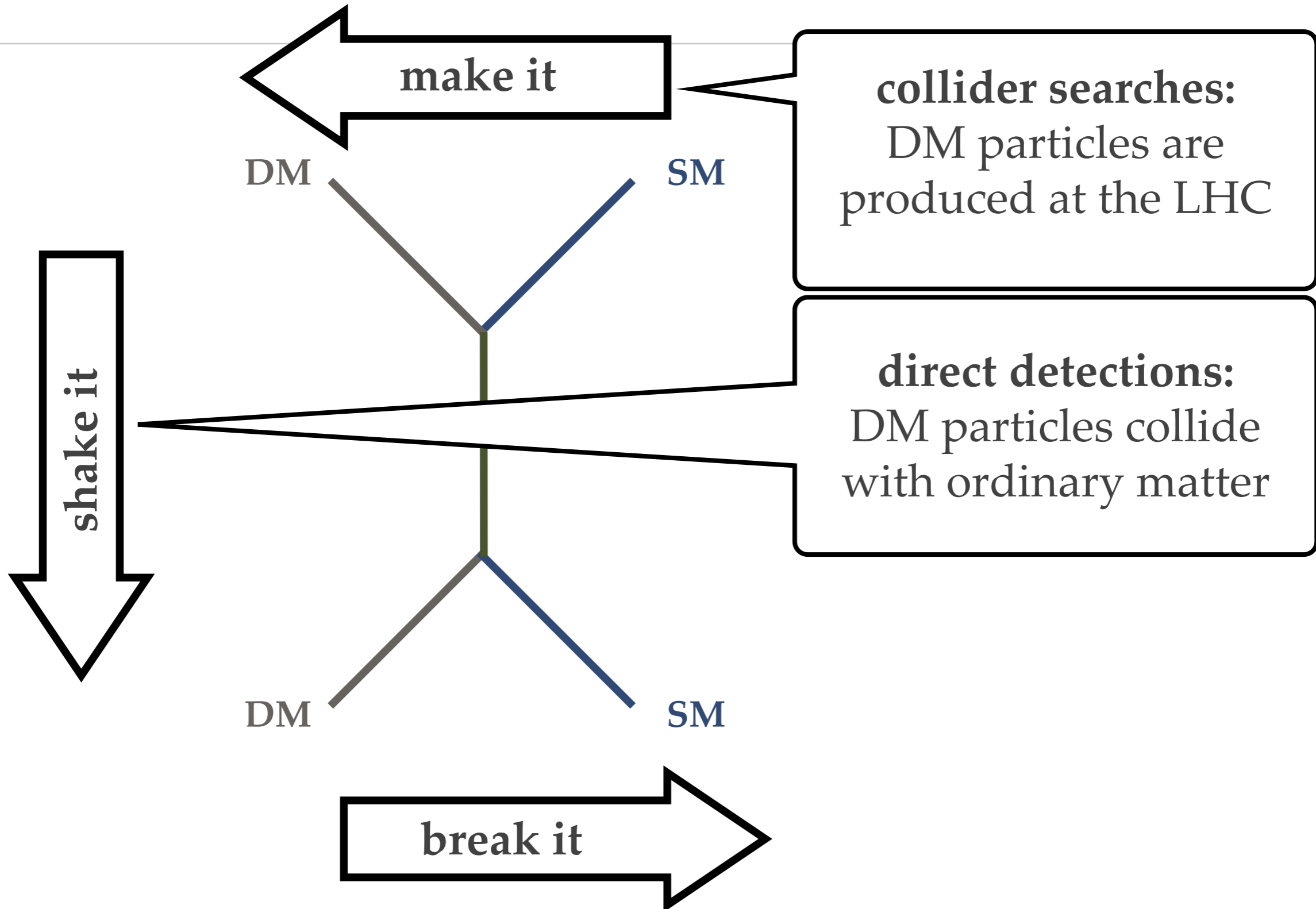


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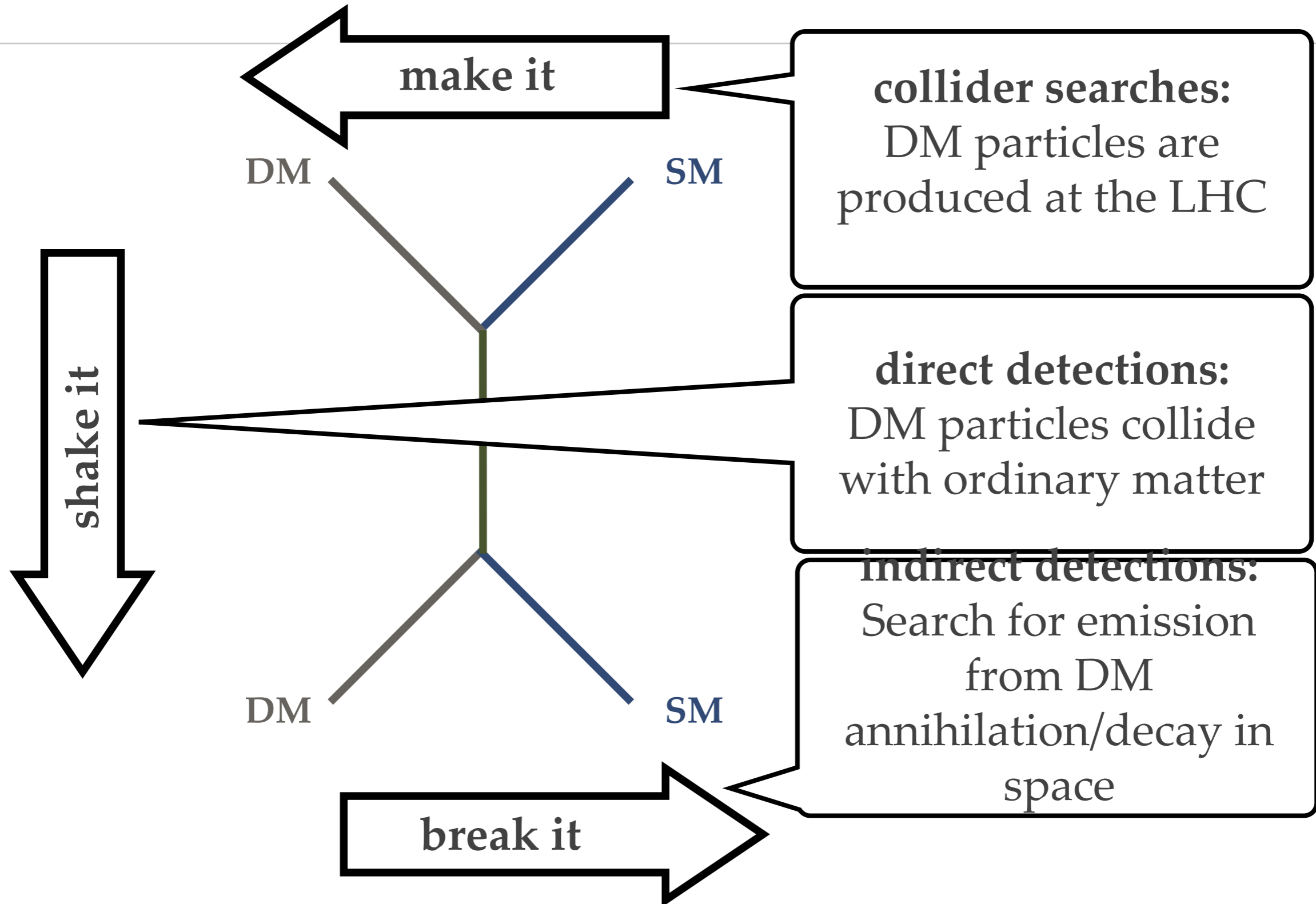




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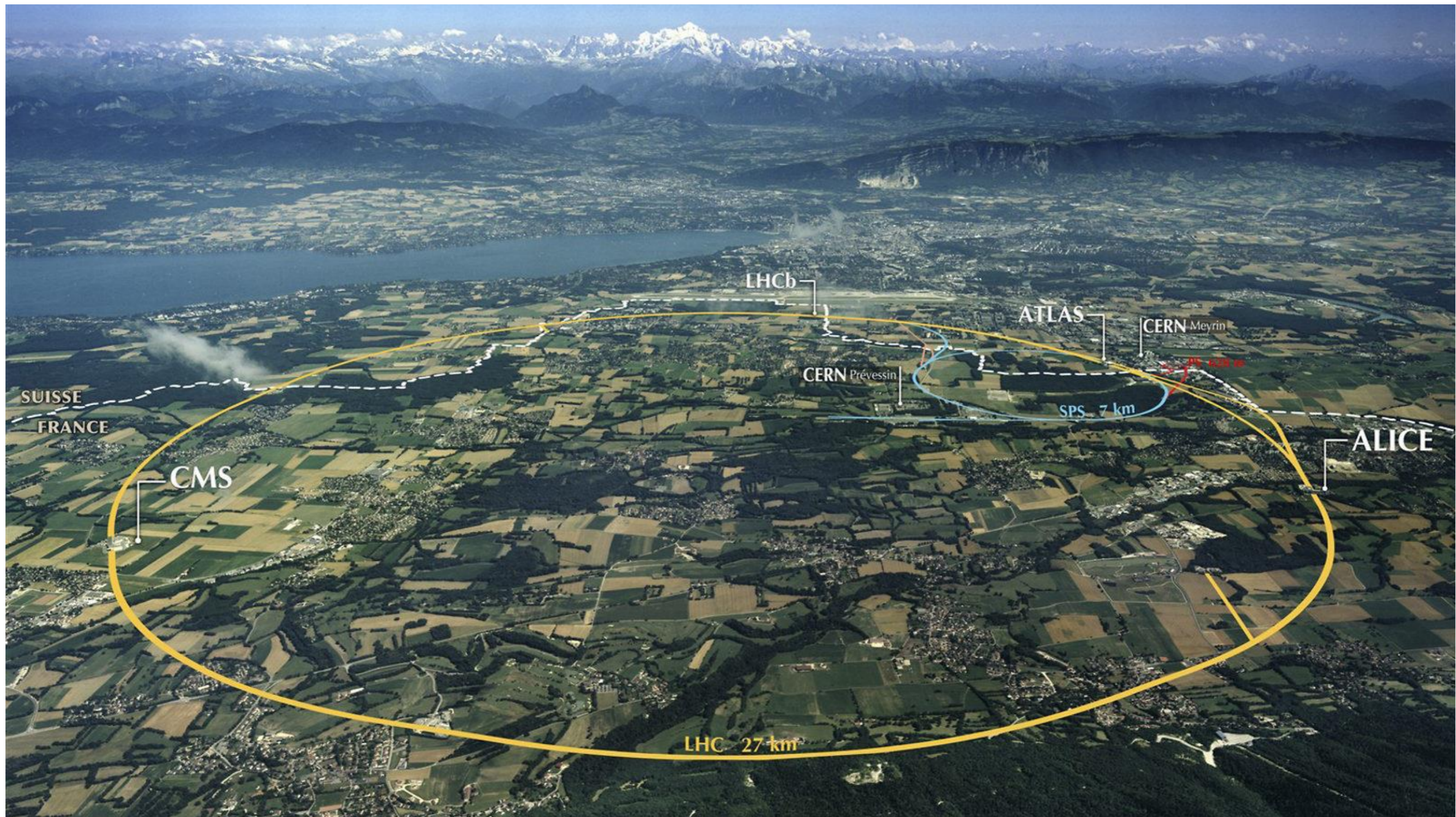


