Radiation-Matter Interaction

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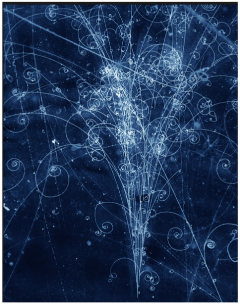


Outline

Section 1

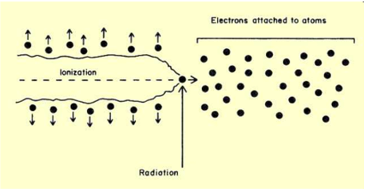
Basic Concepts in Radiation Physics

1. Basic Concepts in Radiation Physics
   Mechanisms of interaction

Mechanisms of interaction

- To understand and well use any kind of detector require to understand the fundamental mechanisms of different radiation interaction with "active" materials.

- Two main mechanism of energy loss inside matter:
  - Excitation: The atom (or molecule) is excited to a higher level
    $\text{atom}^* \rightarrow \text{atom} + \gamma$
    low energy photons of de-excitation → light detection
  - Ionization: The electron is ejected from the atom
    electron/ion pair → charge detection

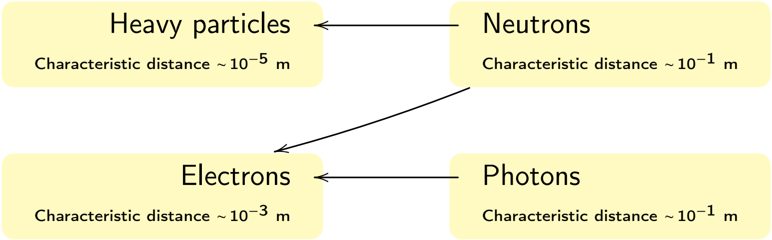


Mechanisms of interaction

- Main interactions we are going to deal with are:
  - Ionization (electromagnetic interaction)
  - Atomic excitation
  - Strong interaction → nuclear reactions
  - Excitation of lattice vibrations
  - Breakup of Cooper pairs in superconductors
  - Formation of superheated droplets in superfluids
  - Excitation of optical states
  - Radiative processes
  - Cerenkov radiation
  - Transition radiation
- All these interactions can be computed at microscopic level

NO INTERACTION = NO DETECTION

Introduction



- Charged particles.
- Coulombian interaction with atomic electrons
- Continuous energy transfer
- In case of $e^{\pm}$ radiative processes should be taken into account

- Neutral particles. Do not subject to electromagnetic interaction
- Need a "catastrophic" interaction
  - May change the nature of radiation
  - Creation of secondary charged particles

Link with macroscopic world

see Module:02m-Cross Sections

- Density of centers in a material $N = \dfrac{N_T}{V} = \dfrac{\rho}{A} N_A$
- Electron number density: $N_e = ZN = \dfrac{Z\rho}{A} N_A$

- The total number of scattering centers in a thickness δx: $N_{(e)} S \delta x$
- Scattering centers per unit perpendicular area: $N_{(e)} \delta x$

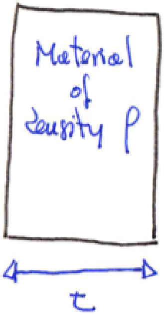
- Number of collisions per unit length: $w = N_{(e)} \sigma$
- Mean free path: $\lambda = \dfrac{1}{w} = \dfrac{1}{N_{(e)}\sigma} = \dfrac{A}{Z\rho N_A \sigma}$
- Macroscopic cross section: $\Sigma_t = \dfrac{1}{\lambda} = N_{(e)} \Sigma(\gamma\beta)$

Surface Density Units

- The thickness of materials is a unit often used in radiation physics.
- Usually is given as a surface density, also know as mass thickness

- We define the mass thickness of a material of density ρ and thickness t as:

$$\text{Mass thickness} = \underbrace{\rho}_{\text{gr/cm}^3} \cdot \underbrace{t}_{\text{cm}}$$



- Most used units are gr/cm^2
- It represents the mass behind unit area

- Why mass thickness is convenient:
  1. Closely related to the density of interaction centres
  2. Normalization of materials with different mass densities
  3. Equal mass thickness → Same effect on the same radiation

Section 2

Energy Loss by Heavy Charged Particles

Energy Loss by Heavy Charged Particles

- Two features characterize the passage of charged particles through matter:
  - Loss of energy by the particle
  - Deflection of the particle from its incident direction
- These effects are the result of various processes
  1. Inelastic collisions with atomic electrons
  2. Elastic scattering from nuclei
     - In general very little energy transfer ($m_p << M_{atom}$)
     - Some exceptions: heavy nuclei in low Z materials
  3. Emission of Cerenkov or Transition radiation (can be important)
  4. Bremsstrahlung. Only important for electrons or VERY high energies
  5. Nuclear reactions.
- These reactions occur many times per unit path length
- Little amount of energy loss/deflection per collision ($\sigma_{EM} \sim 10^{-16} - 10^{-17}\,cm^2$)
- It's their cumulative effect which accounts for effects

Energy Loss by Heavy Charged Particles

- Energy loss has to be treated in an statistical way
  - Nature of the scattering described by QM → probabilistic interpretation
  - Number of collisions is different from particle to particle

- Main energy loss process: collisions with atomic electrons.

see Module:02m-Two Body Scattering

- We can divide them in two groups:
  - SOFT collisions
    Energy transfer small enough to just produce an EXCITATION of the atomic states.
    Consequences emission of characteristic X-RAYS
    emission of Auger e^-
  - HARD collisions
    Energy transfer large enough to produce IONIZATION
    In some cases the electron from ionization has enough energy to produce ionization itself → δ-rays or knock-on electrons.

Stopping Power

- In the study of the atomic inelastic collisions, a probability approach is chosen
- Although probabilistic in nature, there is a large number of interactions
- We can meaningfully work with the average energy loss per unit path length
- This quantity is called STOPPING POWER (S)

$$\frac{dE}{dx} = S_{elec} + S_{nucl}$$

- The mean rate of energy loss is well described by the Bethe-Bloch equation:

$$-\frac{dE}{dx} = K\rho\frac{Z}{A}\frac{z^2}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C(\beta\gamma, I)}{Z}\right] = K\rho\frac{Z}{A}\frac{z^2}{\beta^2}L$$

- Main dependency on z^2 and $1/\beta^2$.
- L=Stopping Number : dimensionless quantity

Bethe-Bloch Formula

$$-\left\langle \frac{dE}{dx} \right\rangle = K\rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C}{Z}\right]$$

K $= 4\pi N_A r_e^2 m_e c^2 = 0.307075\,\mathrm{MeV}\frac{\mathrm{cm}^2}{\mathrm{g}}$

Z : Z of the medium

A : Atomic mass number of the medium

z : charge of the incident particle

I : Mean excitation potential

T_{max} : Maximum energy transfer produced by a head-on collision

$$T_{max} = \frac{2m_e c^2\beta^2\gamma^2}{1 + 2\gamma\frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2} \xrightarrow{2\gamma m_e/M<<1} T_{max} \simeq 2m_e c^2\beta^2\gamma^2$$

δ : Density correction. Important at high energy.

C : Shell correction. Relevant for small β values.

Mass Stopping Power

We can express the stopping power as a function of the mass thickness

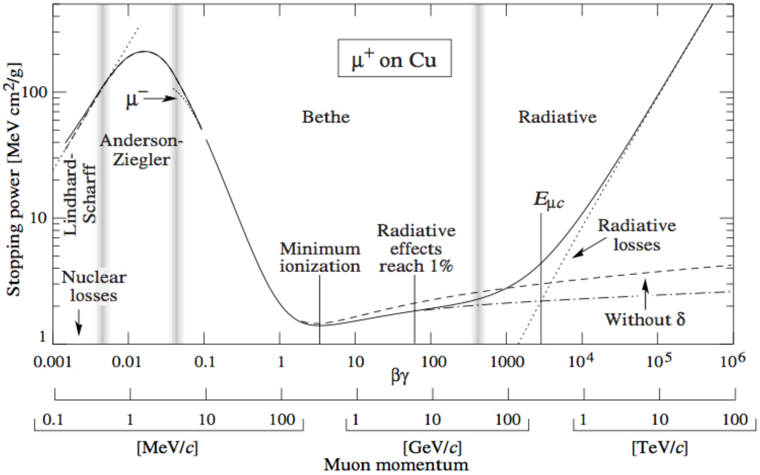
$$-\left\langle \frac{dE}{dx} \right\rangle = z^2 \frac{Z}{A} \rho f(\beta, I)$$

$$\Downarrow$$

$$-\left\langle \frac{1}{\rho} \frac{dE}{dx} \right\rangle = z^2 \frac{Z}{A} f(\beta, I)$$

- For most of the elements Z/A is quite constant
- For close $Z \rightarrow I$ is almost the same
- The mass stopping power is almost independent of the material

Bethe-Bloch Formula



https://physics.nist.gov/PhysRefData/Star/Text/PSTAR.html

Bethe-Bloch Formula: Energy dependence

Bethe region: $0.1 < \gamma\beta < 1000$

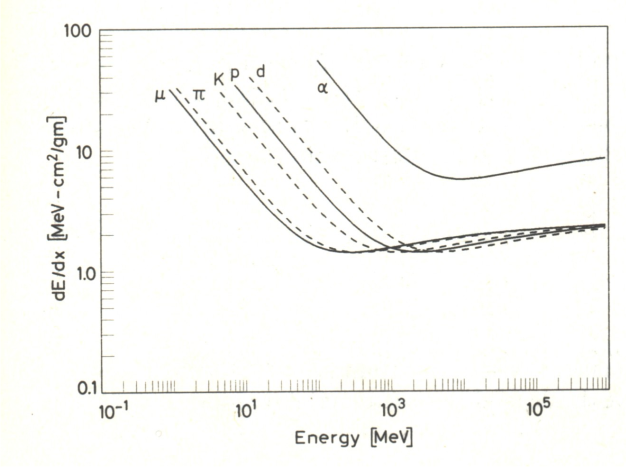
- Accuracy of few %. Uncertainty coming from higher order effects (kinematics, QED, spin, internal structure, electron capture, etc...)
- For non-relativistic particles → main dependence $\propto \frac{1}{\beta^2}$
- There is a minimum in $\frac{dE}{dx}$ around $\gamma\beta = 3.5$ for Z=7
  $\gamma\beta = 3.0$ for Z=100