# How to detect Dark Matter?



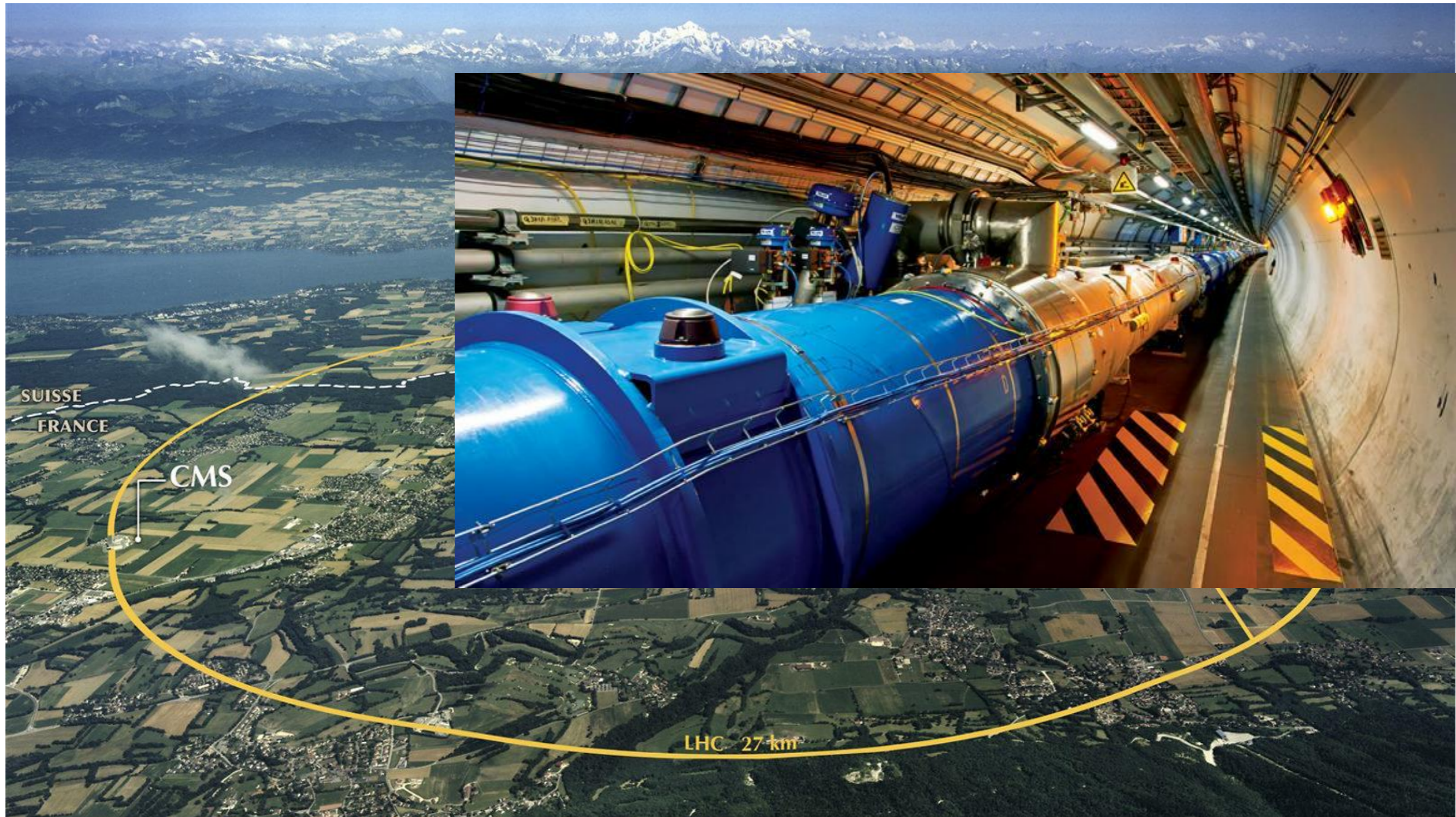


# Make It: Collider Searches



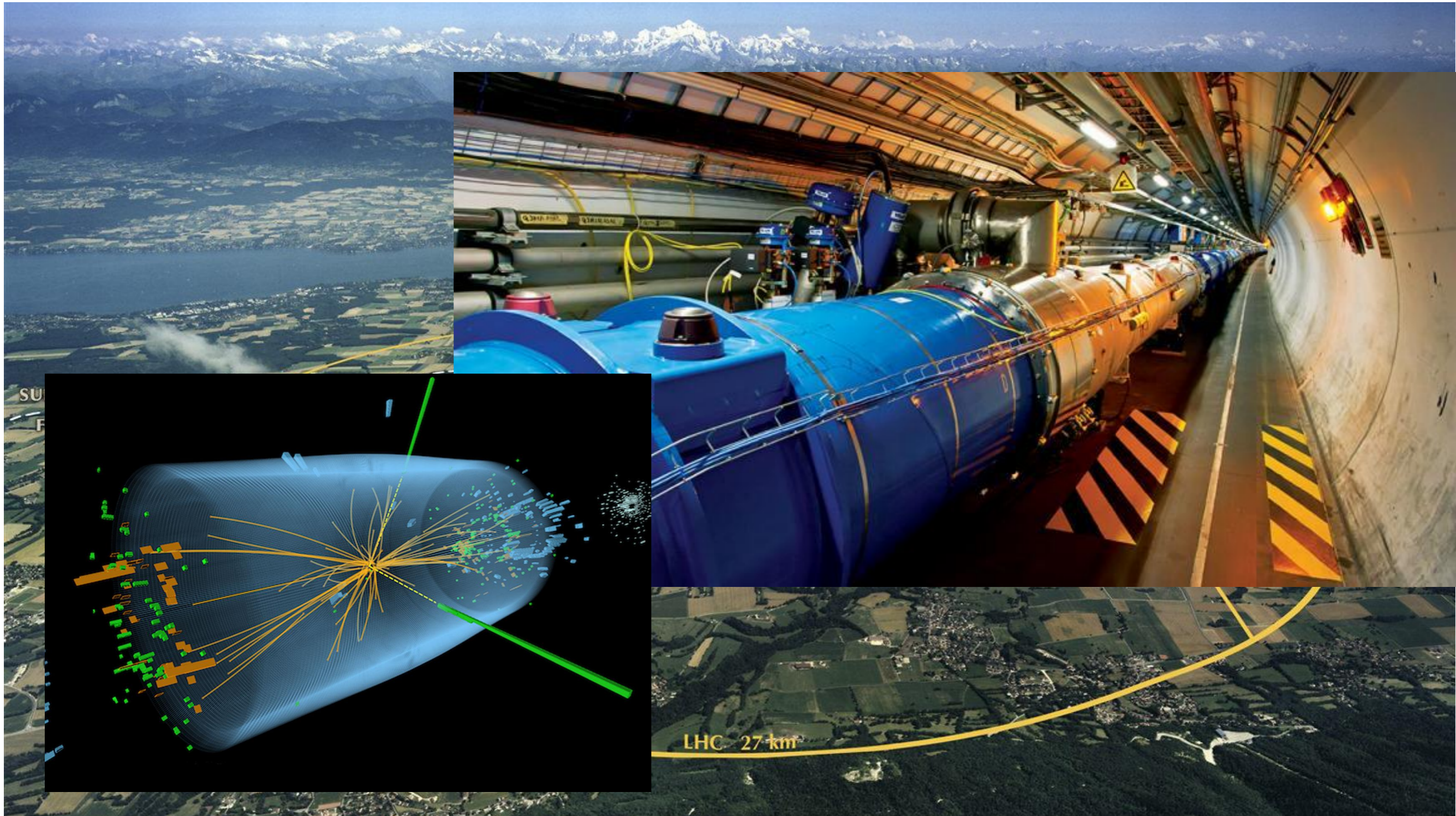


# Make It: Collider Searches



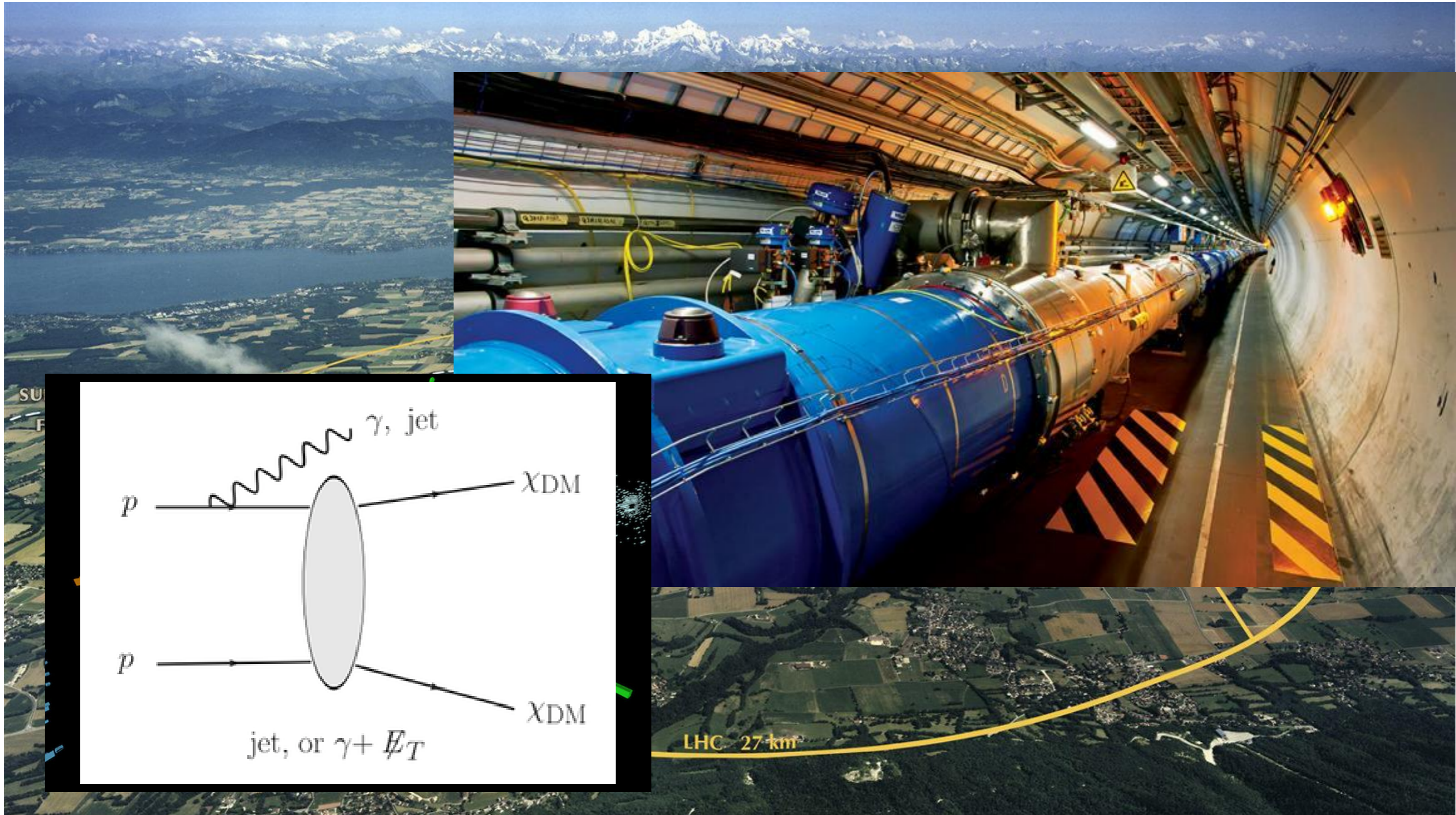


# Make It: Collider Searches





# Make It: Collider Searches



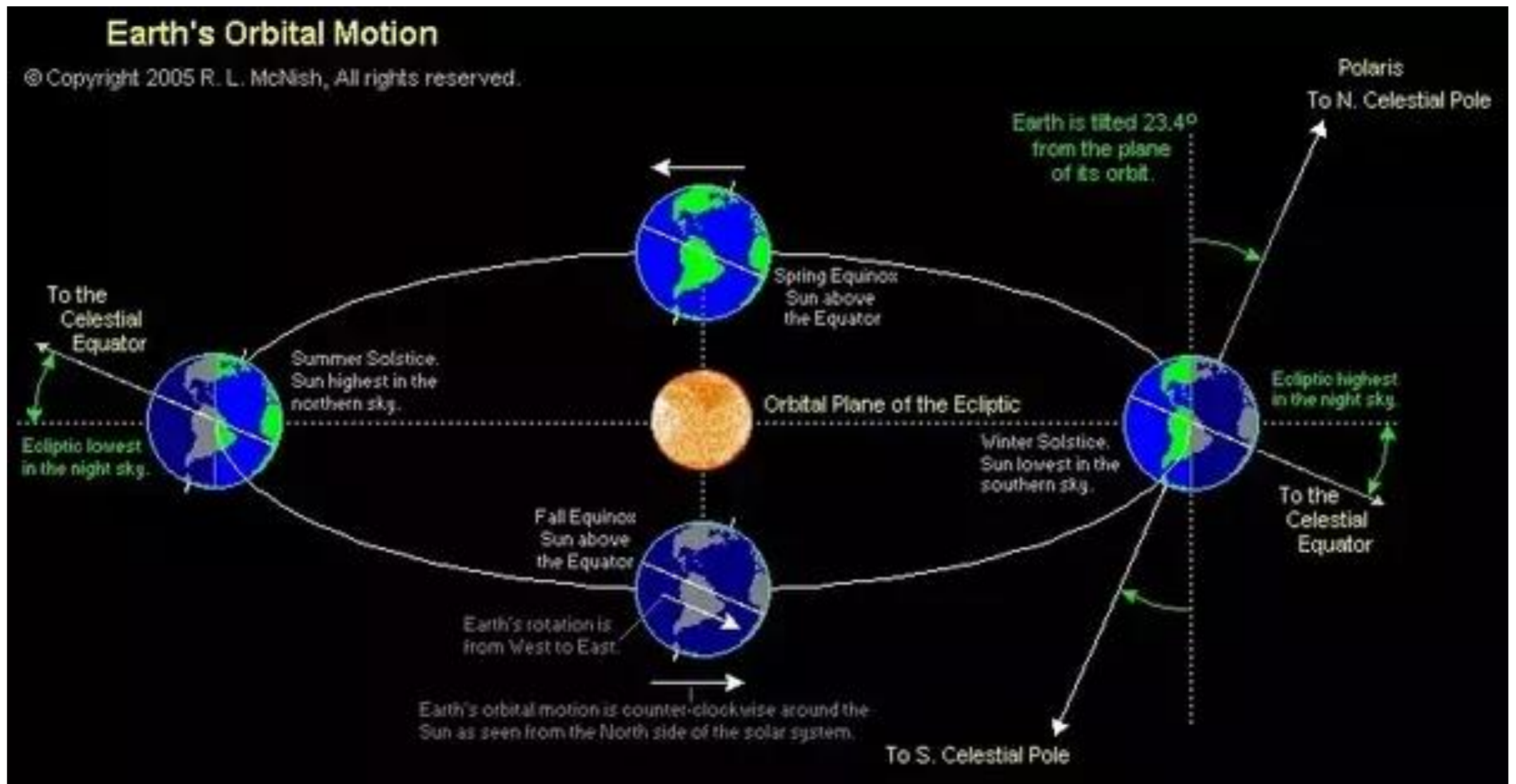
LHC 27 km



# Shake It: Direct Detection

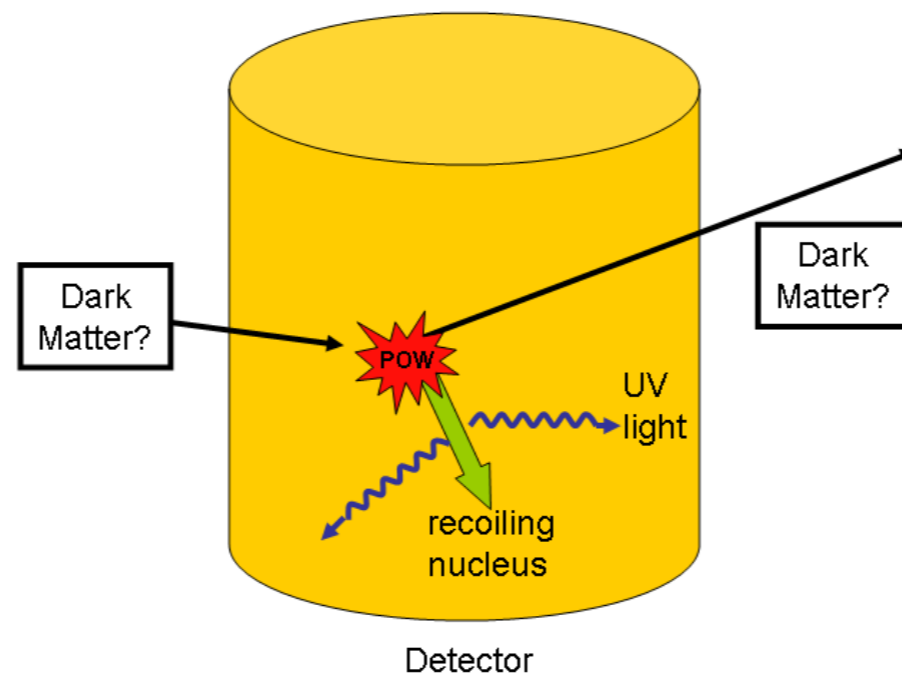
Eppur si muove...

Earth moves through the DM halo, so cold DM particles should hit us from different directions in summer and winter.



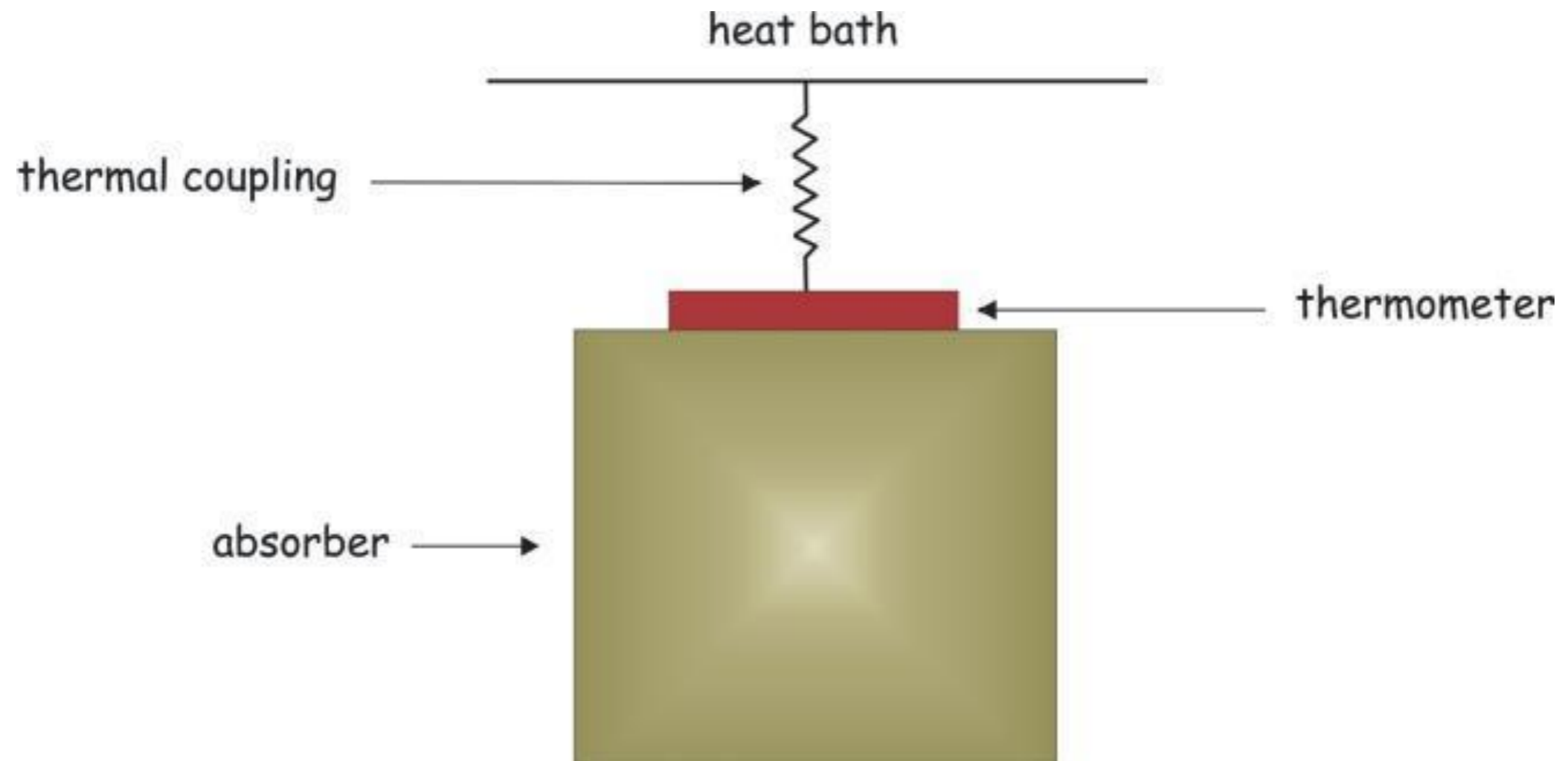
# Shake It: Direct Detection

**Cryogenic crystal detectors** (e.g. CDMS, CRESST, CoGeNT, EDELWEISS):  
Search for heat deposition in very cold crystals ( $T \sim 50$  mK)



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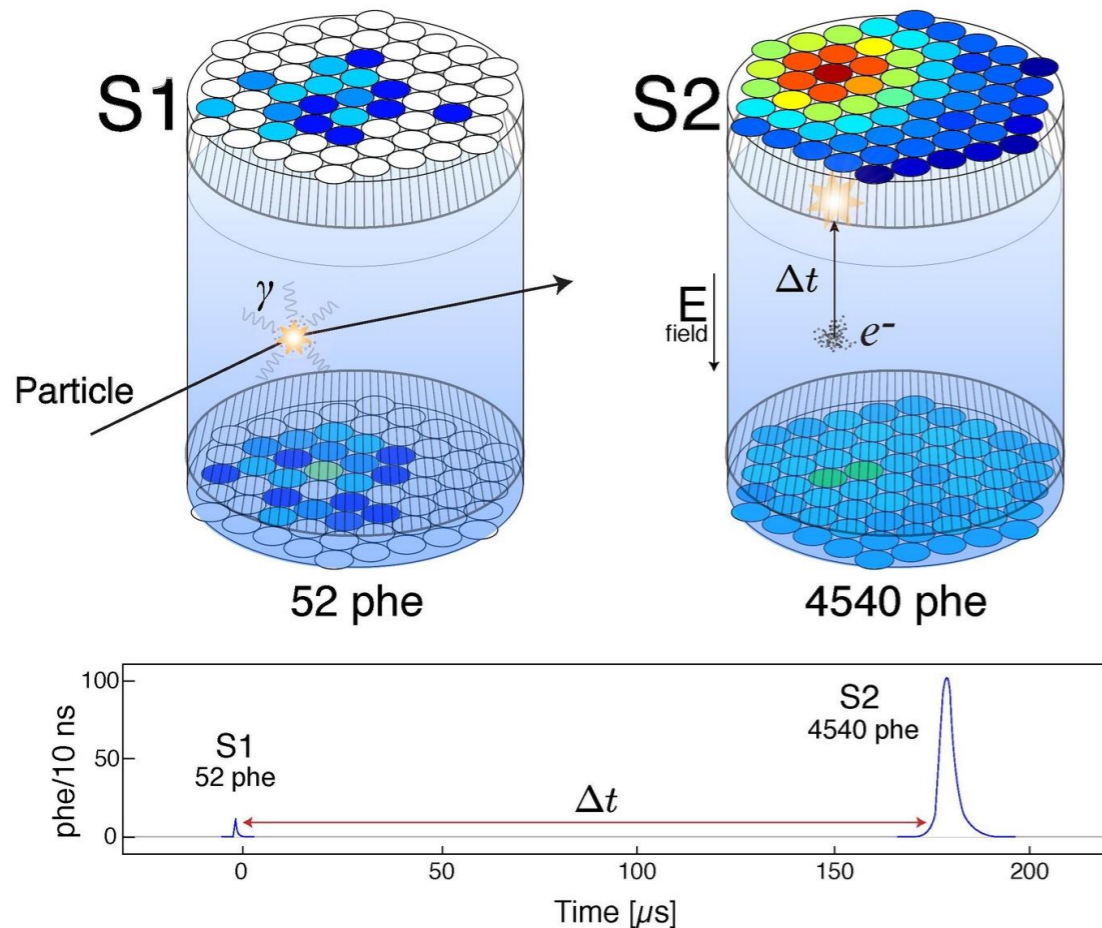


# Shake It: Direct Detection

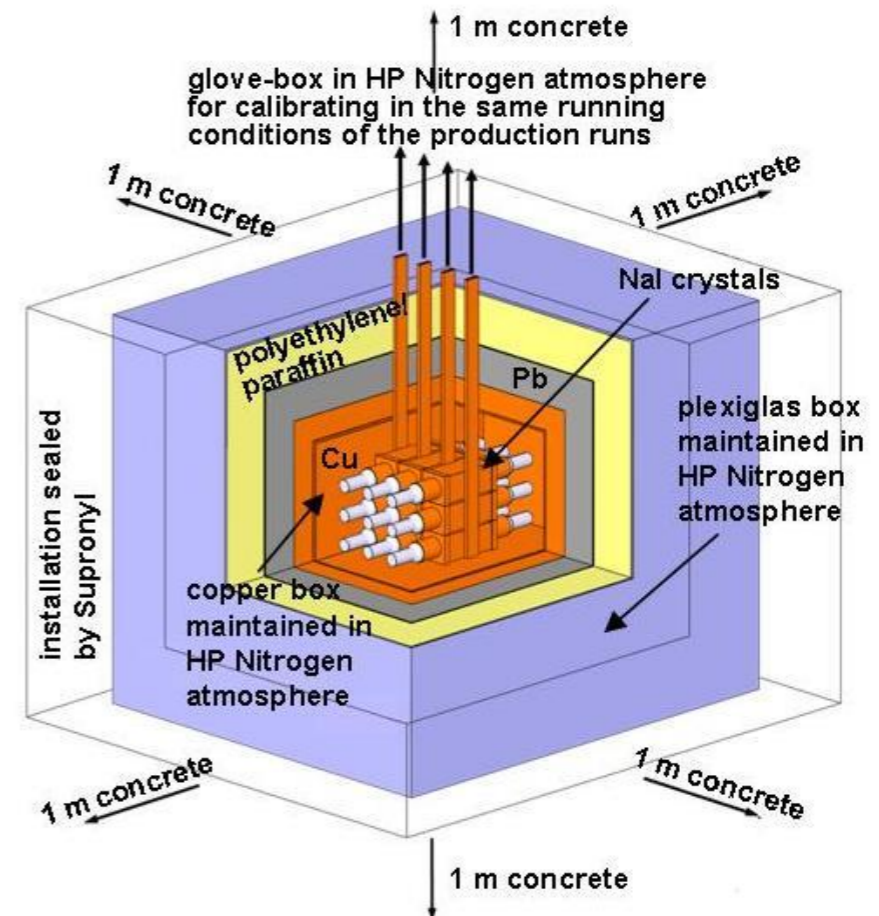
## Scintillator detectors:

Convert kinetic energy from DM “knock” into radiation

**Noble gas scintillator:**  
XENON, LUX, PandaX



**Crystal scintillator:**  
DAMA, ANAIS



Simplified schema of ~ 100 kg NaI(Tl) set-up

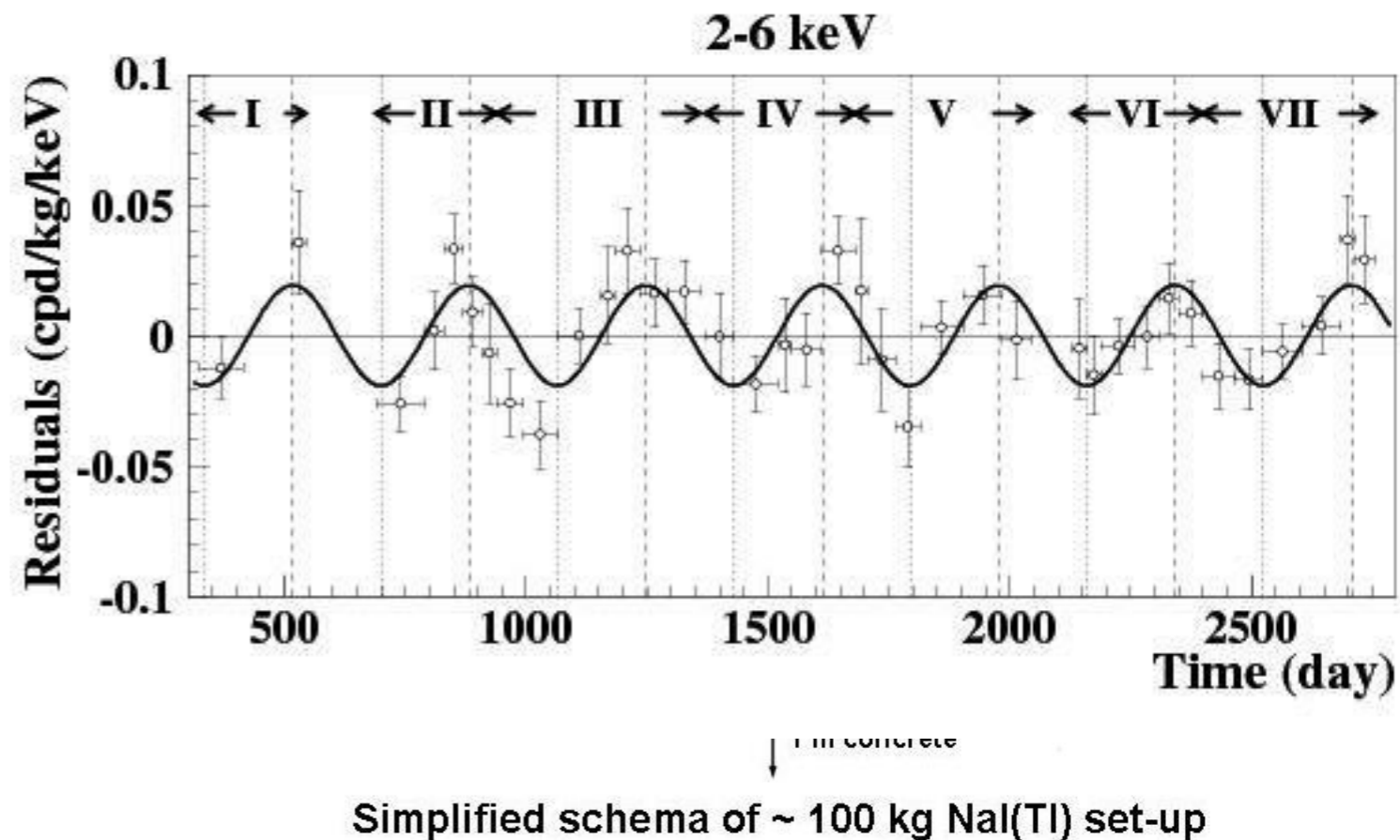
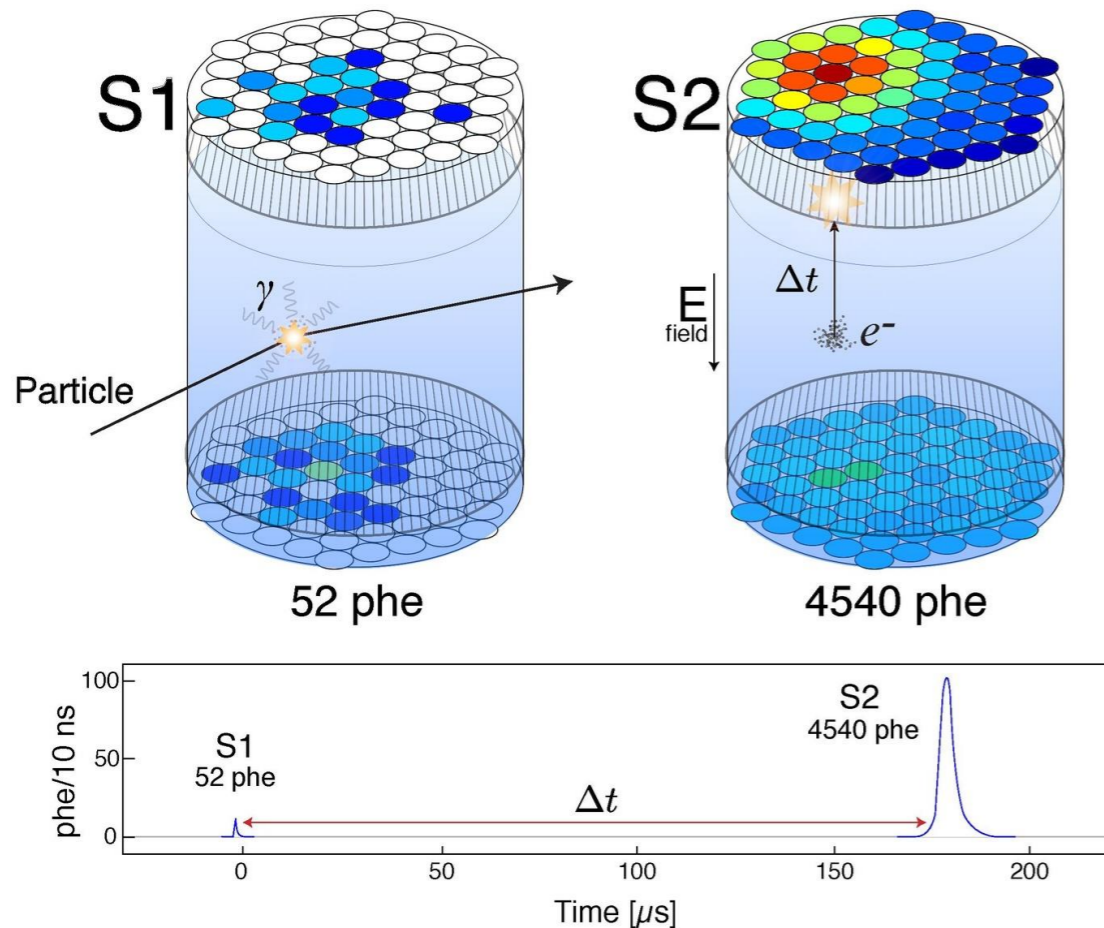
# Shake It: Direct Detection

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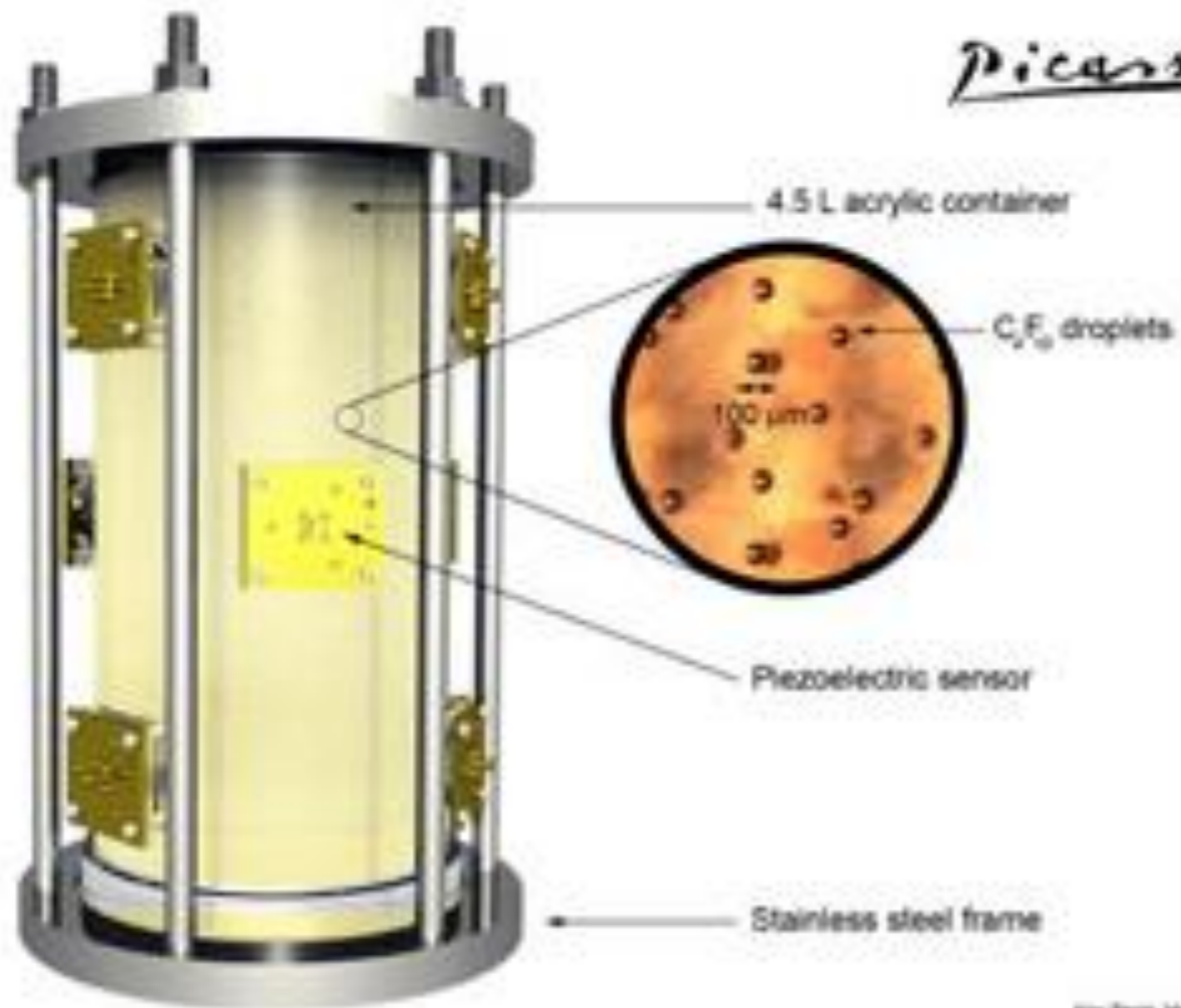
**Noble gas scintillator:**  
XENON, LUX, PandaX

**Crystal scintillator:**  
DAMA, ANAIS



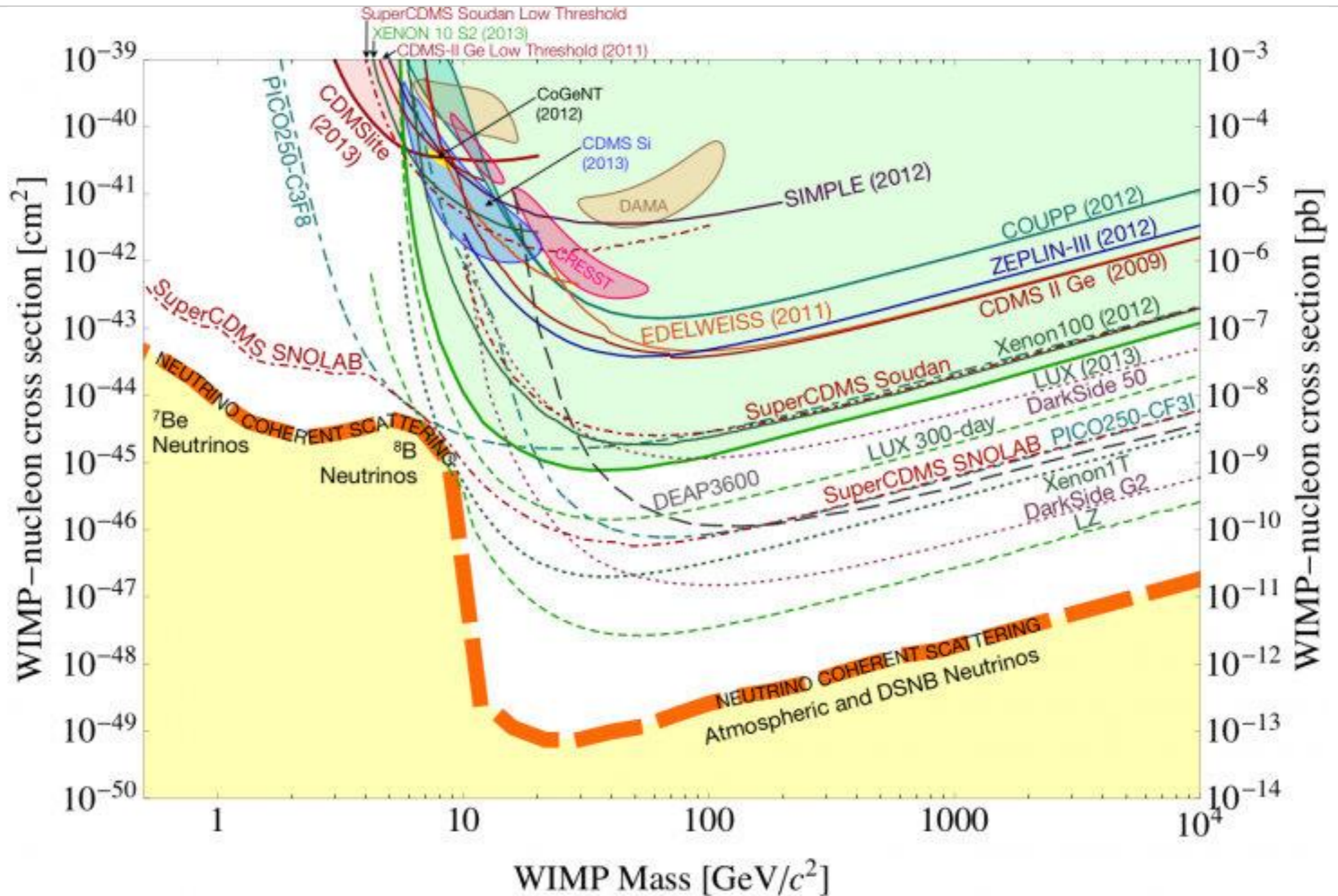
# Shake It: Direct Detection

**Bubble chamber**  
sees tracks of “knocked” particles (PICASSO)





# Current Constraints



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# Break It: Indirect DM Detection

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DM has existed since before the CMB decoupled. That leaves two possibilities:

**1. DM is made of stable particles.**

Examples: standard WIMP, axion, macroscopic black holes...

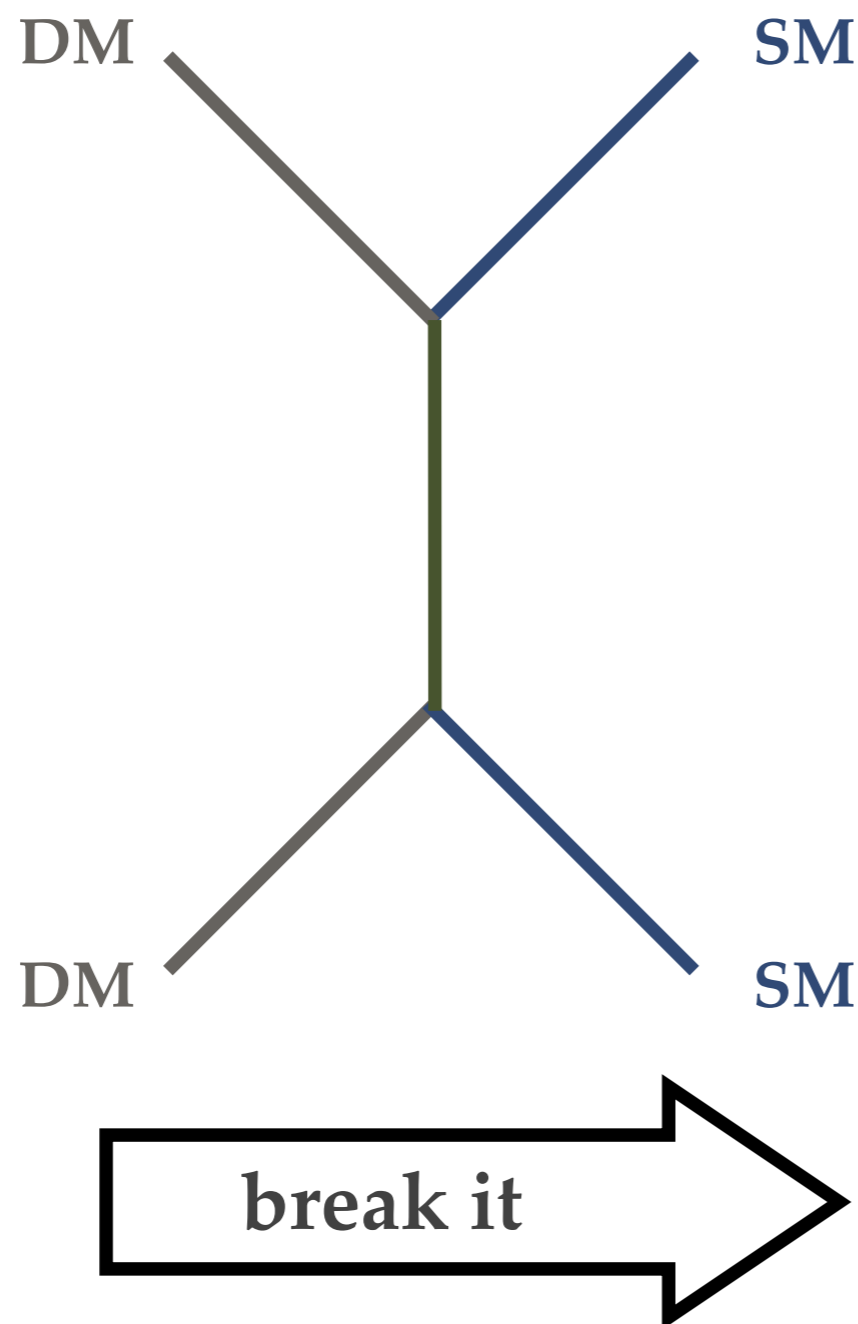
**Expect emission from DM dense region due to DM annihilations**

**2. DM is made of very long lived particles.**

Examples: sterile neutrinos, small black holes...

**Expect emission from DM dense regions due to DM decay**

# Annihilating Dark Matter

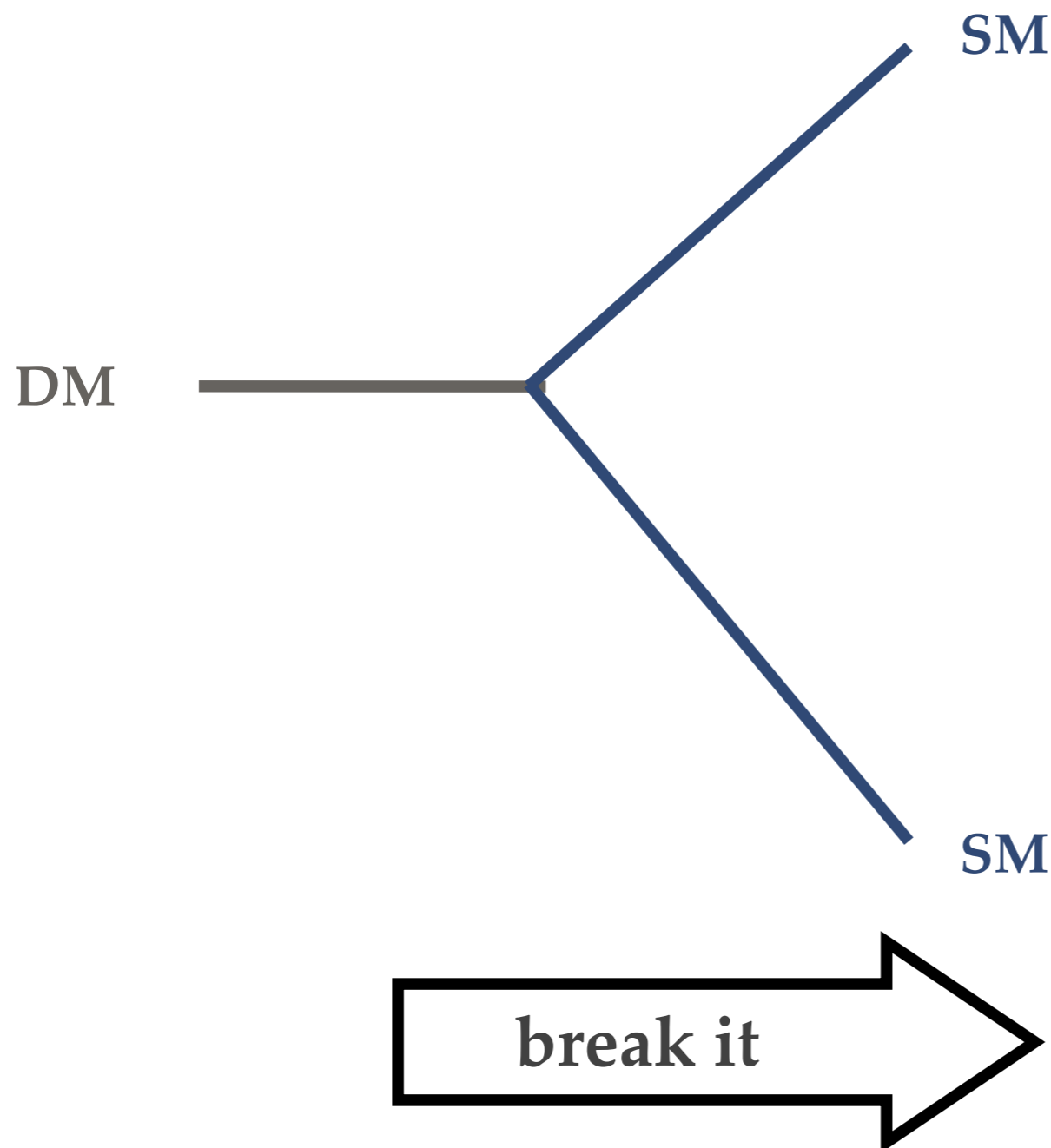


- **Unstable SM particles** further decay in a cascade into stable particles.  
*Examples: mesons, neutrons excited baryons, muons, tauons ...*
- **Stable SM particles** can travel astronomical distances  
*Examples: electrons, protons, neutrinos, photons*

**Signal is proportional to the DM density squared  $\rho^2$**



# Decaying Dark Matter

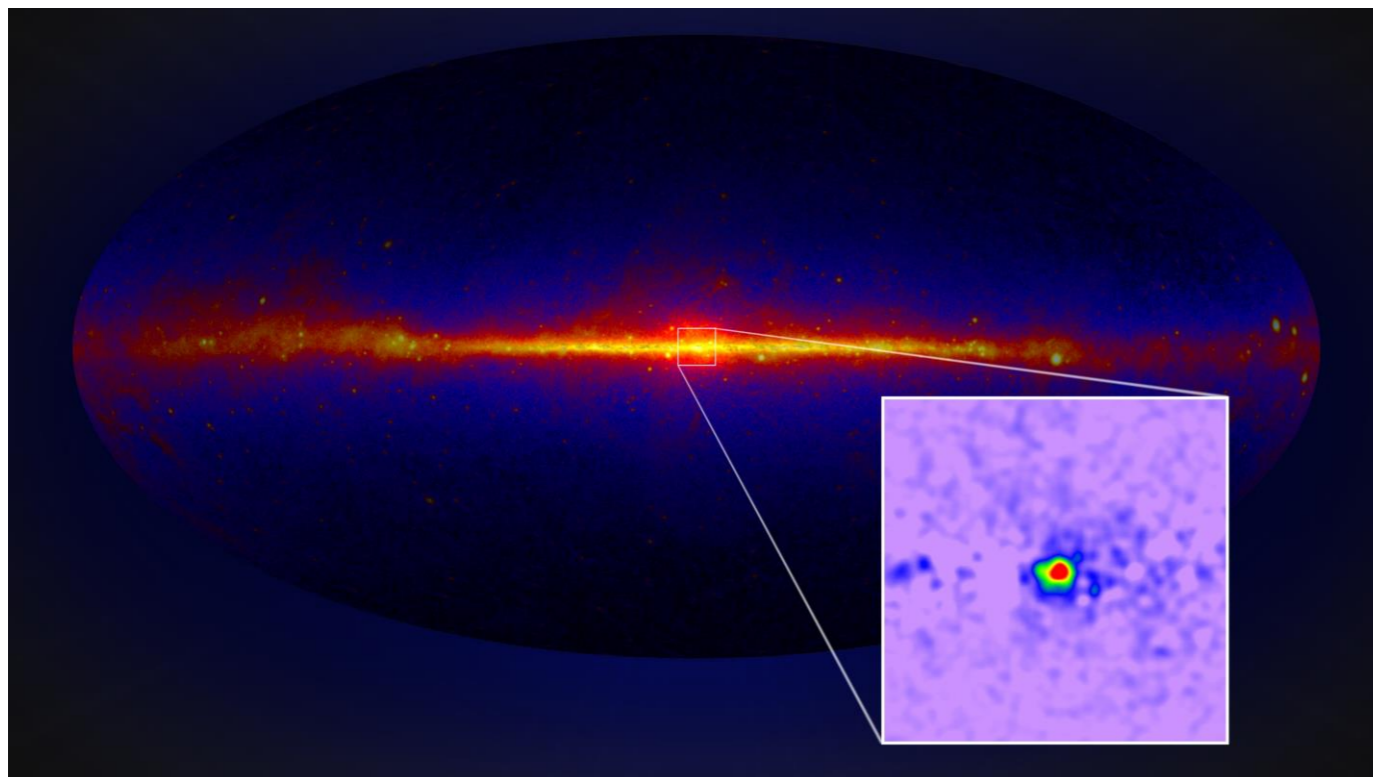


- **Unstable SM particles** further decay in a cascade into stable particles.  
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*Examples: electrons, protons, neutrinos, photons*

**Signal is simply proportional to the DM density  $\rho$**

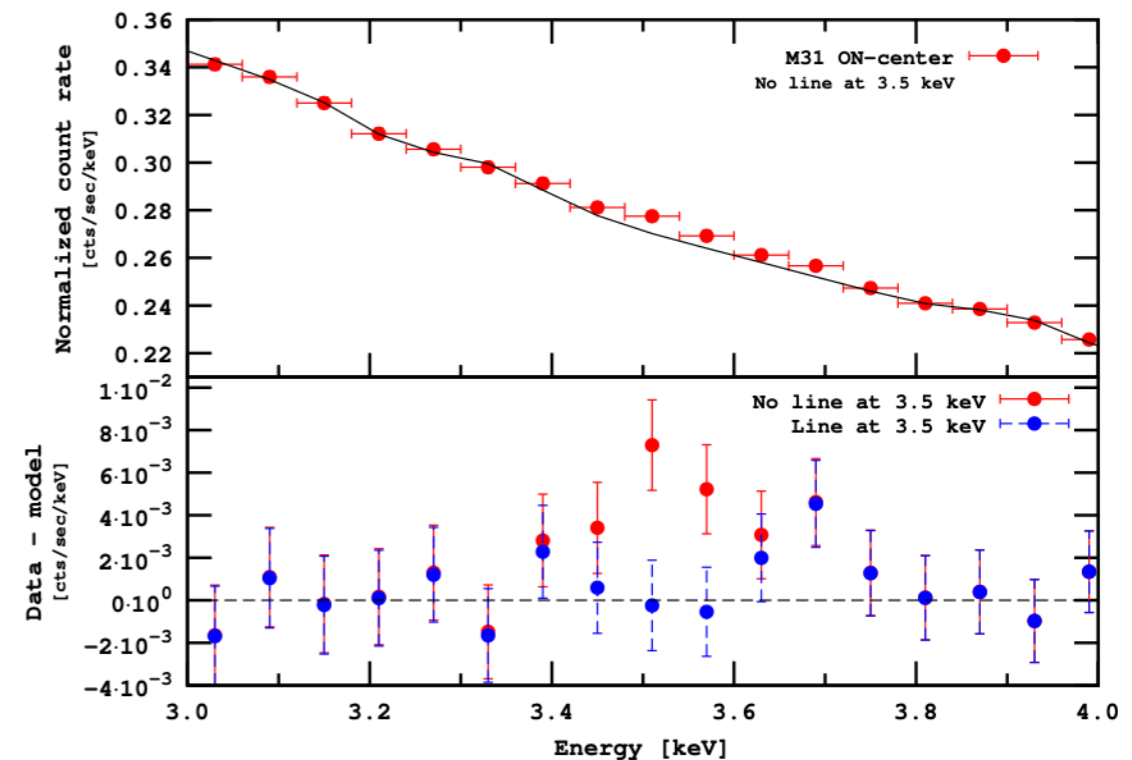
# Photons

- Photons are the most direct messengers, they travel from the source to us on a straight line. This allows to identify the **location of the source** and the **energy released in the decay**.
- For two body decays, the photons form a **monochromatic emission line** and energy directly tells the **DM mass**.