  Particles with this minimum ionizing rates are called mip (minimum ionizing particle)

  In practical cases, most relativistic particles have a mean energy loss rate close to this minimum ionizing rate
- As energy increases $1/\beta^2 \simeq 1$
  $\frac{dE}{dx}$ increases due to the logarithmic dependence → relativistic rise
- From $\gamma\beta \sim 10 - 100$, effects from δ reach 1%

dE/dx particle id

- For particles in the Bethe region each particle species shows a characteristic dE/dx that can be used for particle identification purposes.



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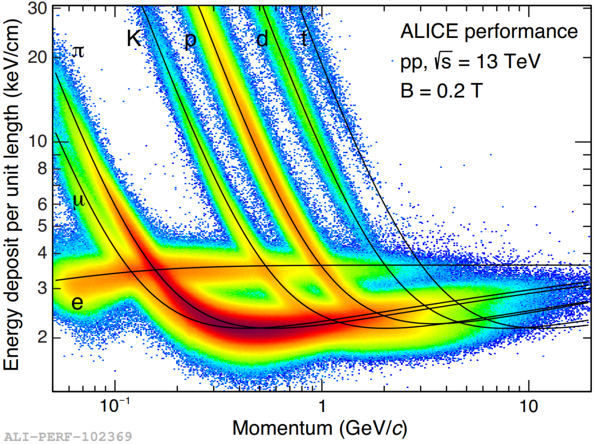


Bethe-Bloch Formula: Energy dependence

$\gamma\beta < 0.1$

- Bethe-Bloch formula is not longer valid
  - Velocities of incident particles comparable to orbital electrons
  - Electrons cannot be considered as stationary → BB estimation breaks down
- A maximum is reached and drops sharply again
  - Most important effect: particle pick up electrons decreasing the effectiveness of stopping power.
- At $\gamma\beta \simeq 0.005$ nuclear losses are not negligible

$\gamma\beta > 1000$

- Radiative corrections no longer can be ignored
- They are not taken into account in BB formula.

https://physics.nist.gov/PhysRefData/Star/Text/PSTAR.html

Mean Excitation Potential

- I is the main parameter of Bethe-Bloch formula
- Non-trivial determination
- In practice I values are determined for most of the materials from dE/dx curves.
- Tabulated values can be found in:

  http://physics.nist.gov/PhysRefData/XrayMassCoef/tab1.html

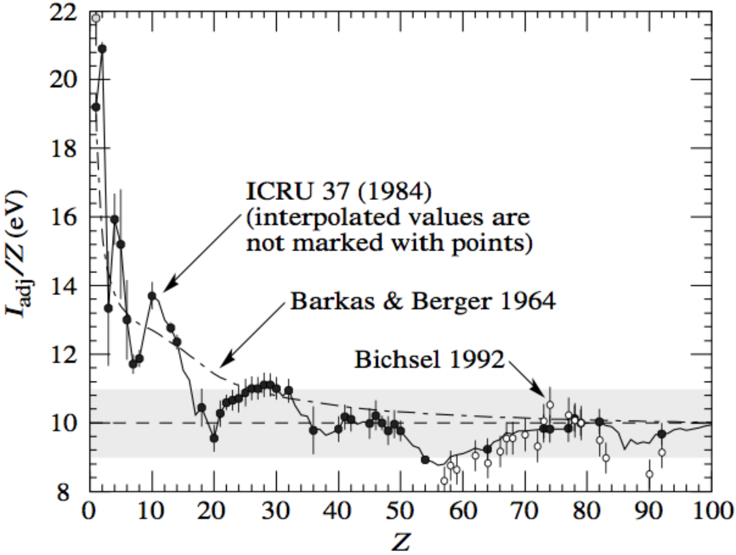
- Semiempirical formula

$$\frac{I}{Z} = 12 + \frac{7}{Z}\,(\text{eV}) \qquad Z < 13$$

$$\frac{I}{Z} = 9.76 + 58.8\,Z^{-1.19}\,(\text{eV}) \qquad Z \geq 13$$

$$I \approx 17.7 Z^{0.85}\,(\text{eV})$$

Mean Excitation Potential



Scaling Laws

- For particles in the same medium, Bethe-Bloch can be seen as

$$-\frac{dE}{dx} = z^2 f(\beta)$$

  Energy loss in a given material depends only with the charge and velocity of the incident particle