**GeV mass Dark Matter?**

e.g. Goodenough/Hooper 2009



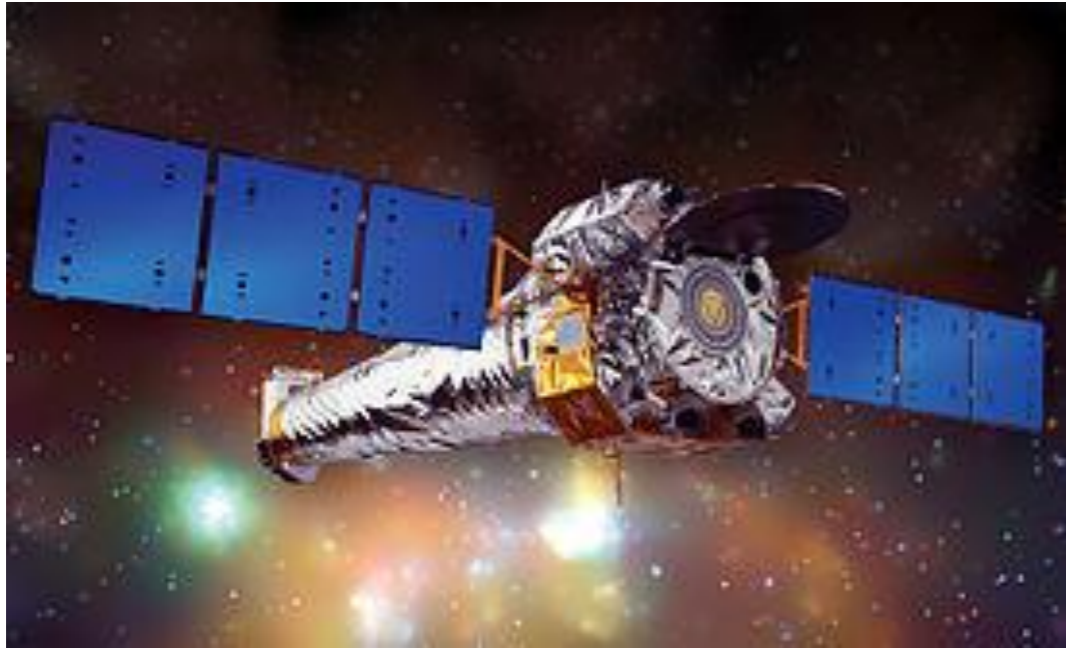
**keV mass Dark Matter?**

Boyarsky et al 2014, Bulbul et al 2014

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# Detecting Photons

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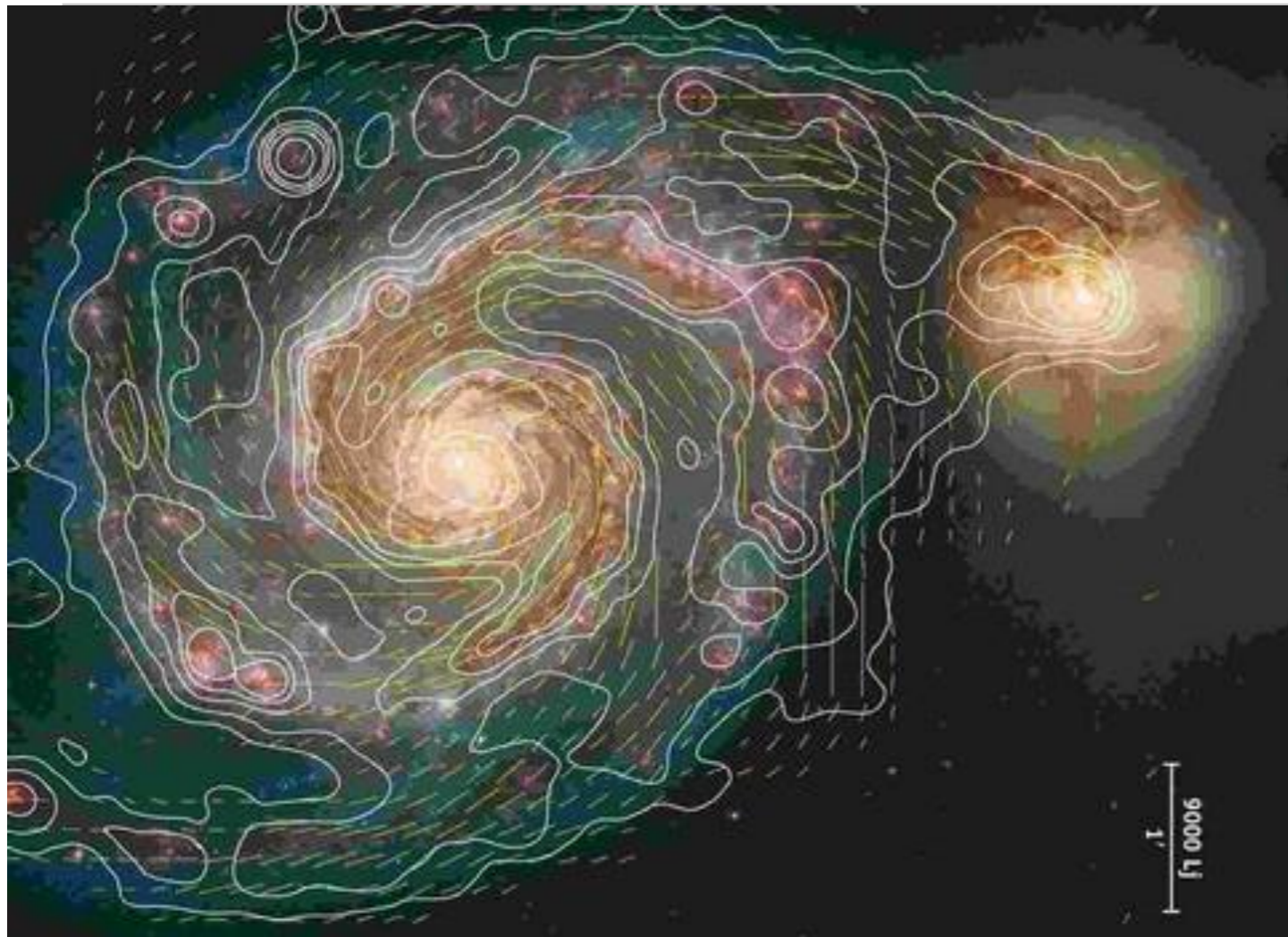
Chandra X -ray Telescope

Fermi Gamma-ray Space Telescope



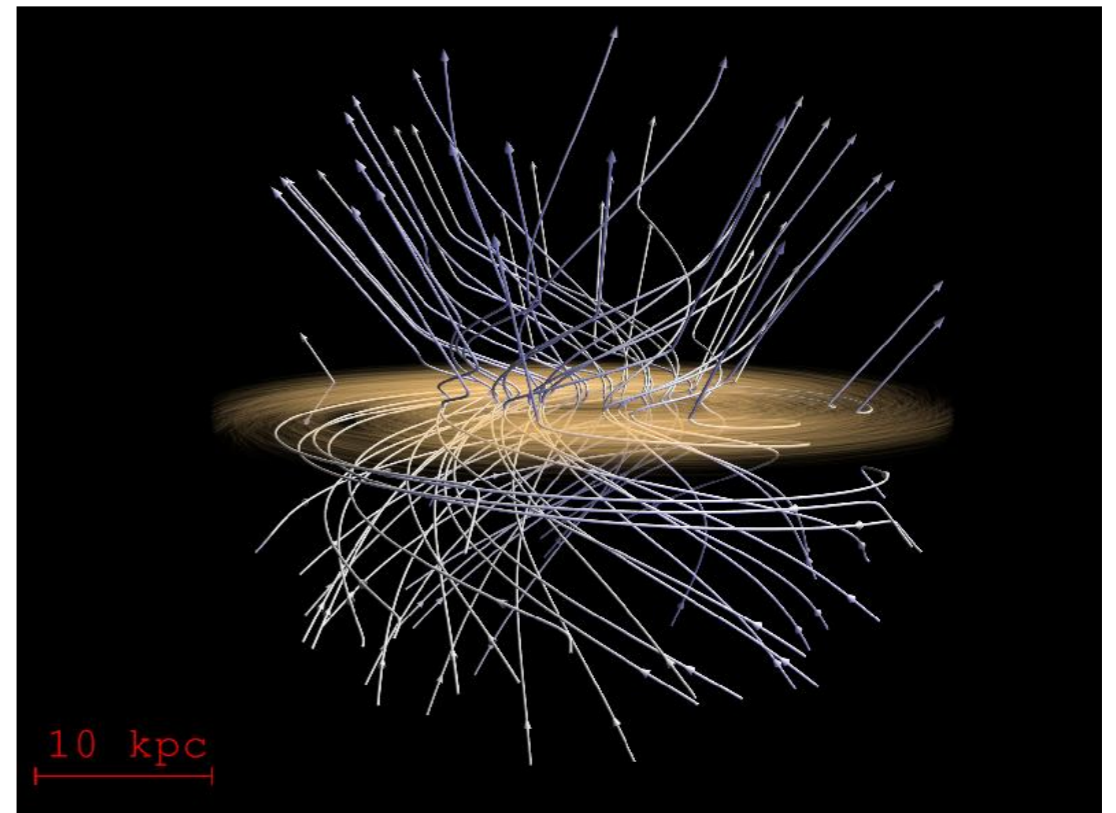


# Charged Cosmic Rays

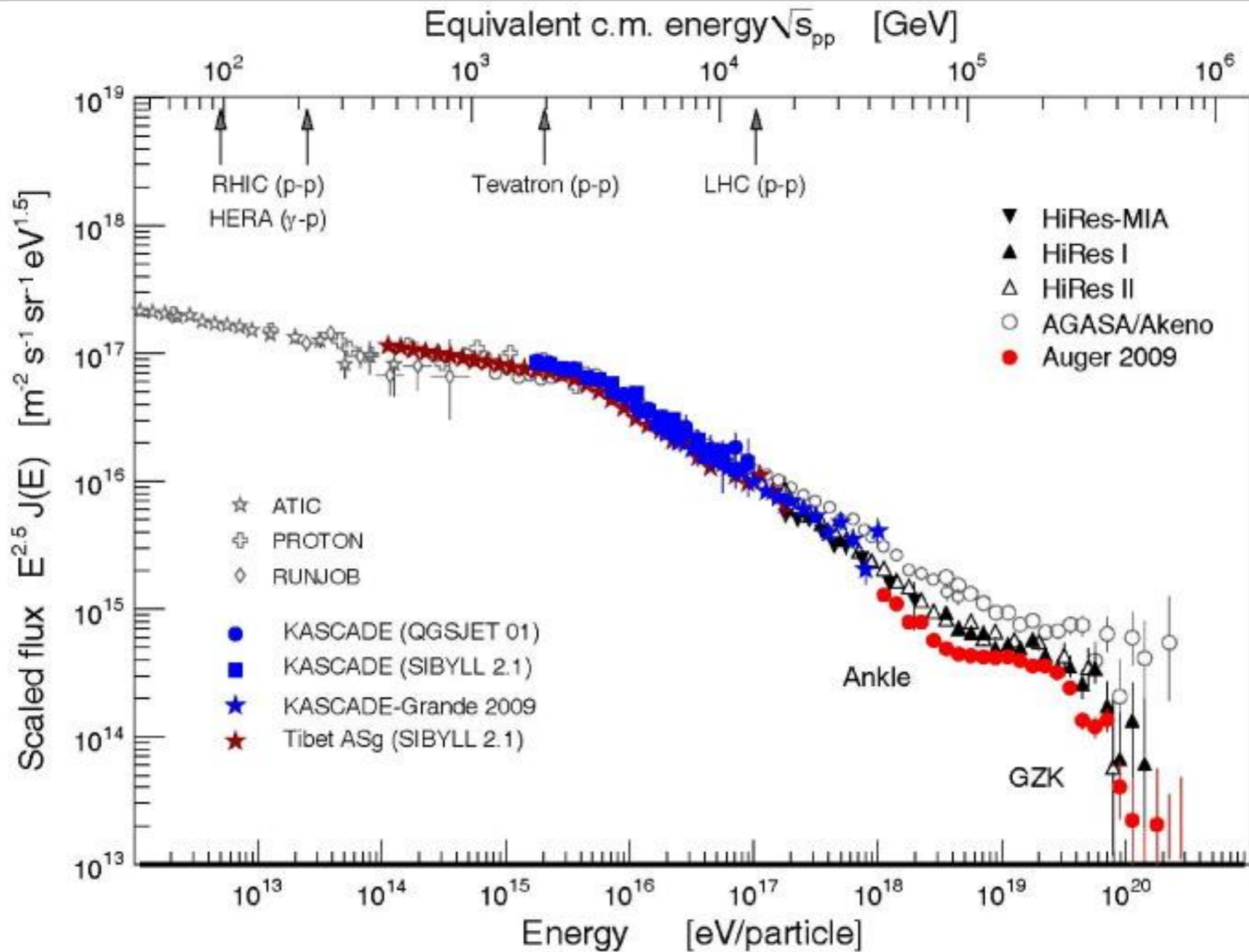


Galaxies (including our own) have magnetic fields. Those fields affect charged cosmic ray propagation.

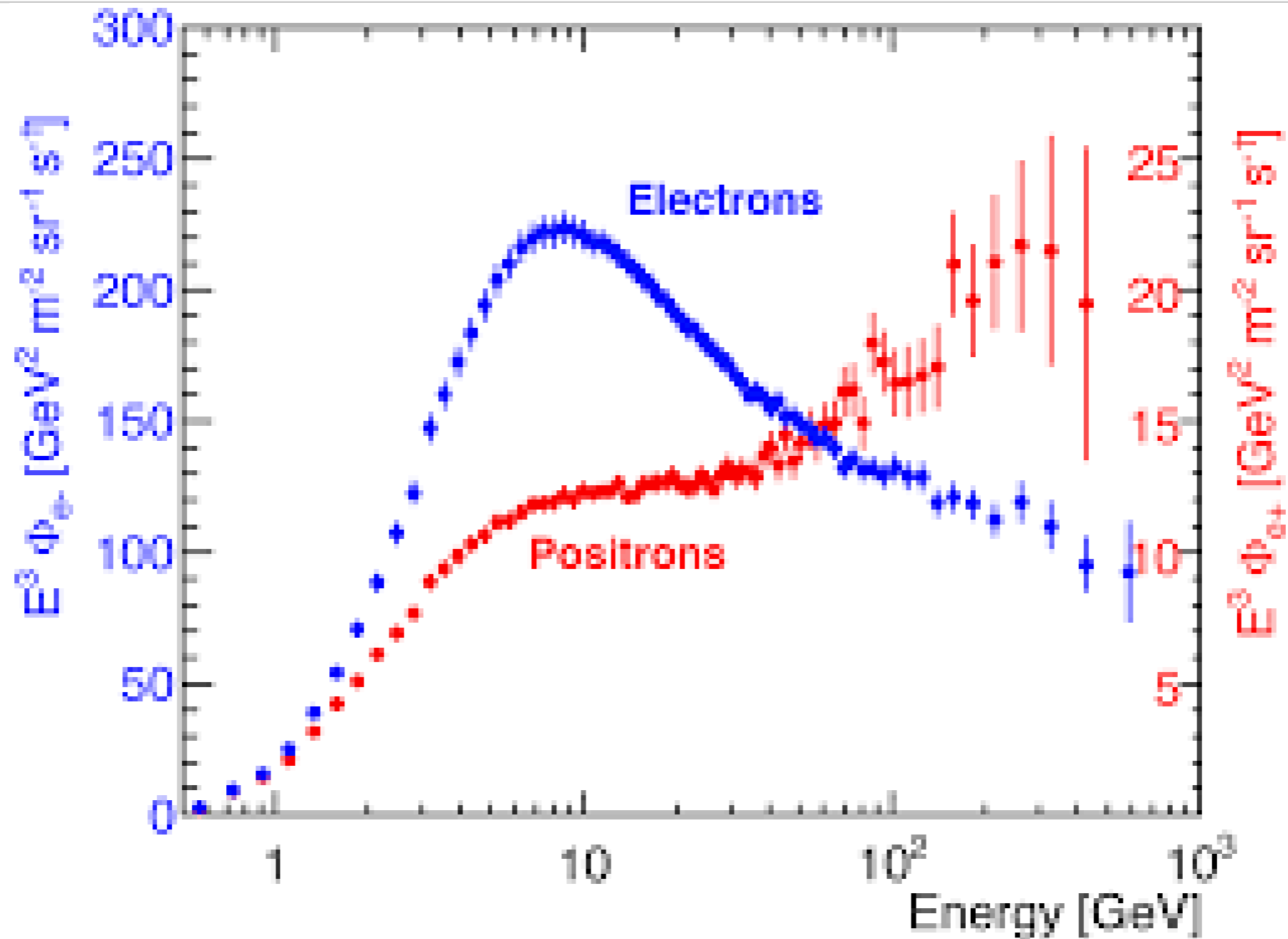
As a result, only limited information about the position of the source can be extracted, **the main observable is the spectrum.**



# Cosmic Ray Spectrum



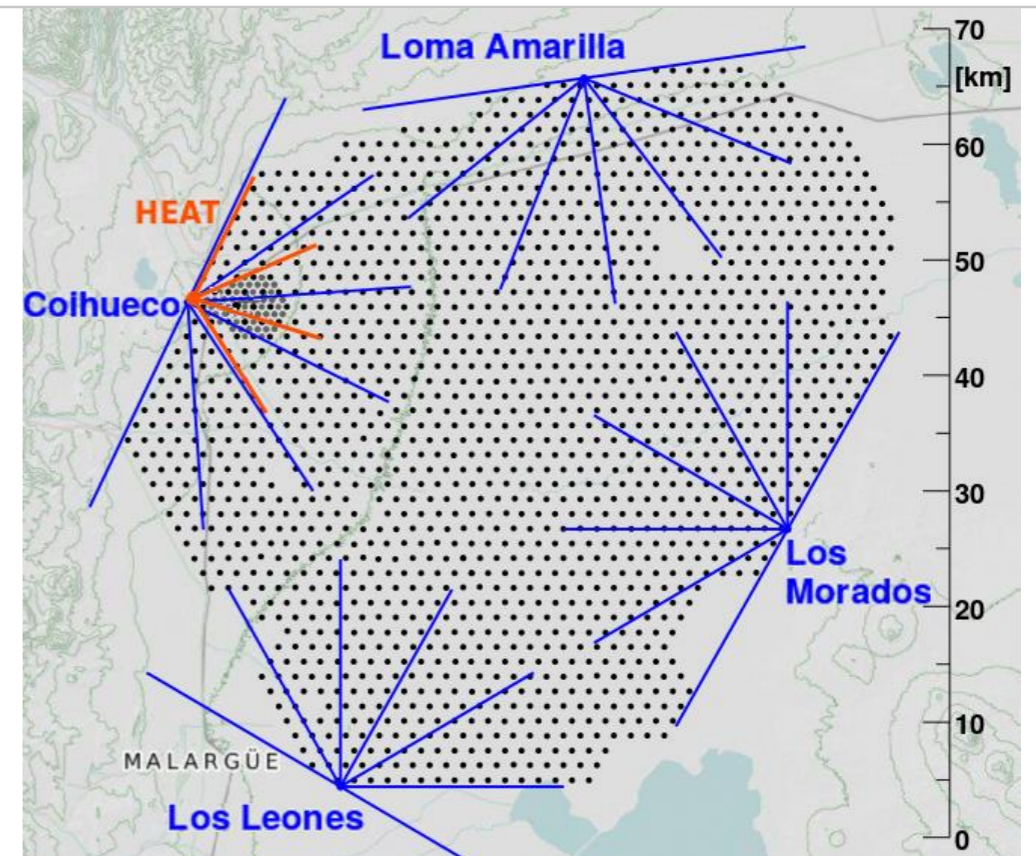
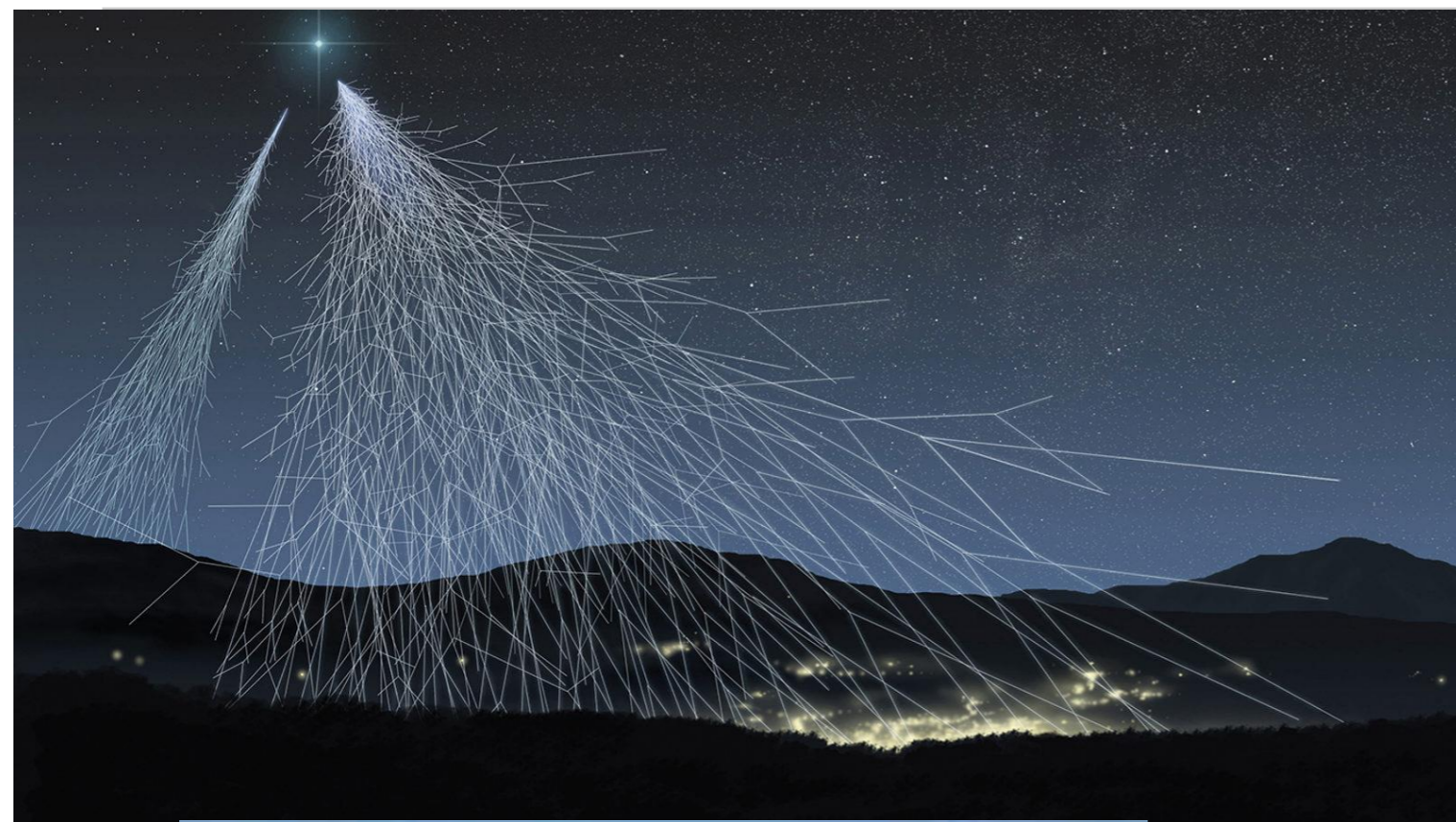
# Positron Excess



Positron spectrum shows funny rise at high energies - from DM?



# Ground Based Cosmic Ray Detection

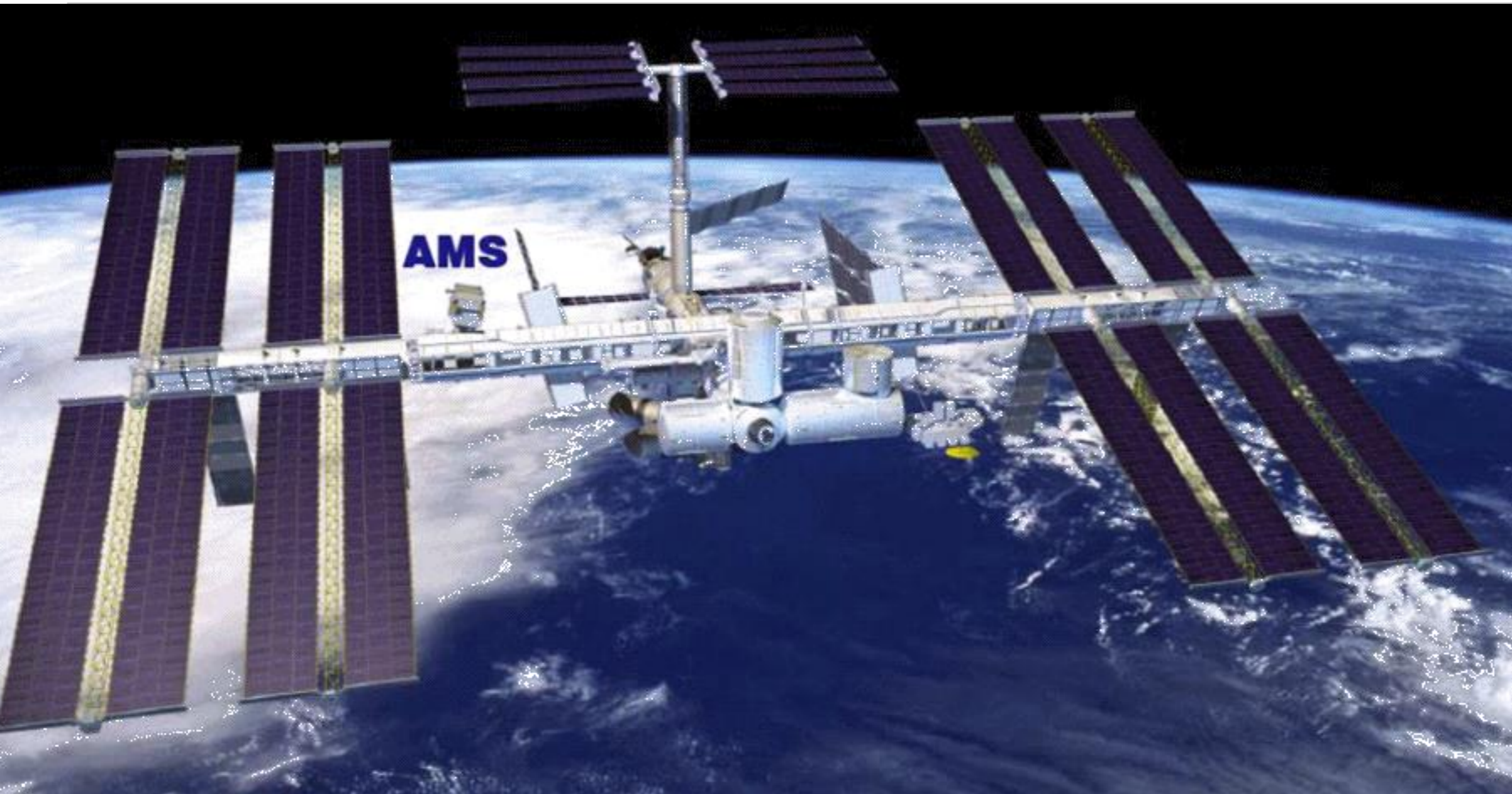


## Pierre Auger Observatory

- Cherenkov detectors (water tanks)
- fluorescence (optical telescopes)
- muon detectors (underground scintillators)
- radio detectors (antenna)

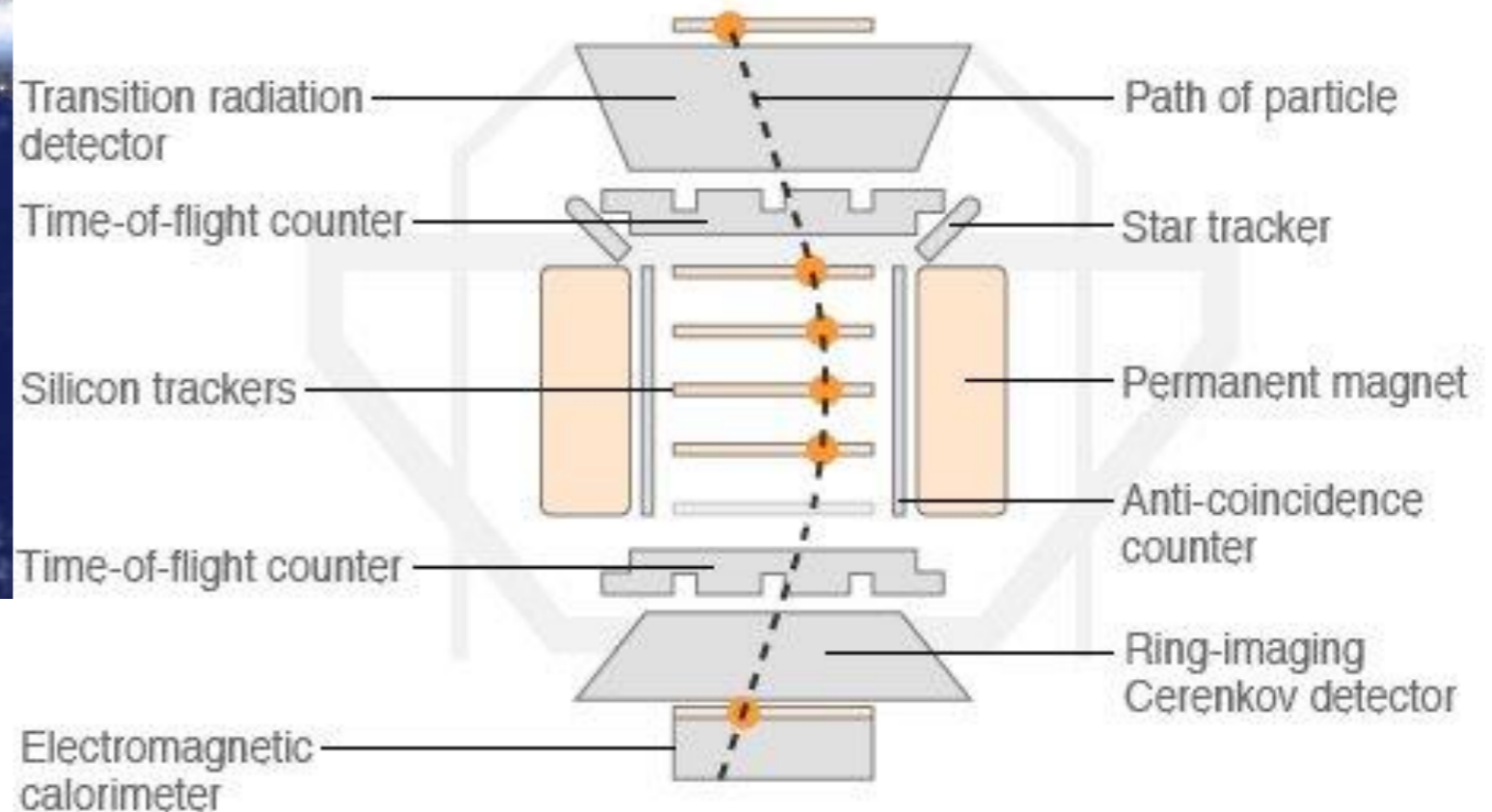
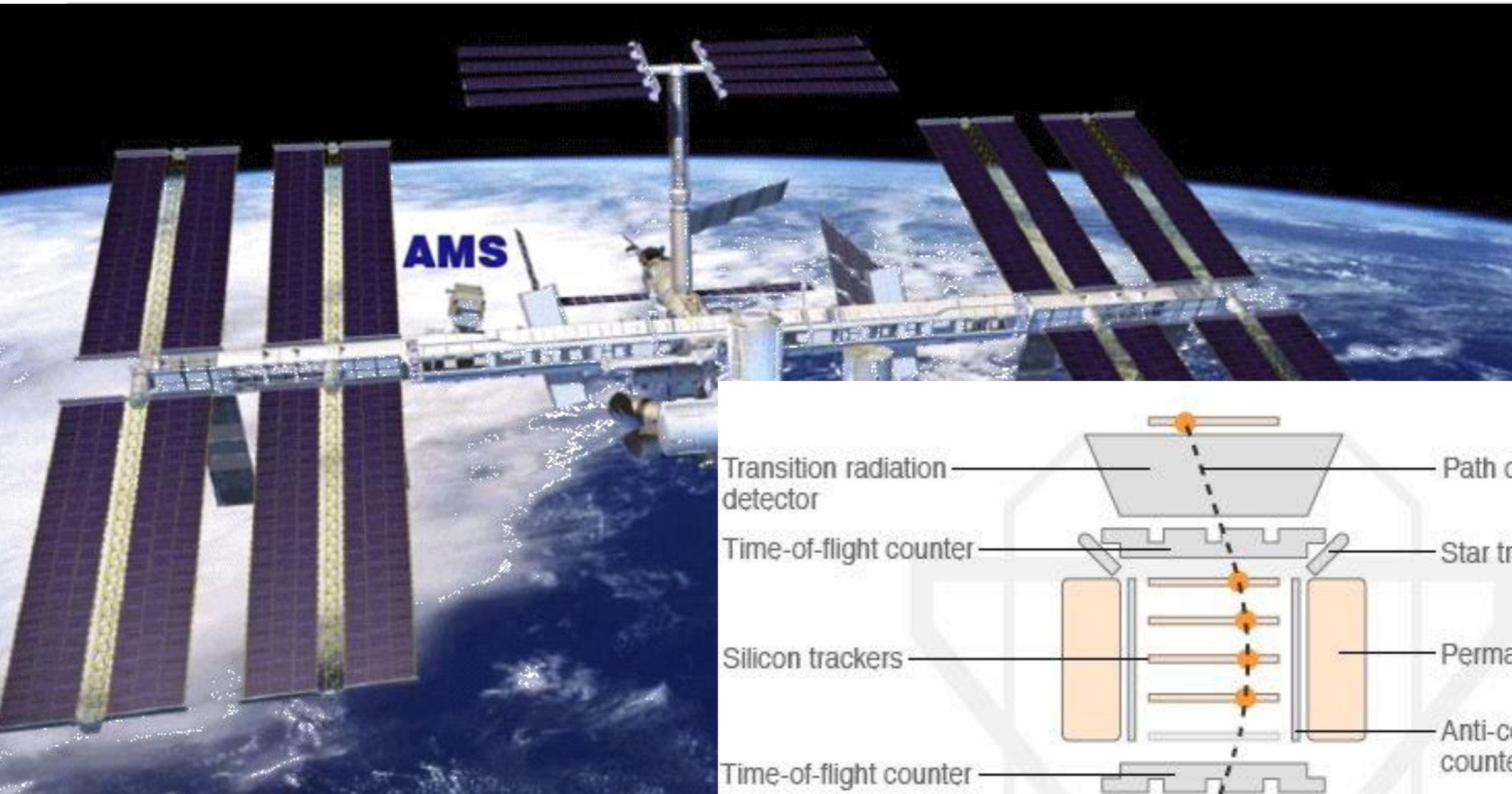


# Space-born Detectors





# Space-born Detectors

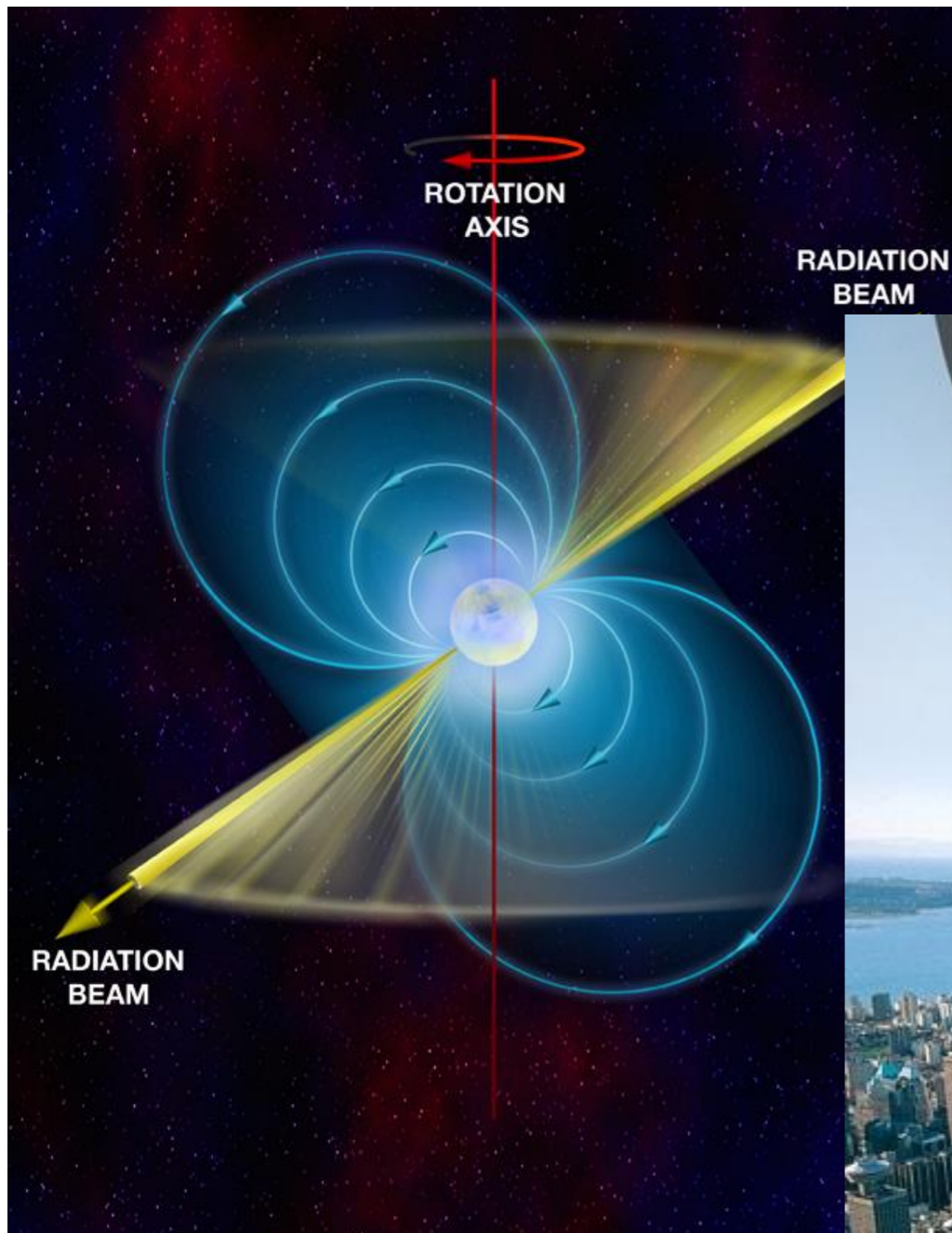


Source: CERN



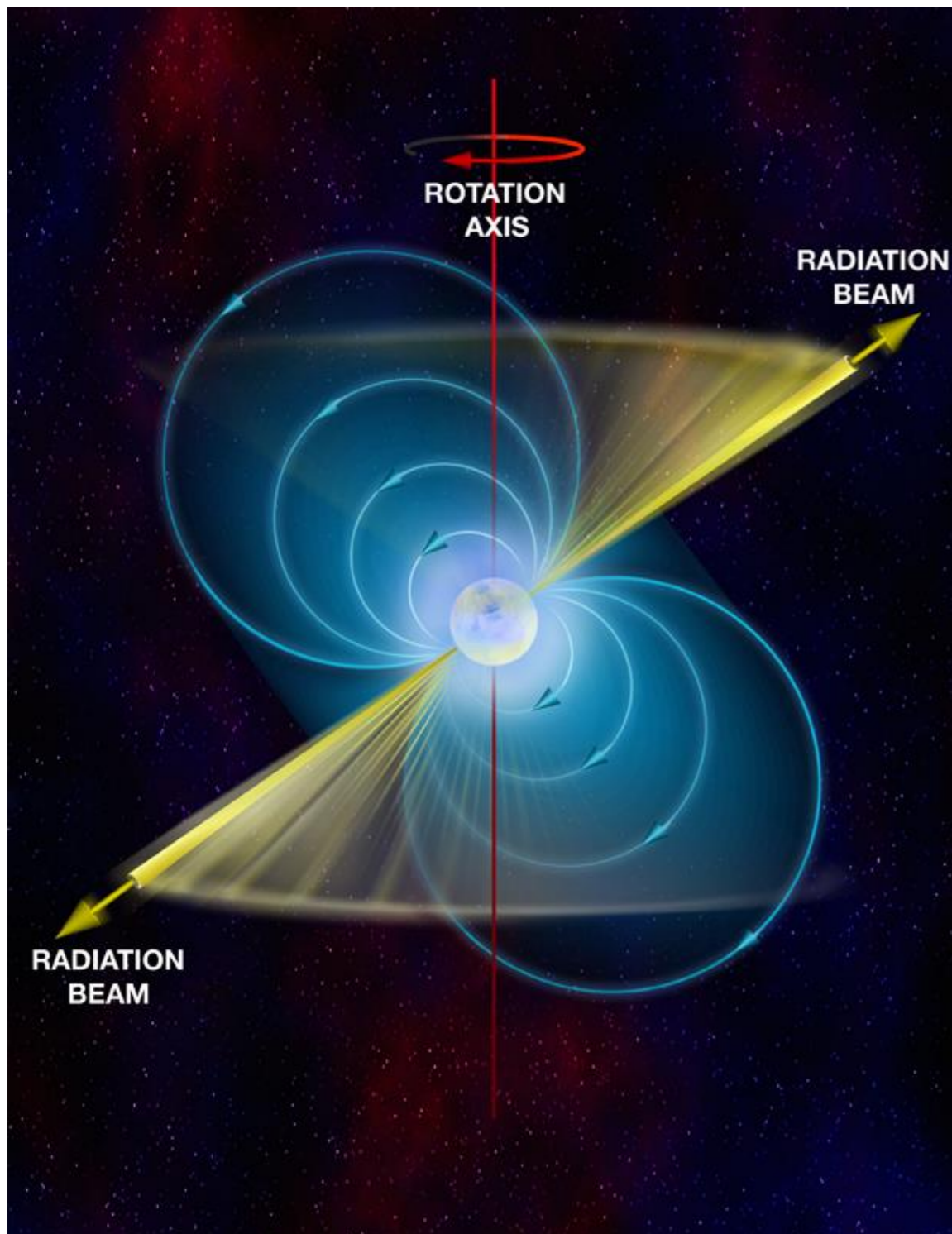
# DM Hunter's Nemesis: The Pulsar

- **Pulsars** are neutron stars, i.e., very compact objects that form at the end of a massive star's life.