- If we take into consideration the kinetic energy

$$T = (\gamma - 1)Mc^2 \rightarrow \beta \propto \frac{T}{M} \rightarrow \beta = g\left(\frac{T}{M}\right)$$

$$-\frac{dE}{dx} = z^2 f'\left(\frac{T}{M}\right)$$

  we see that the dependence is in the ratio $\frac{T}{M}$

Scaling Laws

- If we know the energy loss $(-\frac{dE_1}{dx})$ of a particle of mass M_1 and charge Z_1 then the energy loss of a particle of mass M_2 and charge Z_2 in the same material can be found:
  - Scaling the energy of particle 2 to:

$$\frac{T_1}{M_1} = \frac{T_2}{M_2} \rightarrow T_1 = T_2 \frac{M_1}{M_2}$$

  - Multiplying by the charge ratio Z_2^2/Z_1^2

$$-\frac{dE_2}{dx}(T_2) = -\frac{Z_2^2}{Z_1^2}\frac{dE_1}{dx}\left(T_2 \frac{M_1}{M_2}\right)$$

Example:

$$\begin{aligned} M_\alpha &\simeq 4M_p \\ Z_\alpha &= 2Z_p \end{aligned} \quad \Rightarrow \quad -\frac{dE_\alpha}{dx}(T_\alpha) = -4\frac{dE_p}{dx}\left(T_\alpha \frac{1}{4}\right)$$

$$-\frac{dE_\alpha}{dx}(1\,GeV) = -4\frac{dE_p}{dx}(250\,MeV) \xrightarrow[\text{and astar}]{\text{From pstar}} \begin{array}{l} \frac{dE_\alpha}{dx}(1\,GeV) = 12.61\,MeV\,grcm^{-2} \\ \frac{dE_p}{dx}(250\,MeV) = 3.165\,MeV\,grcm^{-2} \end{array}$$

$\frac{dE}{dx}$ for mixtures and compounds

- Energy loss of a mixture or a compound can be calculated from the energy loss of pure elements

$$\frac{1}{\rho}\frac{dE}{dx} = \frac{w_1}{\rho_1}\left(\frac{dE}{dx}\right)_1 + \frac{w_2}{\rho_2}\left(\frac{dE}{dx}\right)_2 + \cdot$$

where ω_i = fraction by weight of elements

$$\omega_i = \frac{a_i A_i}{A_m}$$

A_i = atomic weight of ith element

a_i = Number of atoms of element ith

$A_m = \sum a_i A_i$

Example: H_2O $a_H = 2$ $Z_H = 1$ $A_H = 1$
$a_O = 1$ $Z_O = 8$ $A_O = 16$

- We can define effective values that can be used directly in the Bethe-Bloch formula

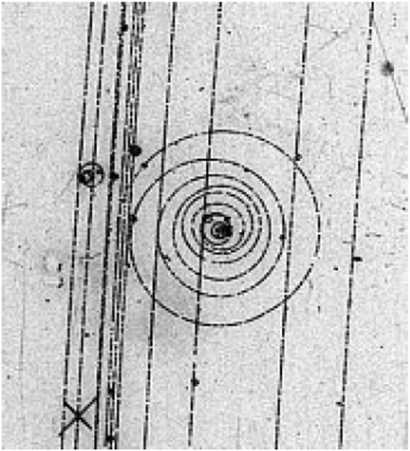
$$Z_{eff} = \sum a_i Z_i$$
$$A_{eff} = \sum a_i A_i$$
$$\ln I_{eff} = \sum \frac{a_i Z_i \ln I_i}{Z_e ff}$$

$$\delta_{eff} = \sum \frac{a_i Z_i \delta_i}{Z_{eff}}$$
$$C_{eff} = \sum a_i C_i$$

δ electrons

see Module:02m-DeltaElectrons

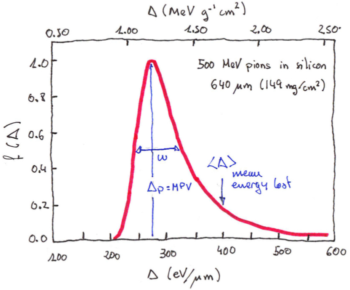
- So called δ electrons, or high energy knock-on electrons or δ-rays are kicked-off electrons with relatively high-energy.
  - Creation of ionization clusters along the δ electron's path
  - Main reason of non-locality energy-loss
  - Deteriorate spatial resolution of the original particle.
  - Kinematical relation between emission angle and energy



Energy straggling

- Bethe-Bloch gives us the mean energy loss ($= \langle\Delta\rangle$).
- Energy loss is a probabilistic phenomena, characterized by a pdf
- The pdf's of energy loss in a media are called straggling functions or Landau functions

- $\Delta E = \sum_{n=1}^{N} \delta E_n$
- Is the mean, $\langle\Delta\rangle$ the best estimator?
  - Δ_p, seems a best estimator
- What about the spread?
  - Chosen the FWHM=w