# DM Hunter's Nemesis: The Pulsar



- **Pulsars** are neutron stars, i.e., very compact objects that form at the end of a massive star's life.
- They do not shine like normal stars, but have enormous magnetic fields that cause polar lights which are visible on earth. Because they spin rapidly, they blink like a lighthouse.
- **The enormous electromagnetic fields can also act as a particle accelerator and generate high energy cosmic rays.**

# Sterile neutrino Dark Matter



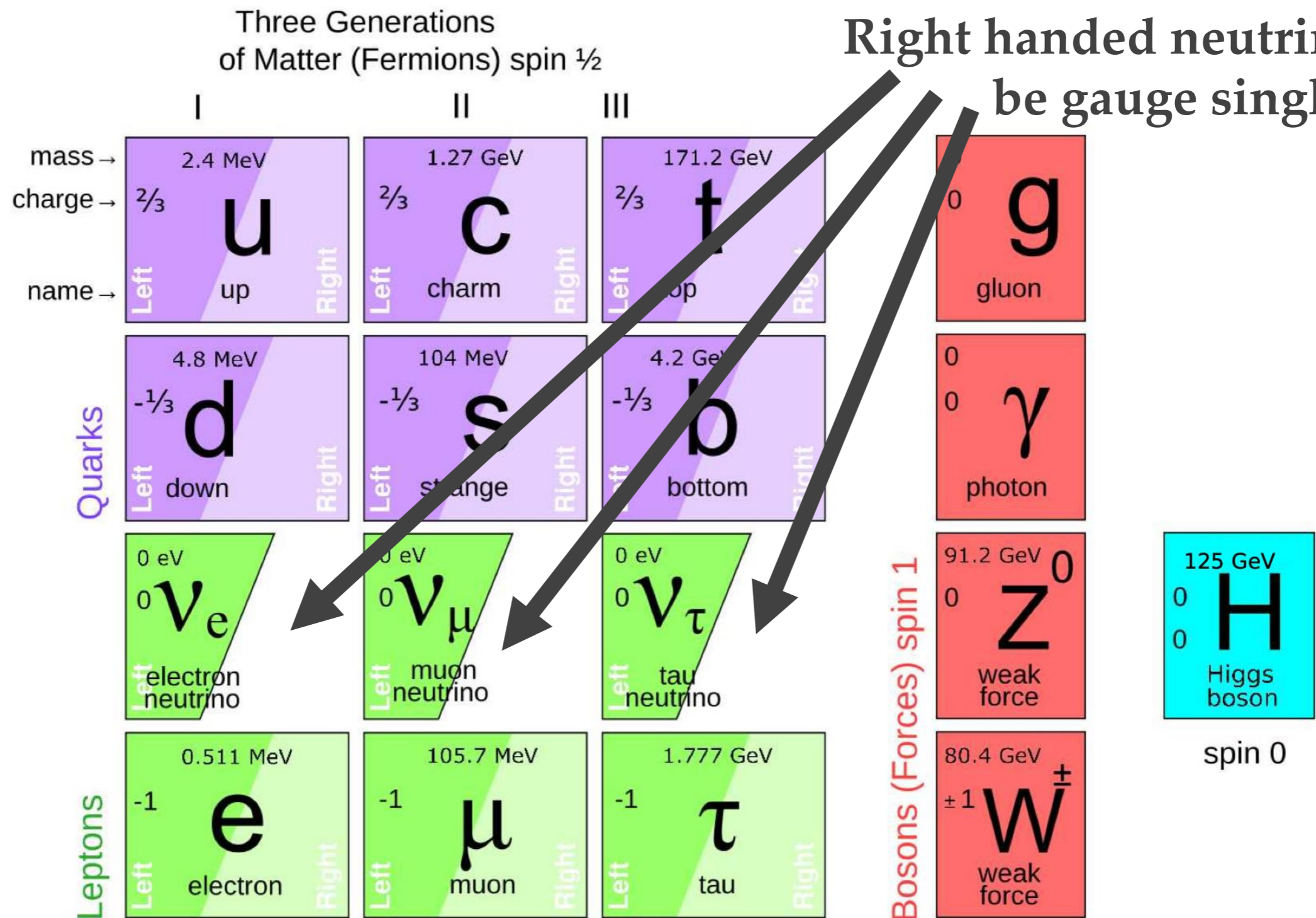
# Sterile Neutrinos

Three Generations  
of Matter (Fermions) spin  $\frac{1}{2}$

	I	II	III		
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0	
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	
name →	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	
	Left Right	Left Right	Left Right		
				0	
				0	
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	
Quarks	Left Right	Left Right	Left Right		
	0 eV	0 eV	0 eV	91.2 GeV	
	0	0	0	0	
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>Z<sup>0</sup></b> weak force	125 GeV
	Left Right	Left Right	Left Right		0
					0
	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV	
	-1	-1	-1	$\pm 1$	
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>W<sup>±</sup></b> weak force	<b>H</b> Higgs boson
Leptons	Left Right	Left Right	Left Right		spin 0

Bosons (Forces) spin 1

# Sterile Neutrinos



---

# Sterile neutrino - mixing

---

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

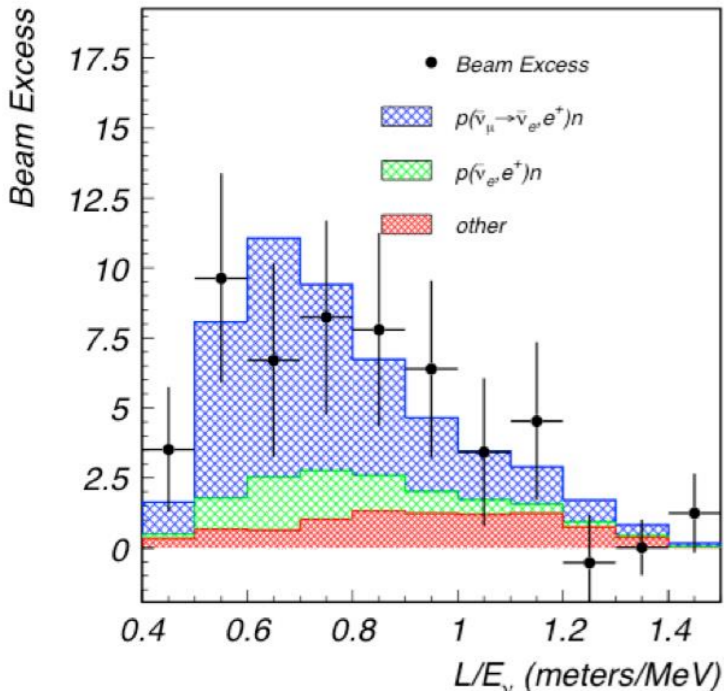
**Sterile neutrino is gauge singlet, feels no electromagnetic or nuclear forces!**

**But can mix with ordinary neutrinos...**

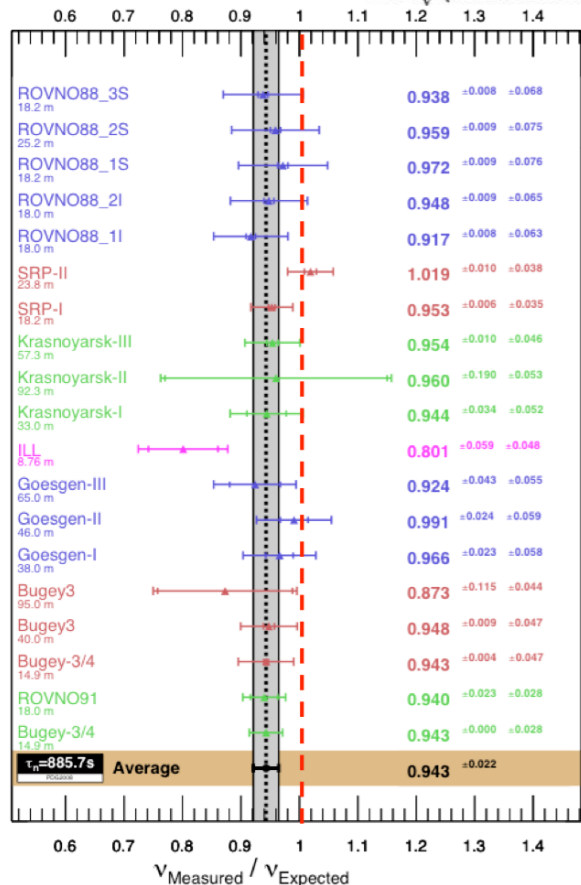
**...this would affect neutrino oscillation data**



# Have we seen it?

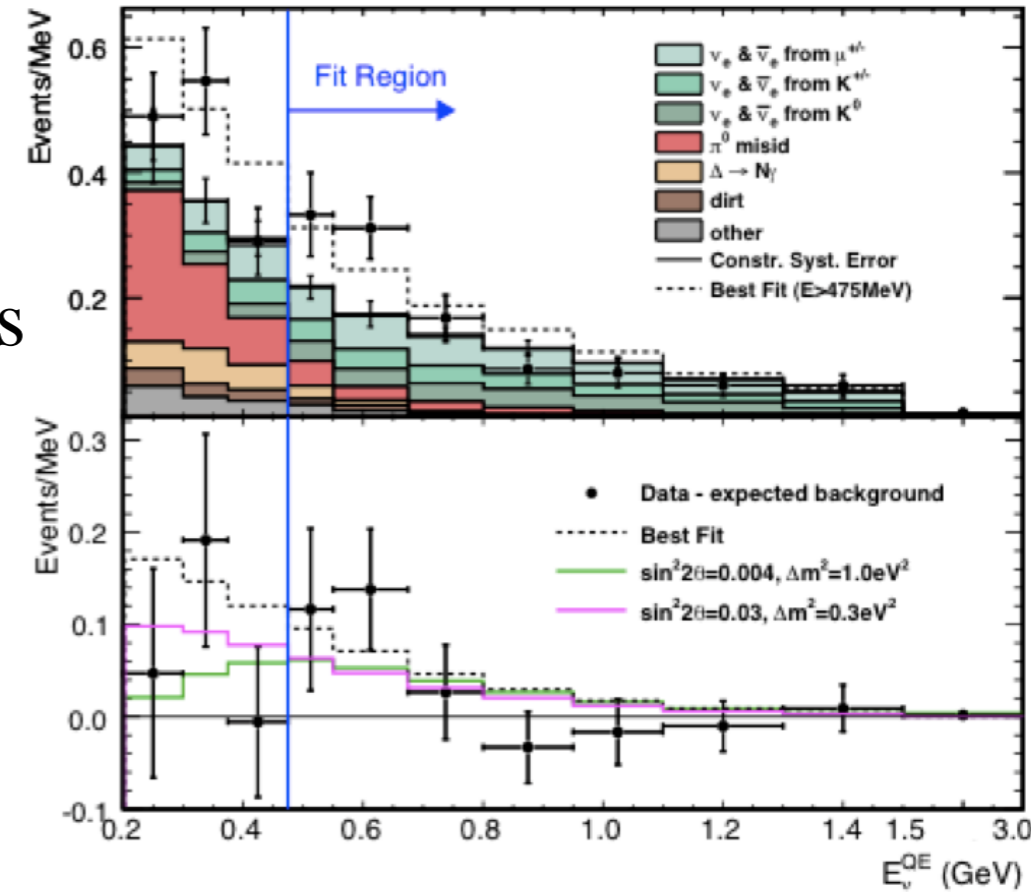


**LSND:**  
excess of  
electron  
antineutrinos

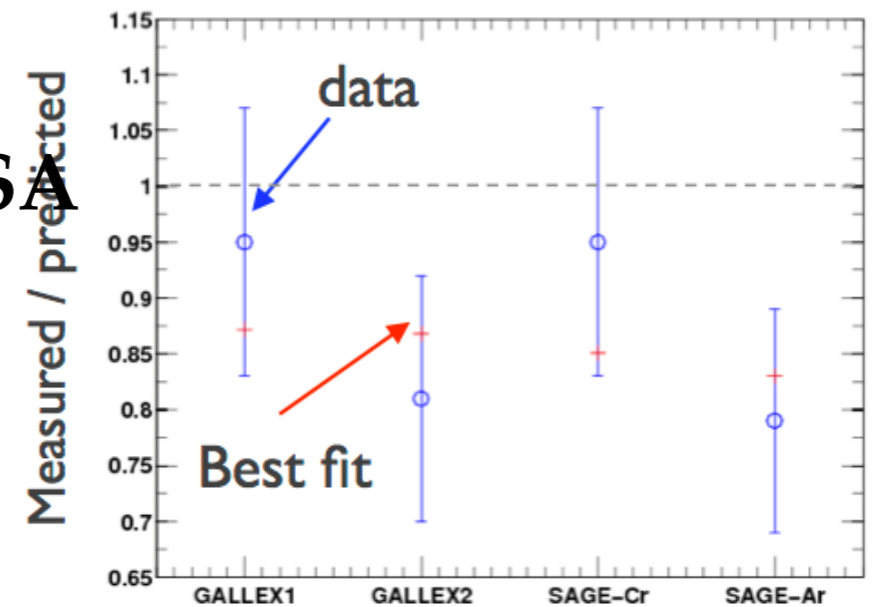


**Reactor  
anomaly:**  
too few  
neutrinos from  
reactors?

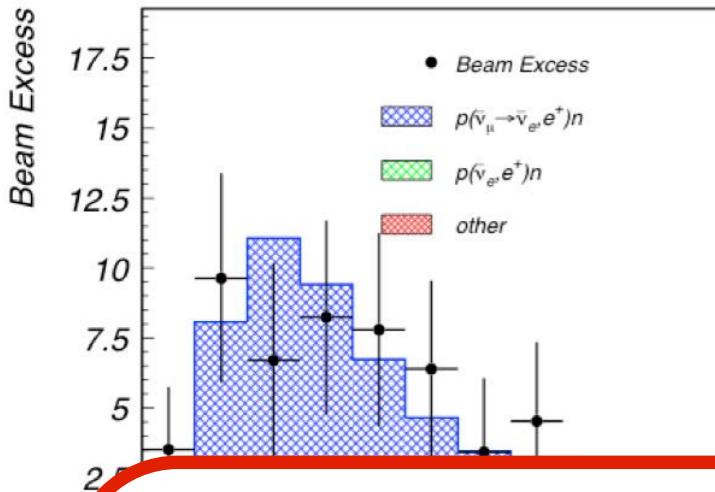
**MiniBoone:**  
extra electron  
(anti)neutrinos  
at low energy



**GALLEX/SA  
GE:**  
missing  
electron  
neutrinos

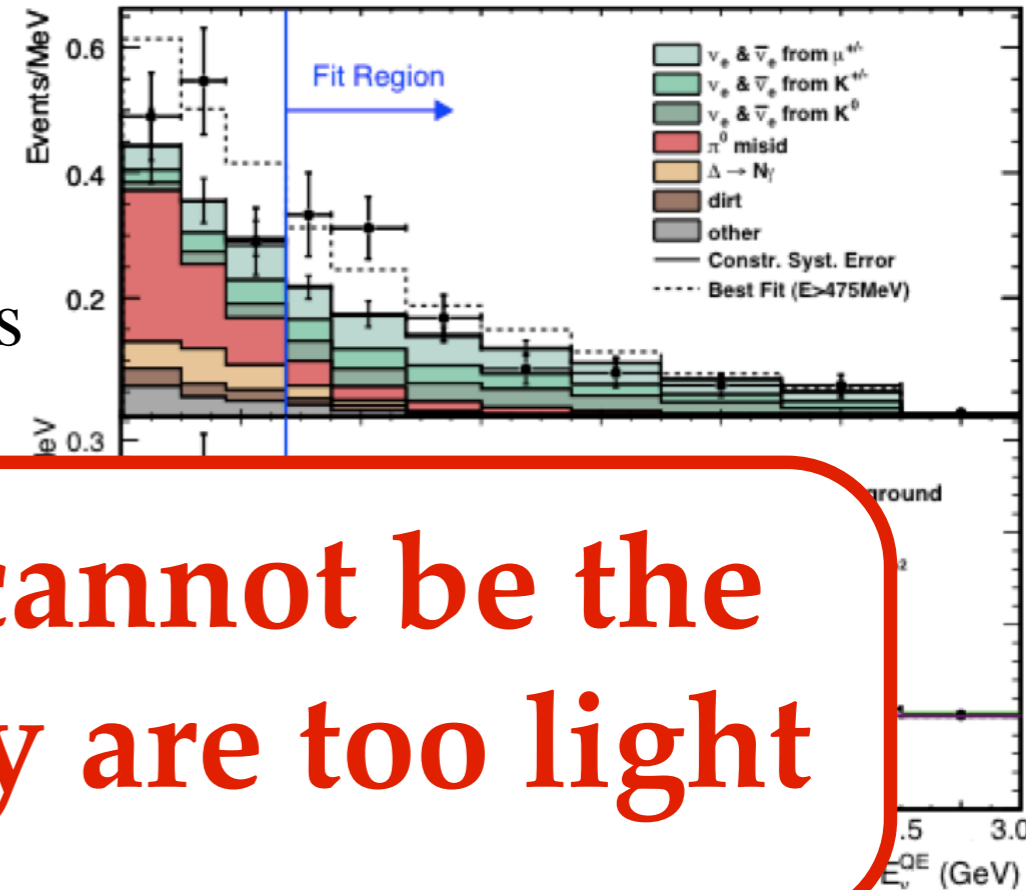


# Have we seen it?

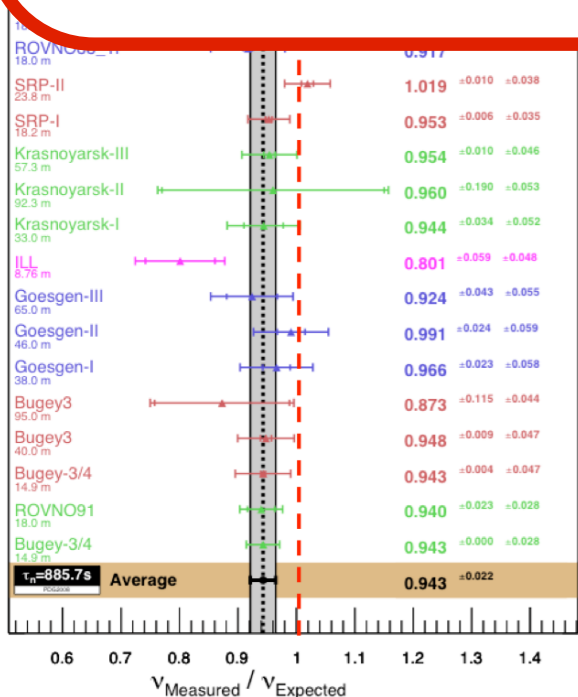


**LSND:**  
excess of  
electron

**MiniBoone:**  
extra electron  
(anti)neutrinos  
at low energy

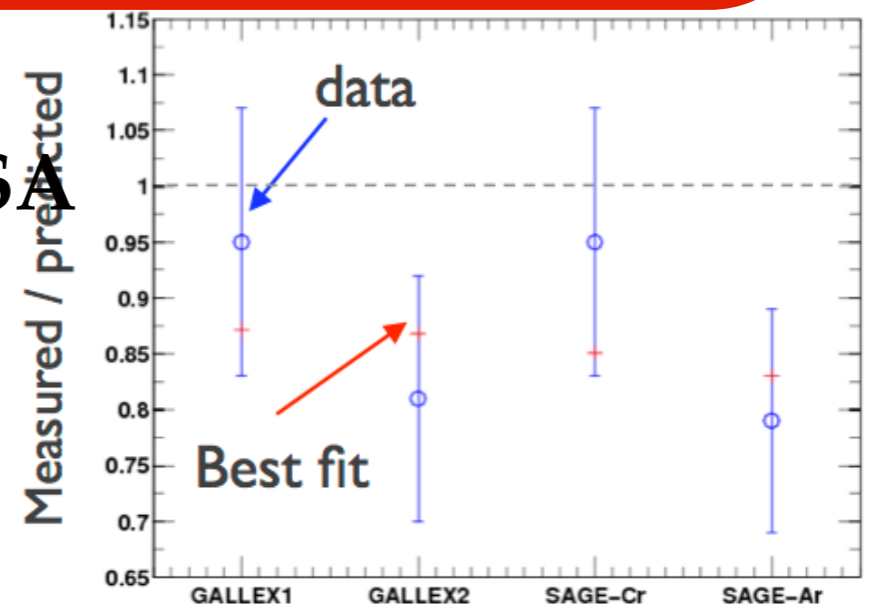


**These sterile neutrinos cannot be the Dark Matter because they are too light**



**anomaly:**  
too few  
neutrinos from  
reactors?

**GALLEX/SAGE:**  
missing  
electron  
neutrinos



# Heavy “Sterile” Neutrino Dark Matter

Dark Matter Particles are

- heavy
- long lived
- neutral
- feebly interacting

❖ **What is the Dark Matter made of?**

It makes up most of the mass in the universe.





# Heavy “Sterile” Neutrino Dark Matter

Dark Matter Particles are

- heavy
- long lived
- neutral
- feebly interacting



Neutrinos are the only known particles that fulfil three conditions...

...but they are too light

## ❖ What is the Dark Matter made of?

It makes up most of the mass in the universe.