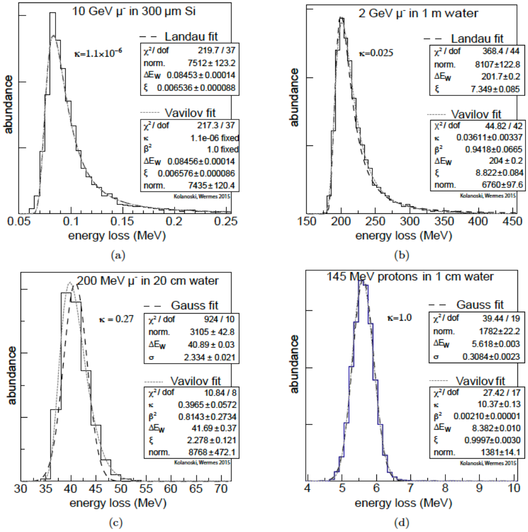
- Assymetric distribution:
  - Gaussian core: Many ionisation processes with small δE
  - Tail: Few processes with large energy losses

Energy straggling

- $\langle \Delta \rangle$ is ill-defined because of large fluctuations
- It cannot be used to describe energy loss if very few collisions occur
- Main effect of energy straggling: an incident beam of monoenergetic particles gets a finite width.
- Two main randomness contributions
  - Number of collisions
  - Energy loss spectra for multiple collisions

- First analytic derivation by Landau (1944) generalized in 1954 by Vavilov.
  - Landau-Vavilov distribution
  - Two parameters: Δ_p and w

Landau-Vavilov distribution



Energy straggling: Estimation of Δ_p and w

Thick absorbers

- The total number of collisions is large enough to apply a gaussian approximation
- In this case, the mean energy loss can be used safely.

$$f(\Delta, x) \propto \exp\left(-\frac{\delta - \langle\Delta\rangle)^2}{\sigma_0}\right)$$

- The spread is estimated as:

non-relativistic $\quad \sigma_0^2 = 0.1569\rho\frac{Z}{A}x\,[MeV]$

relativistic $\quad \sigma_0^2 = \frac{(1-\frac{1}{2}\beta^2)}{1-\beta^2}$

- It exists always a skew to large energy transfers.

Energy straggling: Estimation of Δ_p and w

Thin absorbers

- Based on theories from Landau-Vavilov. Later refined by Bischel
- For moderate thickness Δ_p can be estimated as:

$$\Delta_p = \xi \left[\ln \frac{2mc^2\beta^2\gamma^2}{I} + \ln \frac{\xi}{I} + 0.2 - \beta^2 - \delta(\gamma\beta) \right]$$

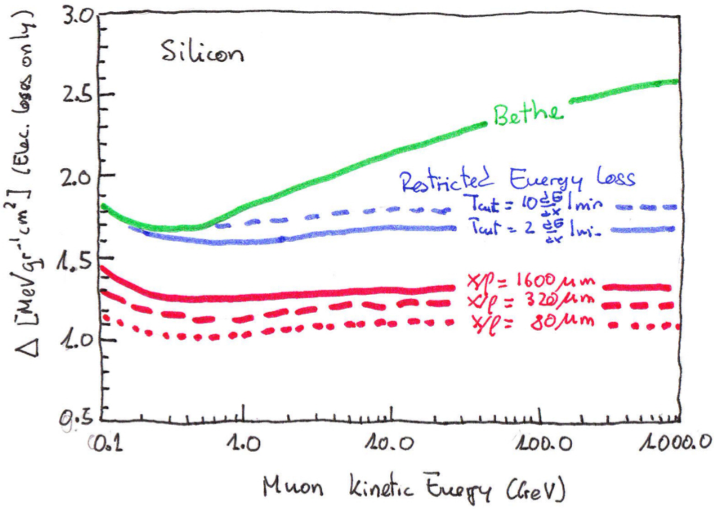
$$\xi(MeV) = \frac{K}{2} \left\langle \frac{Z}{A} \right\rangle \frac{x}{\beta^2} \qquad x \text{ in } g\, cm^{-2}$$

  - Δ_p scales with $\ln x$ while $\langle \Delta \rangle$ is independent of thickness
  - Δ_p reaches a plater in the relativistic region

$$\Delta_p \xrightarrow[\gamma\beta \gtrsim 100]{\xi} \left[\ln \frac{2mc^2\xi}{(\hbar\omega_p)^2} + 0.2 \right]$$

- The width can be estimated by $w = 4\xi$.
  - For very thin segments distribution the width is significantly wider

Energy straggling: Estimation of Δ_p and w



Restricted Energy loss

- We can define the mean energy deposit by an ionizing particle when energy transfers are restricted to $T \leq T_{cut} \leq T_{max}$ as:

$$-\frac{dE}{dx}\bigg|_{T<T_{cut}} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{cut}}{I^2} - \frac{\beta^2}{2}\left(1+\frac{T_{cut}}{T_{max}}\right) - \frac{\delta}{2}\right]$$

- When $T_{cut} \rightarrow T_{max}$ we recover Bethe-Bloch
- T_{cut} replaces T_{max} on the logarithm:
  - Not any more a relativistic rising
  - Plateau at high energies
- For low energy no difference
- In case of thin absorbers where not all energy lost can be absorbed, the restricted energy loss is a better estimator for energy transfer.
- Same behavior as Δ_p estimators