# Heavy “Sterile” Neutrino Dark Matter

Dark Matter Particles are

- heavy
- long lived
- neutral
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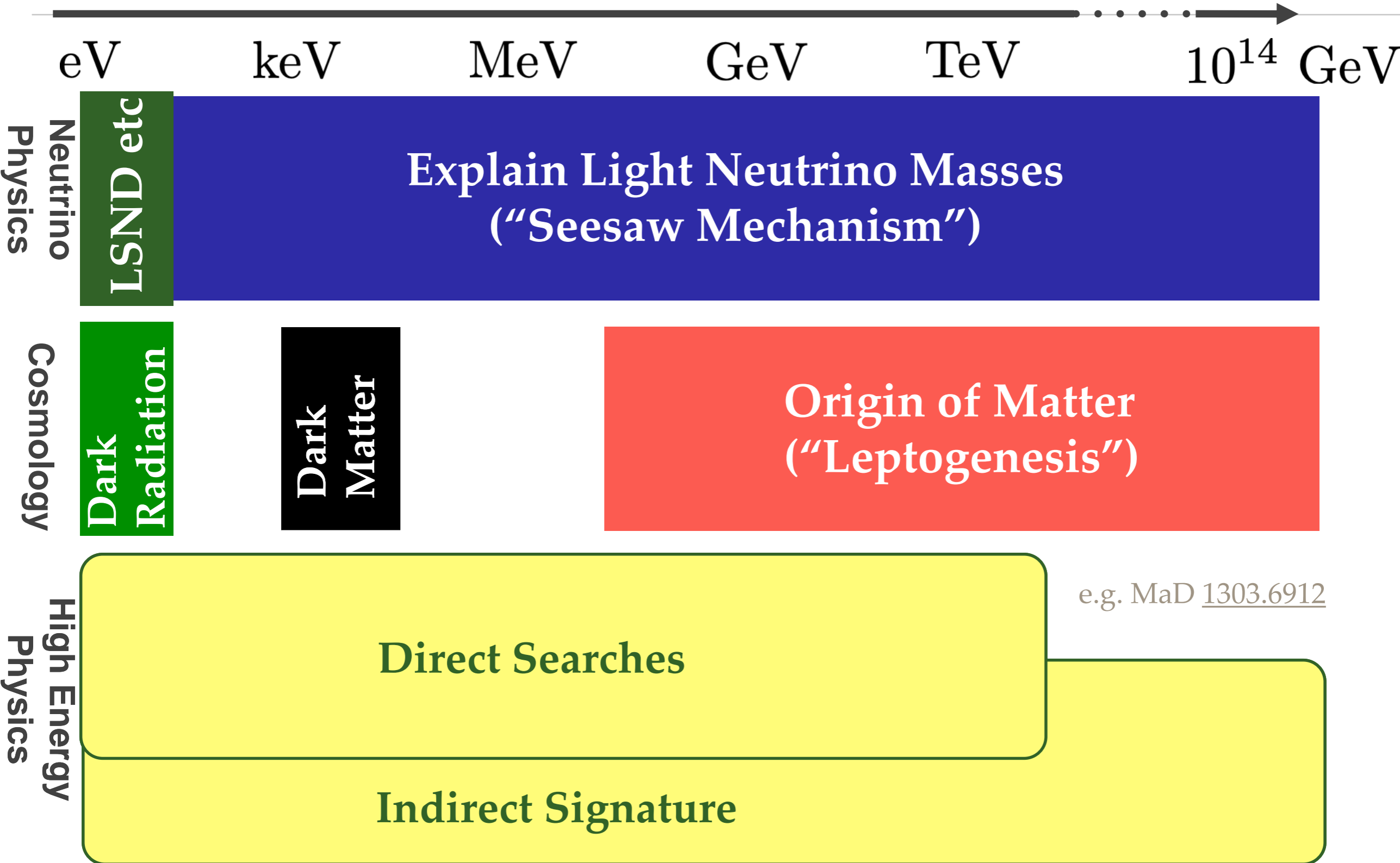
} heavy sterile neutrinos  
can fulfil all conditions!

## ❖ What is the Dark Matter made of?

It makes up most of the mass in the universe.



# Heavy Neutrino Mass Scale





# How heavy do they have to be?

velocity distribution for DM particles:

$$F_X(\mathbf{v}) = \frac{1}{(\sqrt{2\pi} M_X \sigma_X)^3} \exp\left(-\frac{\mathbf{v}^2}{2\sigma_X^2}\right),$$

the maximum number density must be consistent with Pauli principle

$$f_X^{\max}(\mathbf{v}, \mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} F_X(0)$$

$$f_F^{\text{crit}} \equiv \frac{g_X}{(2\pi)^3},$$

$$\frac{(2\pi)^{3/8}}{g_X^{1/4}} \left(\frac{\rho_X}{\sigma_X^3}\right)^{1/4} \leq M_X$$

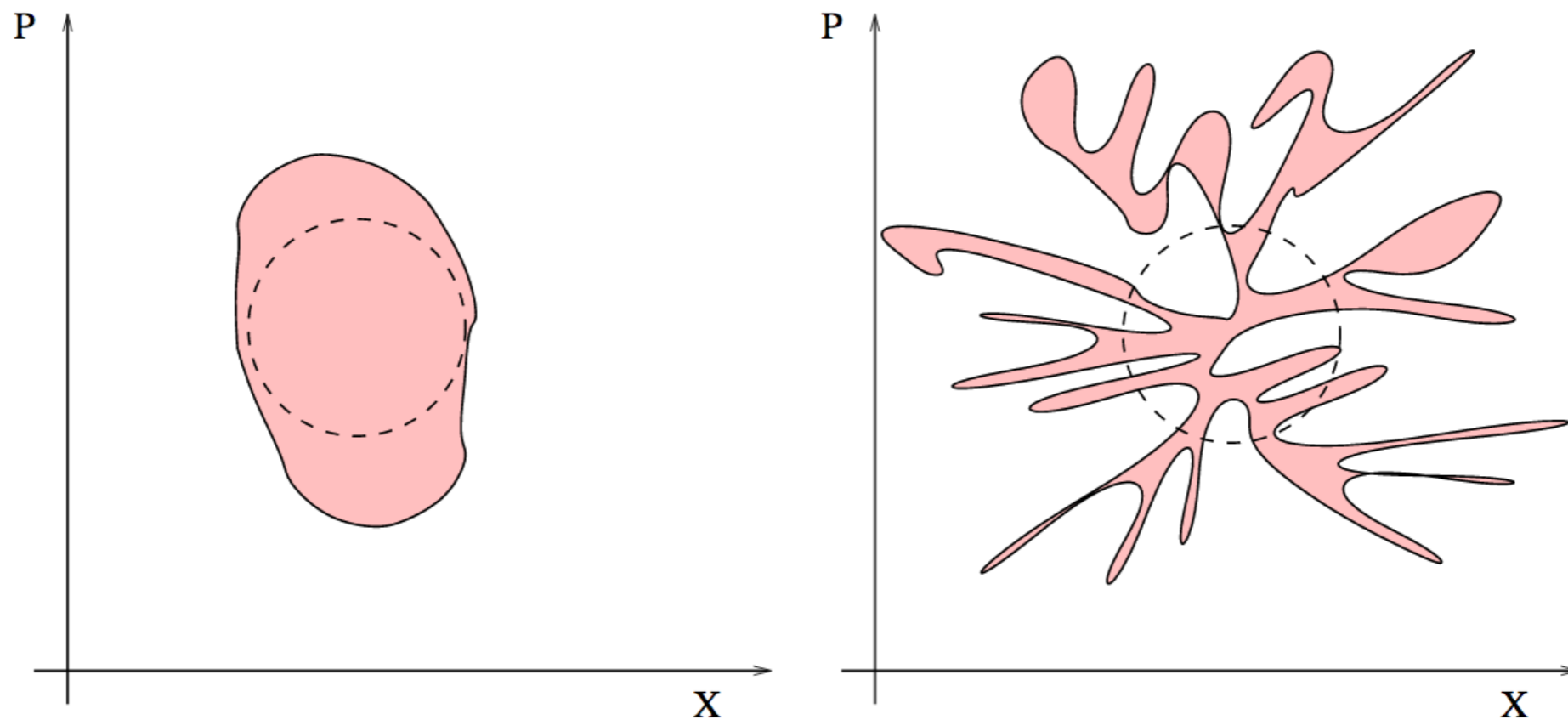
**Pauli limit on fermionic  
Dark Matter mass**

for milky way:

$$M_X \gtrsim 25 \text{ eV}$$

# DM Phase Space Density

Liouville's theorem: phase space volume constant



But coarse grained phase space density decreases in dense regions

$$\tilde{f}(\mathbf{k}, \mathbf{x}, t) \leq \max_k f_i(\mathbf{k}),$$

# Tremaine Gunn Bound

Astronomical data constraints the quantity

$$Q \equiv \frac{\rho_0}{\langle \mathbf{v}_{\parallel}^2 \rangle^{3/2}}$$

For spheroidal dwarf galaxies:

$$\langle \mathbf{v}_{\parallel}^2 \rangle = \langle \mathbf{v}^2 \rangle / 3, \quad \rho_0 = M_X n_X \quad \langle \mathbf{p}^2 \rangle = M_X^2 \langle \mathbf{v}^2 \rangle$$

Combining the equations

$$Q = 3^{3/2} M_X^4 \frac{n}{\langle \mathbf{p}^2 \rangle^{3/2}} \simeq 3^{3/2} M_X^4 \tilde{f}(\mathbf{p}, \mathbf{X}, t_0)$$

using coarse grained  
phase space  
distribution

**Tremaine  
Gunn  
bound**

$$M_X \gtrsim \left( \frac{Q}{3^{3/2} \max \tilde{f}_i} \right)^{1/4}$$



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# Dark Matter Decay

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primary decay channel

$$N \rightarrow 3\nu$$

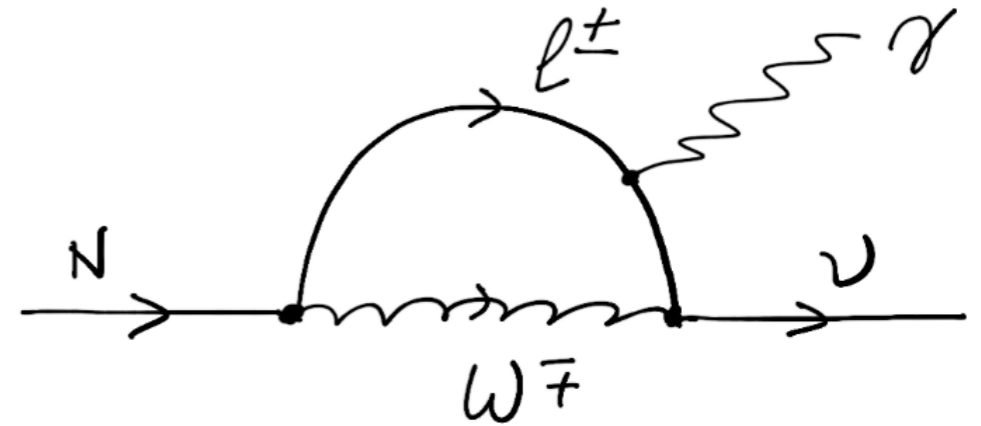
$$\Gamma_{N \rightarrow 3\nu} = \frac{G_F^2 M^5}{96\pi^3} \sum_{\alpha} |\theta_{\alpha}|^2 \approx \frac{1}{1.5 \times 10^{14} \text{ sec}} \left( \frac{M}{10 \text{ keV}} \right)^5 \sum_{\alpha} |\theta_{\alpha}|^2$$

lifetime must be longer than the age of the universe

$$\theta^2 < 3.3 \times 10^{-4} \left( \frac{10 \text{ keV}}{M} \right)^5$$

# Indirect DM Searches

loop level decay into photons

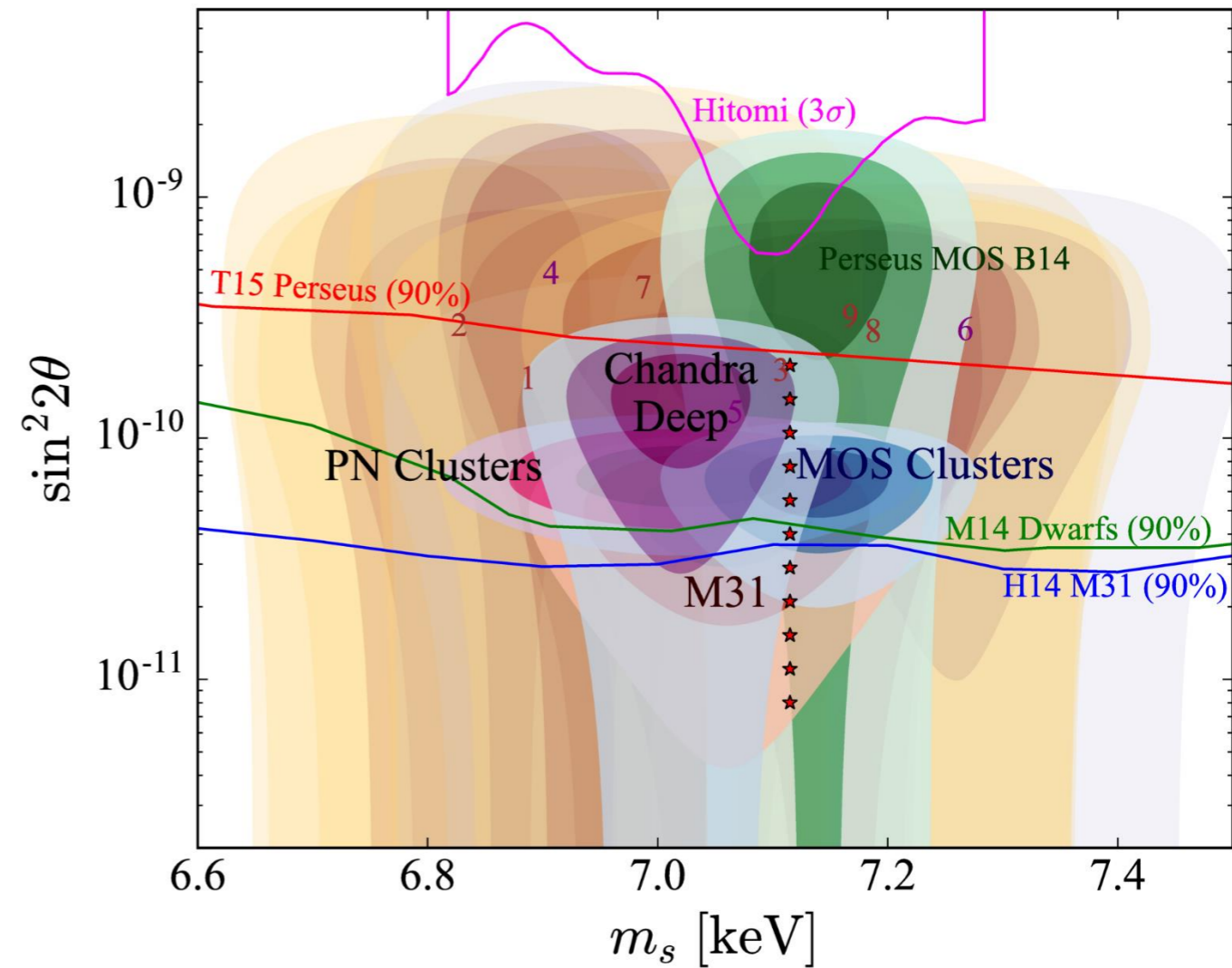
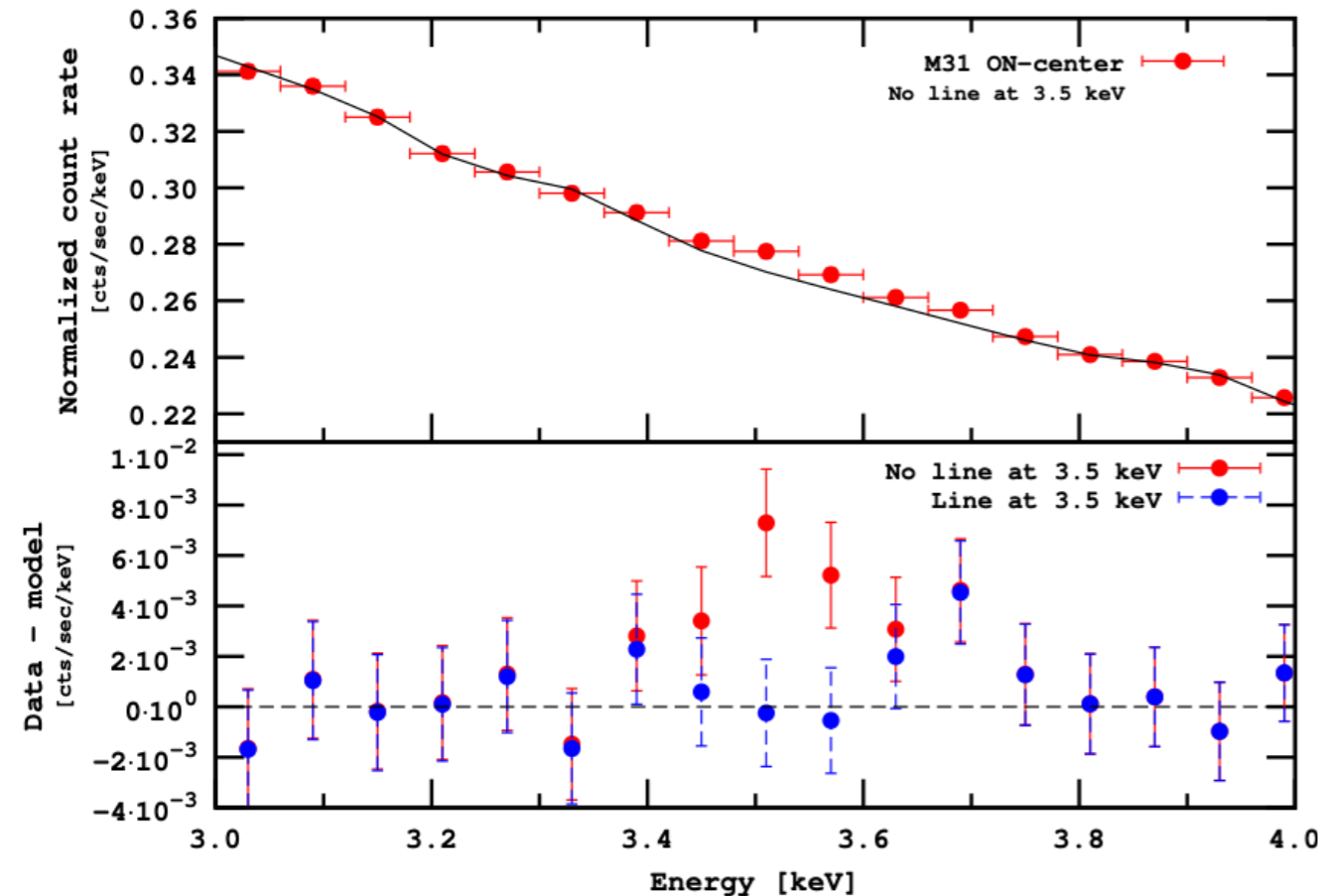


$$\Gamma_{N \rightarrow \gamma \nu} = \frac{9 \alpha G_F^2}{256 \pi^4} \theta^2 M^5 = 5.5 \times 10^{-22} \theta^2 \left[ \frac{M}{1 \text{ keV}} \right]^5 \text{ sec}^{-1} .$$

**One can search for an emission line!**



# Has the line been seen?



Boyarsky/Ruchayskiy/Iakubovskiy/Franse 2014  
see also  
Bulbul/Markevitch/Foster/Smith/Loewenstein/Randall  
2014

**Situation unclear...**

**need better spectral resolution (XARM and ATHENA will help)**



---

# How to make Sterile Neutrino DM?

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1. **thermal production through mixing**
2. **thermal production through new interactions at high energy**
3. **non-thermal production in decay of heavy particles**

# Production through Mixing

Consider system with one active and one sterile neutrino

$$\begin{aligned} |\nu_a\rangle &= \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle, \\ |\nu_s\rangle &= -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle. \end{aligned}$$

In the primordial plasma there is an effective mixing angle

$$\begin{aligned} |\nu_a\rangle &= \cos\theta_m(t) |\nu_1(t)\rangle + \sin\theta_m(t) |\nu_2(t)\rangle, \\ |\nu_s\rangle &= -\sin\theta_m(t) |\nu_1(t)\rangle + \cos\theta_m(t) |\nu_2(t)\rangle \end{aligned}$$

**Thermal production rate** :  $\Gamma_N \sim G_F^2 T^5 \sin^2(2\theta_m)$

---

# Effective Mixing Angle

---

$$\sin^2(2\theta_m) = \frac{\Delta^2(p) \sin^2(2\theta)}{\Delta^2(p) \sin^2(2\theta) + [\Delta(p) \cos(2\theta) - V_D - V_T]^2}.$$

The active-sterile mass splitting enters via

$$\Delta(p) = \Delta m^2 / (2p)$$

And the “matter potentials” are

$$V_T \simeq -\frac{8}{3} \sqrt{2} G_F \left[ \frac{\rho_\nu}{m_Z^2} + \frac{\rho_\ell}{m_W^2} \right] E_\nu,$$

$$V_D \simeq 2\sqrt{2} G_F n_\gamma l_\nu = 2\sqrt{2} G_F \frac{2\zeta(3)}{\pi^2} T^3 l_\nu,$$



# Non-resonant Production

$$\sin^2(2\theta_m) = \frac{\Delta^2(p) \sin^2(2\theta)}{\Delta^2(p) \sin^2(2\theta) + [\Delta(p) \cos(2\theta) - V_D - V_T]^2}$$

The active-sterile mass splitting enters via

$$\Delta(p) = \Delta m^2 / (2p)$$

And the “matter potentials” are

$$V_T \simeq G_{\text{eff}}^2 T^4 p \quad G_{\text{eff}}^2 \sim 10^2 G_F^2$$

$$V_D \simeq 0$$

Thermal production rate peaks at  $T \sim 0.1 - 1 \text{ GeV}$

$$\Gamma_N \sim G_F^2 T^5 \sin^2(2\theta_m)$$

# Non-resonant Production

$$\sin^2(2\theta_m) = \frac{\Delta^2(p) \sin^2(2\theta)}{\Delta^2(p) \sin^2(2\theta) + [\Delta(p) \cos(2\theta) - V_D - V_T]^2}$$

The active-sterile mass splitting enters via

$$\Delta(p) = \Delta m^2 / (2p)$$

**vacuum mixing angle smaller than  $10^{-6}$   
(X-ray searches)**

And the “matter potentials” are

$$V_T \simeq G_{\text{eff}}^2 T^4 p \quad G_{\text{eff}}^2 \sim 10^2 G_F^2$$

$$V_D \simeq 0$$

**Thermal production rate peaks at  $T \sim 0.1 - 1 \text{ GeV}$**

$$\Gamma_N \sim G_F^2 T^5 \sin^2(2\theta_m)$$

# Non-resonant Production

$$\sin^2(2\theta_m) = \frac{\Delta^2(p) \sin^2(2\theta)}{\Delta^2(p) \sin^2(2\theta) + [\Delta(p) \cos(2\theta) - V_D - V_T]^2}$$

The active-sterile mass splitting enters via

$$\Delta(p) = \Delta m^2 / (2p)$$

at high T the matter potential suppresses the effective mixing angle

And the “matter potentials” are

$$V_T \simeq G_{\text{eff}}^2 T^4 p \quad G_{\text{eff}}^2 \sim 10^2 G_F^2$$

$$V_D \simeq 0$$

Thermal production rate peaks at  $T \sim 0.1 - 1 \text{ GeV}$

$$\Gamma_N \sim G_F^2 T^5 \sin^2(2\theta_m)$$



# Non-resonant Production

$$\sin^2(2\theta_m) = \frac{\Delta^2(p) \sin^2(2\theta)}{\Delta^2(p) \sin^2(2\theta) + [\Delta(p) \cos(2\theta) - V_D - V_T]^2}$$