Ionization yield

- Sometimes it's useful to relate the energy loss to the number of ion pairs produced near particle's track.
- This relation is not straight forward due to the energetic δ-rays and the non-locality energy deposition
- The ionization yield or specific ionization is defined them as the number of ion pairs that a particle produces per unit distance

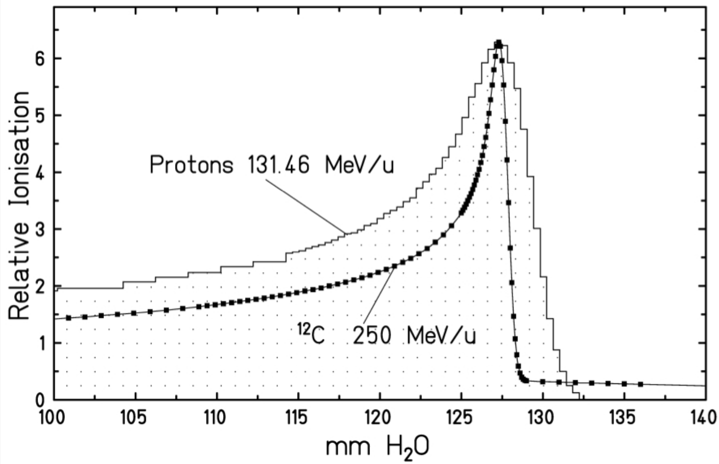
$$\frac{dN}{dx} = \frac{-\frac{dE}{dx}}{\overline{\omega}}$$

- $\frac{dE}{dx}$: is the stopping power
- $\overline{\omega}$ is the mean ionization energy.
  $\overline{\omega} \simeq 30$ eV
  But depends on the material and the β of the particle

Bragg curves

- Heavy charged particles starts to slow down since they enter in contact with matter
- Energy loss is progressive.
  - Its rate depends on β and T_{max}
  - At high energy, energy loss is relatively moderated and constant
  - Only in the end of it's path, the energy loss rate goes up
- The peak at the end of particles path is called Bragg peak
- Particles stop when $\gamma\beta << 0.01$ and starts to pick up electrons
- Energy straggling appears clearly in the tail.
- Important feature for medical applications

Bragg curves



Range of charged heavy particles

- From stoping powers we can compute how far particles can penetrate before they lose all their energy.

$$
\begin{aligned}
R(T_0) &= \int_0^{T_0} \left[-\frac{dE}{dx} \right]^{-1} dE \\
&= R_0(T_{min}) + \int_{T_{min}}^{T} \left[-\frac{dE}{dx} \right]^{-1} dE
\end{aligned}
$$

where T_{min} = minimal energy at which $\frac{dE}{dx}$ is valid
$R_0(T_{min})$ = constant which accounts for low energy part

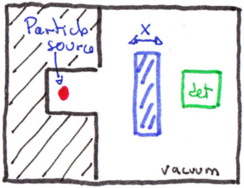
- From the dependency of Bethe-Bloch formula we get:

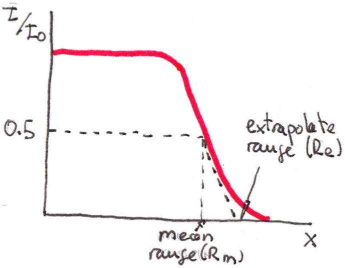
$$
-\frac{dE}{dx} \propto \beta^{-2} \propto T^{-1} \qquad \rightarrow \qquad R = aE^2
$$

- The range computed in this way is called CSDA range (Continuous Slow-Down Approximation)

Range of charged heavy particles

- Experimentally we can measure the range through transmission measurements:

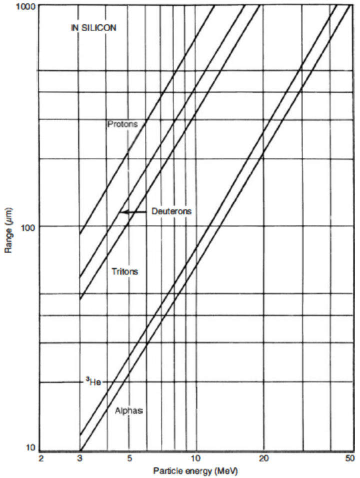




- The effect of energy straggling is seen here in a non clear definition of the range
- We can define two experimental definitions of range:
  - Mean range: Range in which $\frac{I}{I_0} = 0.5$
  - Extrapolated range: linear extrapolation at this point.

Range: experimental measurements

- The relation $R = aE^b$ is confirmed
- Theoretically expected $b = 2$, but measured $b = 1.73$
- Same b for different particles.



Range-Bragg curve relationship

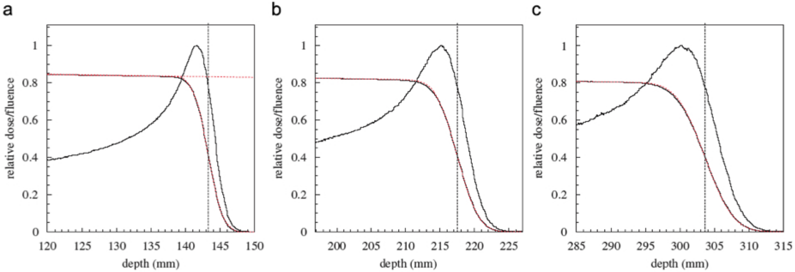


Fig. 6. Bragg and fluence curves obtained by the GEANT4 Monte Carlo simulations with the *I*-value of 75 eV for (a) 150 MeV, (b) 190 MeV, and (c) 230 MeV. The error function with a correction for the inverse square effect fitted to the fluence curves is drawn with dashed lines. Dash-dotted lines denote the ranges obtained by the fit.

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Range scaling laws

- Different particles in the same medium

$$R_2(T_2) = \frac{M_2}{M_1}\frac{Z_1^2}{Z_2^2}R_1\left(T_2\frac{M_1}{M_2}\right)$$

- Same particle in diffetent media. Bragg-Kleeman rule

$$\frac{R_1}{R_2} = \frac{\rho_2}{\rho_1}\frac{\sqrt{A_1}}{\sqrt{A_2}}$$

- In case of a mixture or a compound

$$R_{comp} = \frac{A_{comp}}{\sum \frac{a_i A_i}{R_i}}$$

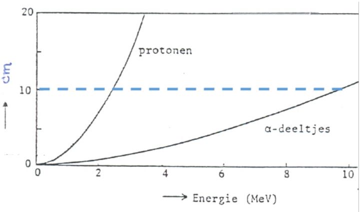
Multiple Scattering

- Charged particle traversing material will suffer multiple interactions:
  - Scattering with electrons (energy loss)
  - Scattering with nuclei (mainly deflections) → multiple scattering
- Multiple scattering: large number of scattering processes with very low deviations
- Distribution of scattering angles described by Moliere's theory
  - Small scattering angles: normally distributed around $\Theta = 0$
  - Large scattering angles: more frequent than expected from Gaussian distribution.

$$\langle\Theta^2\rangle = \Theta_\theta^2 + \Theta_\phi^2$$

$$\Theta_{rms}^{proj} = \sqrt{\langle\Theta^2\rangle} = \frac{13.6\,MeV}{\beta cp[MeV/c]} z \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln \frac{x}{X_0}\right]$$

Protection against heavy charged particles



Dracht in lucht van alfadeeltjes en protonen

Reichweite von α-Strahlen

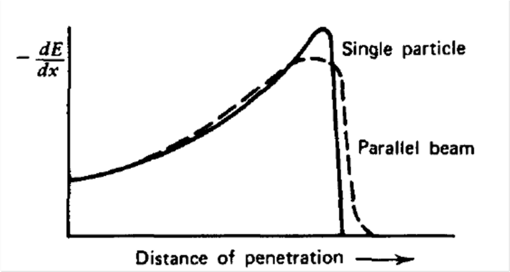
Energie MeV	Reichweite in biologischem Gewebe µm	Reichweite in Aluminium µm	mg cm^{-2}
4,0	31	16	4,6
4,5	37	20	5,4
5,0	43	23	6,2
5,5	49	26	7,0
6,0	56	30	8,1
6,5	64	34	9,2
7,0	72	38	10,2
7,5	81	43	11,6
8,0	91	48	13,0
8,5	100	53	14,3
9,0	110	58	15,6
9,5	120	64	17,3
10,0	130	69	18,6

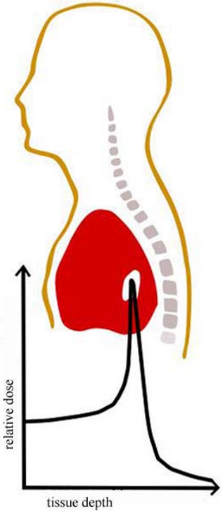
- keep distance to source (10 cm is OK)
- sheet of paper or thin plate of aluminium stops the α-particles
- outer layer of our skin stops α particles and protects the inner layers

- α-source very dangerous when ingested !! (not traceable from outside; large damage in small region of body)

Proton-theraphy: Putting all together

- Energy loss is a stochastic process:
  - different for every particle
  - Energy straggling and multiple scattering
- The energy of different identical particles in a beam shows a range of values after traversing a certain distance x in matter





Section 3

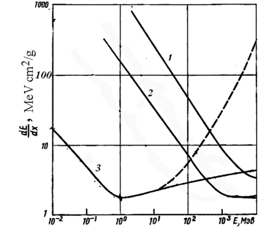
Energy loss by electrons and positrons

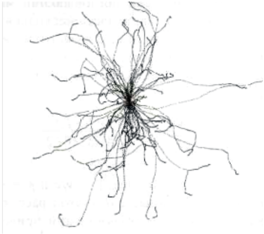
Introduction

- Energy loss by electrons and positrons have two major features and some particularities wrt energy loss of heavy charged particles
  - Radiative effects (bremsstrahlung) cannot be ignored. This is the main cause of energy loss at high energies