The active-sterile mass splitting enters via

$$\Delta(p) = \Delta m^2 / (2p)$$

at low T the light neutrino flux is too low

And the “matter potentials” are

$$V_T \simeq G_{\text{eff}}^2 T^4 p \quad G_{\text{eff}}^2 \sim 10^2 G_F^2$$

$$V_D \simeq 0$$

Thermal production rate peaks at  $T \sim 0.1 - 1 \text{ GeV}$

$$\Gamma_N \sim G_F^2 T^5 \sin^2(2\theta_m)$$

# Resonant Production

$$\sin^2(2\theta_m) = \frac{\Delta^2(p) \sin^2(2\theta)}{\Delta^2(p) \sin^2(2\theta) + [\Delta(p) \cos(2\theta) - V_D - V_T]^2}.$$

The active-sterile mass splitting enters via

$$\Delta(p) = \Delta m^2 / (2p)$$

resonance condition

$$\Delta(p) \cos(2\theta) - V_D - V_T = 0$$

**resonance condition strongly depends on lepton asymmetries**

$$M^2 - 2 \frac{4\sqrt{2}\zeta(3)}{\pi^2} G_F l_\nu p T^3 + 2G_{\text{eff}}^2 p^2 T^4 = 0, \quad l_\nu \equiv (n_\nu - n_{\bar{\nu}}) / n_\gamma.$$

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# Resonance Condition

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resonance for mode with  $x \equiv p/T$  occurs at

$$x_{res} = \frac{G_F}{G_{\text{eff}}^2 T^2} \frac{4\zeta(3)}{\sqrt{2}\pi^2} l_\nu \left[ 1 \pm \sqrt{1 - \frac{1}{2} \frac{M^2}{T^2} \frac{G_{\text{eff}}^2}{G_F^2} \frac{\pi^4}{8\zeta(3)^2} \frac{1}{l_\nu^2}} \right]$$

resonance requires a lepton asymmetry

$$|l_\nu| > \frac{1}{2} \frac{M}{T} \frac{G_{\text{eff}}}{G_F} \frac{\pi^2}{2\zeta(3)},$$

**this is several orders of magnitude larger than the baryon asymmetry!  
(but well below the observational bound)**

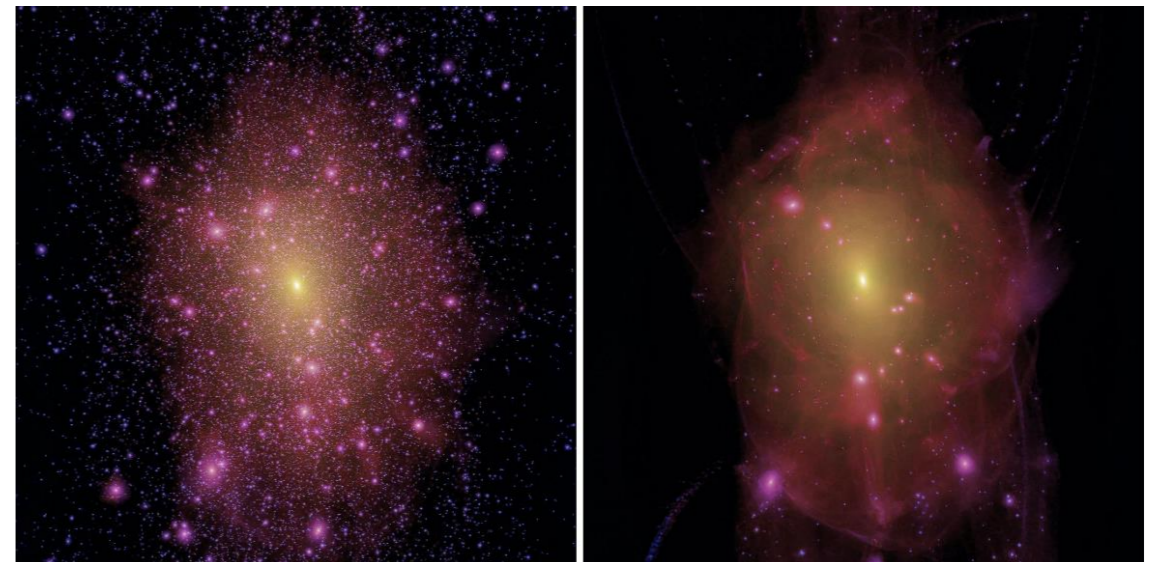
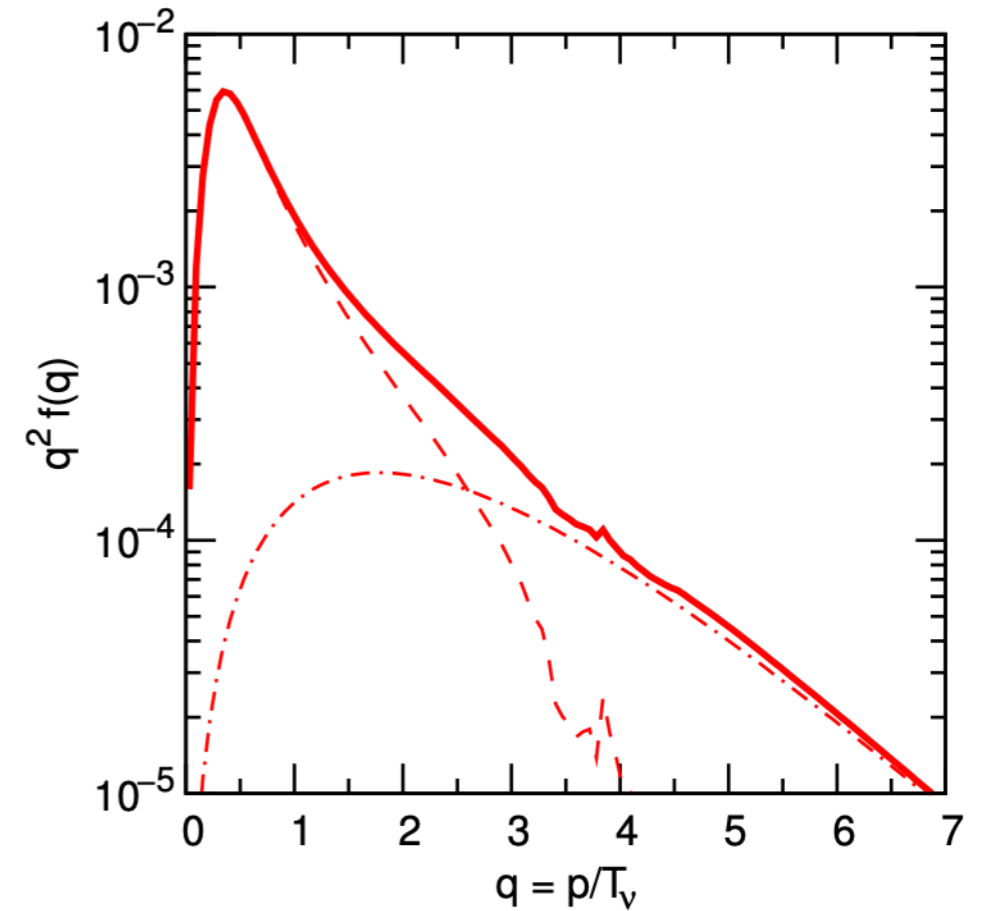
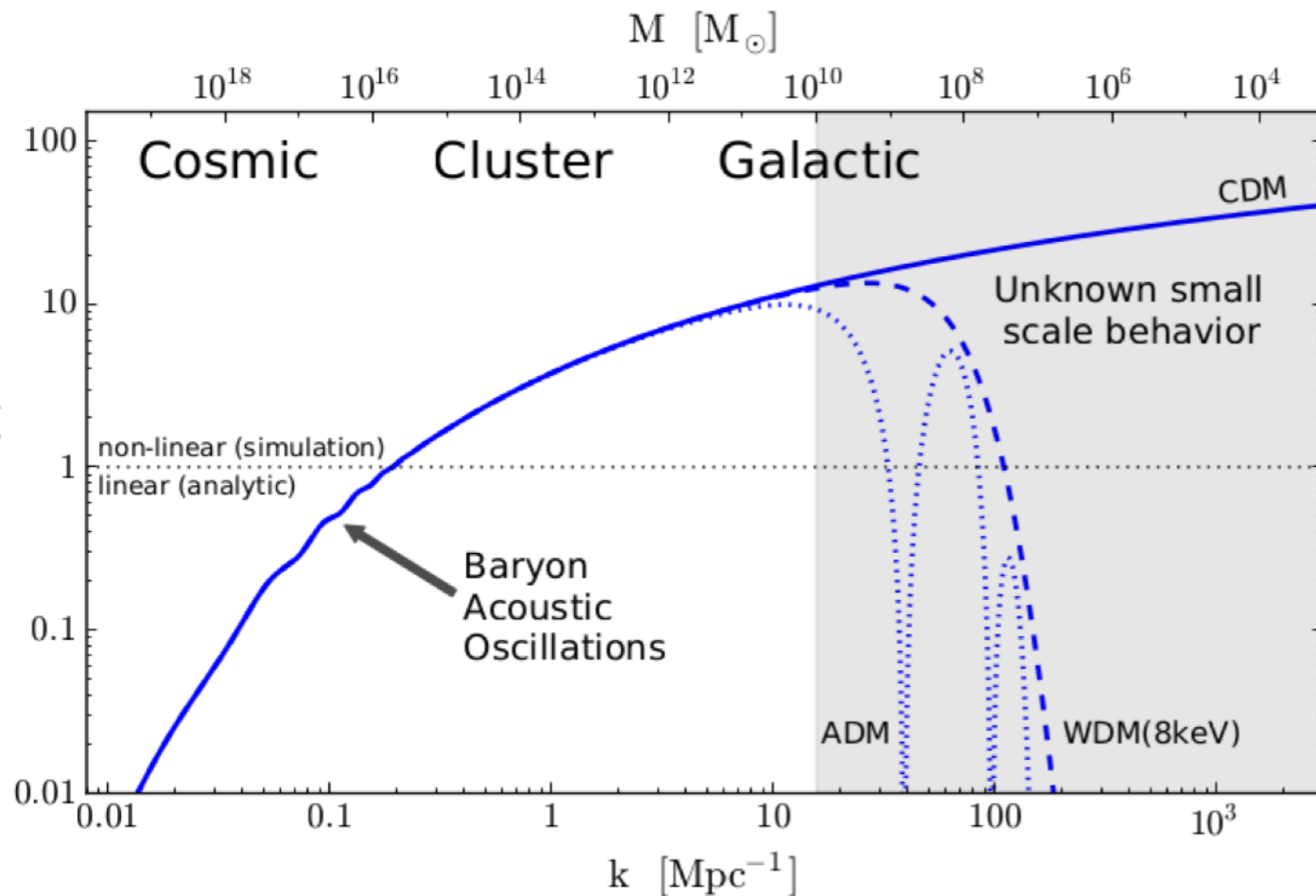


# Structure Formation

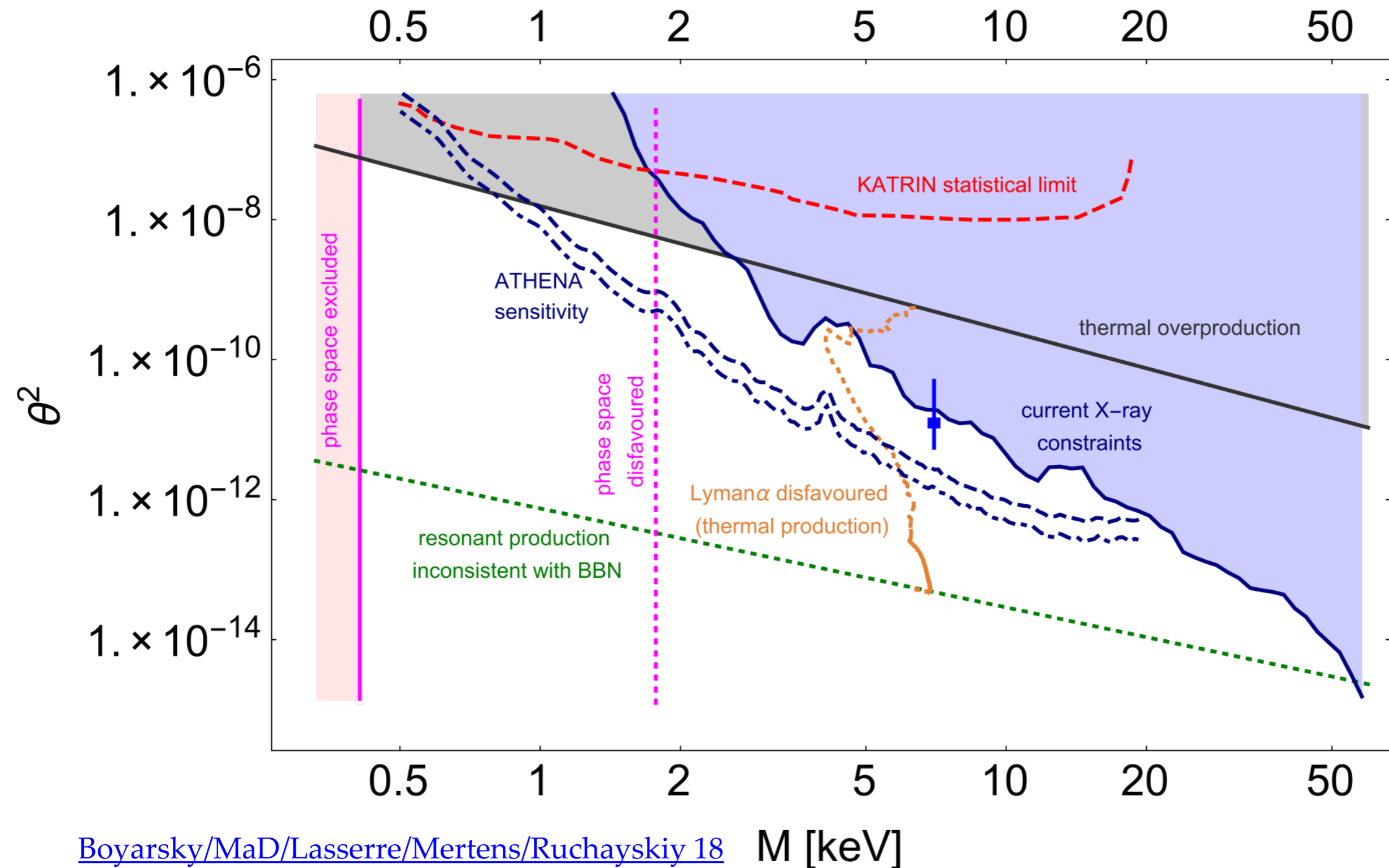
DM free streaming length

$$\lambda_{\text{fs}}(t) \equiv a(t) \int_{t_i}^t dt' \frac{v(t')}{a(t')} \approx 1 \text{ Mpc} \frac{\text{keV}}{M} \frac{\langle p_{\text{DM}} \rangle}{\langle p_{\nu} \rangle}$$

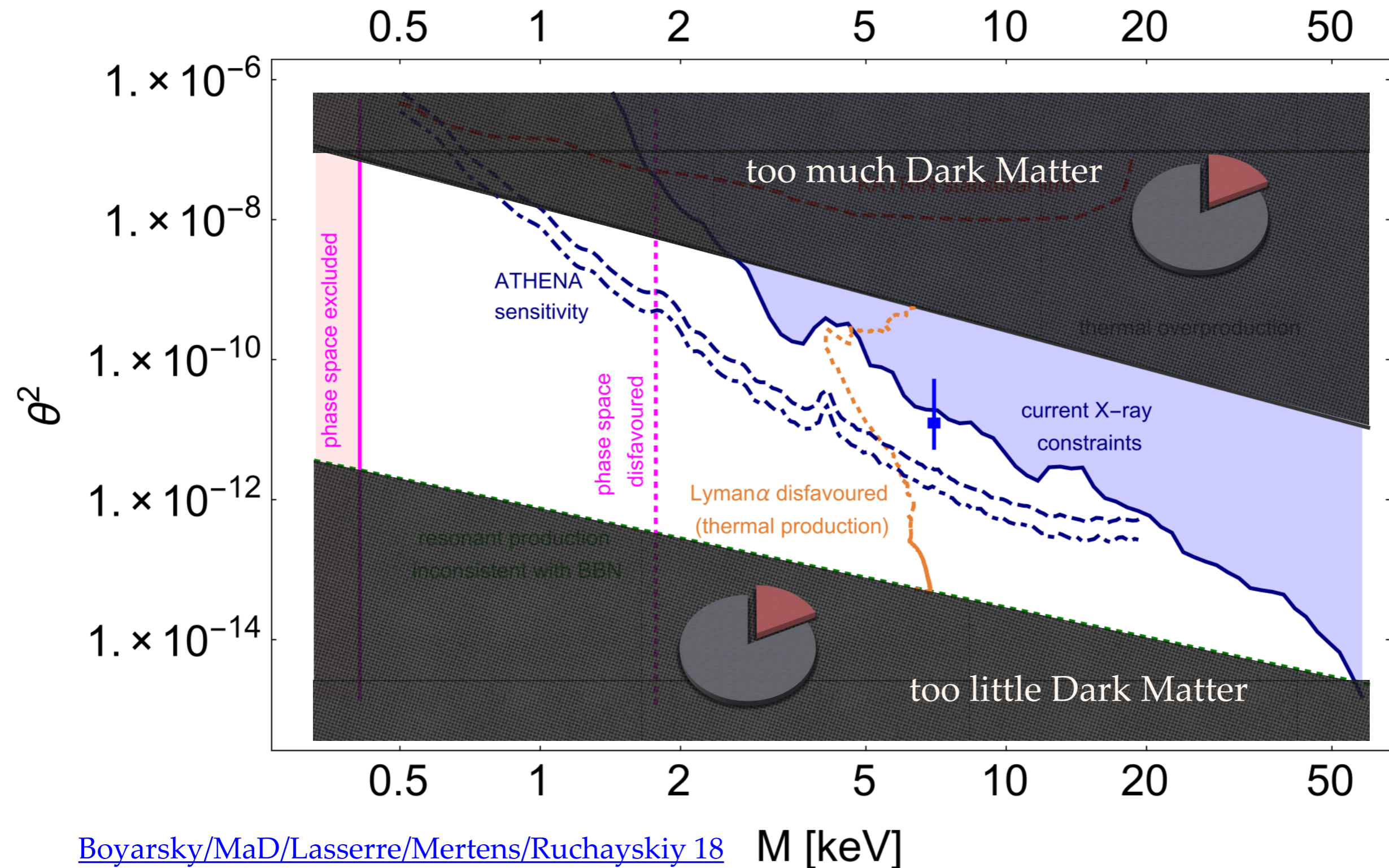
affects matter power spectrum



# Sterile Neutrino Dark Matter

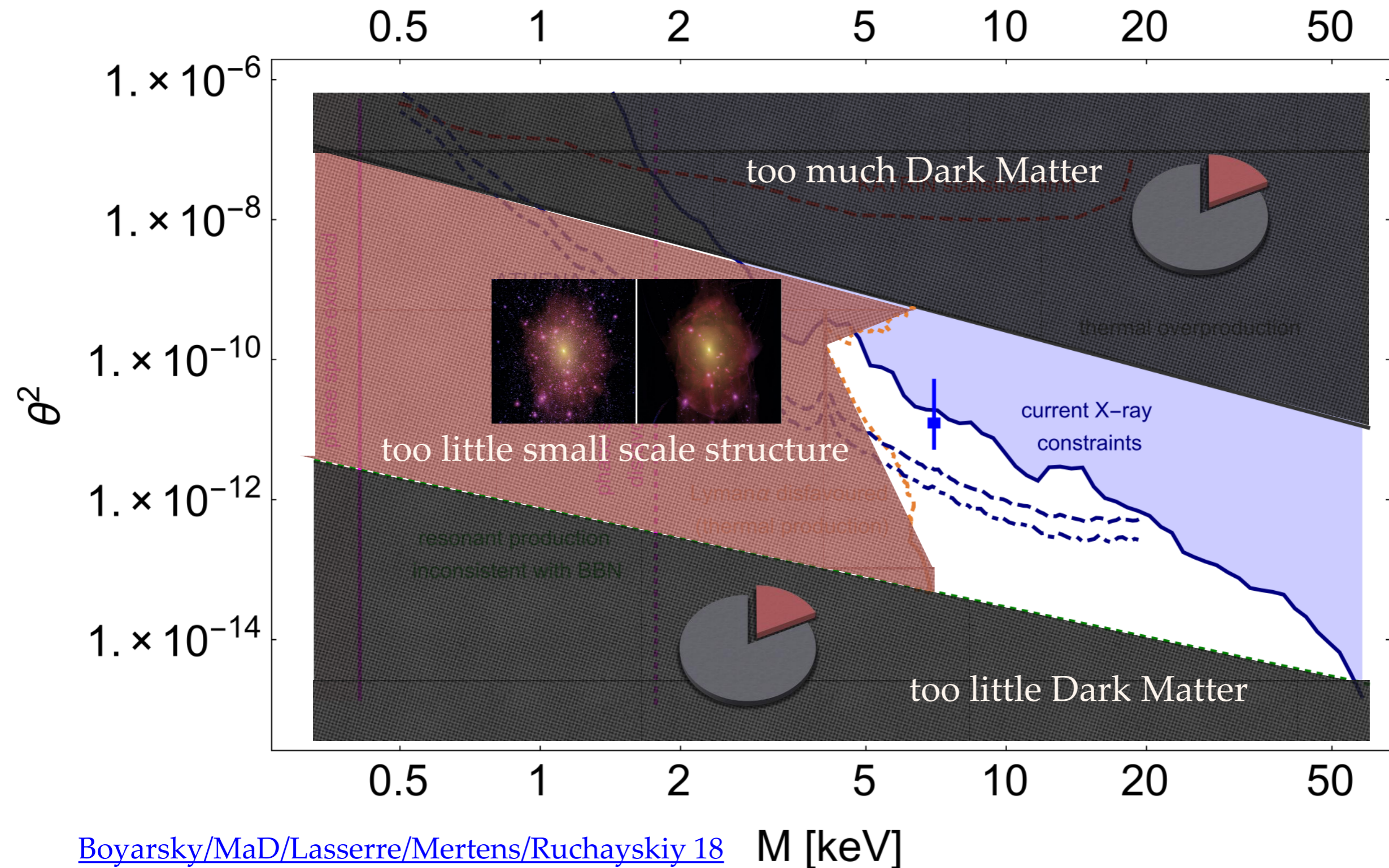


# Sterile Neutrino Dark Matter





# Sterile Neutrino Dark Matter





# Sterile Neutrino Dark Matter