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{coll} + \left(\frac{dE}{dx}\right)_{rad}$$

  - Multiple scattering is very important: non-rectilinear path





- At low energy ionization is 100-1000 less than α particles

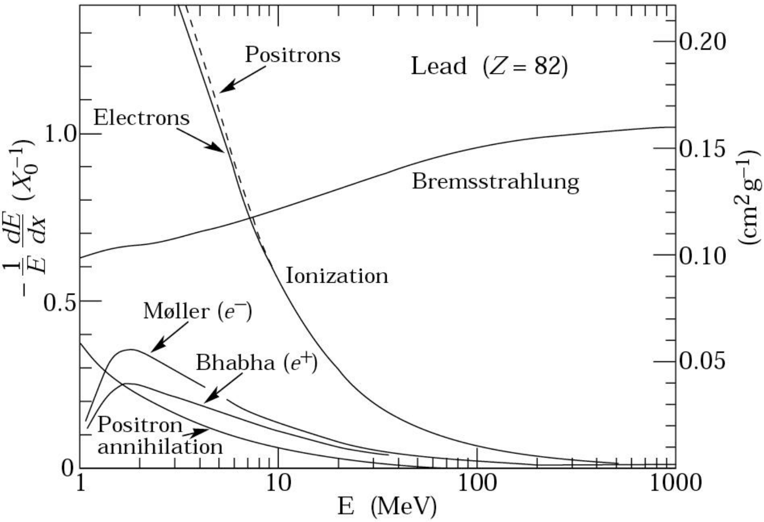
Electron-Positron Energy Loss: Particularities

- In case of electron-electron collision we deal with indistinguishable particles
  - Convention: scattered electron is the one with less energy

- Incident electrons/positrons have the same mass as orbital electron. In this case the maximum energy transfer in a collision is

$$T_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2} \xrightarrow{m_e = M} T_{max} = \begin{cases} \text{electrons} = \dfrac{T_{e^-}}{2} \\ \text{positrons} = T_{e^+} \end{cases}$$

Electron-Positron Energy Loss



https://physics.nist.gov/PhysRefData/Star/Text/ESTAR.html

Collision Energy Loss

- Main energy loss mechanism at low energy. Four contributions.
  - Moller Scattering: elastic scattering $e^- + e^- \to e^- + e^-$
  - Bhabha Scattering: elastic scattering $e^- + e^+ \to e^- + e^+$
  - Electron-Positron annihilation: $e^+ + e^- \to \gamma\gamma$
  - Inelastic scattering.
- Moller, Bhabha and annihilation are relatively important at low energy.
- In any case inelastic is always dominating.

$$-\frac{1}{\rho}\frac{dE}{dx} = K\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} + \frac{1}{2}F(\tau) - \frac{\delta}{2}\right]$$

$$F(\tau) = \begin{cases} 1-\beta^2 + \dfrac{\frac{\tau^2}{8}-(2\tau+1)\ln 2}{(\tau+1)^2} & \text{for } e^- \\ 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right) & \text{for } e^+ \end{cases} \qquad \tau = \frac{T_e}{m_e c^2}$$

Bremsstrahlung

- Electrons and positrons are the only particles in which bremsstrahlung contributes in a significant way at energies below few hundred GeV

$$\sigma_b \propto \frac{1}{m^2} \quad \rightarrow \quad \begin{array}{l} \dfrac{\sigma_b^e}{\sigma_b^\mu} = \left(\dfrac{m_\mu}{m_e}\right)^2 \simeq 40000 \\ \dfrac{\sigma_b^e}{\sigma_b^p} = \left(\dfrac{m_p}{m_e}\right)^2 \simeq 4 \times 10^6 \end{array}$$

- Bremsstrahlung depends on the strength of the electric field felt by the electrons
- Sensitive to the amount of screening (ξ)

$$\xi = \frac{100 m_e c^2 h\nu}{E_0\, E\, Z^{1/3}} \qquad \begin{array}{lll} \xi >> 1 & m_0 c^2 < E_0 < 137 m_0 c^2 Z^{1/3} & \text{no screening} \\ \xi = 0 & E_0 >> 137 m_0 c^2 Z^{1/3} & \text{total screening} \end{array}$$

where E_0: Initial electron energy, E: Final electron energy, $h\nu$: Photon energy, Z: of the material

Bremsstrahlung

$$-\left(\frac{dE}{dx}\right)_{rad} = \frac{\rho\, N_A}{A} E_0 \phi_{rad}$$

no screening $$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln \frac{2E_0}{m_e c^2} - \frac{1}{3} - f(Z)\right)$$

total screening $$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln(183\, Z^{1/3}) + \frac{1}{18} - f(Z)\right)$$

$$f(Z) = a^2 \left[(1+a^2)^{-1} + 0.20206 - 0.369a^2 + 0.0083a^4 - 0.002a^6\right] \qquad a = \frac{Z}{137}$$

Electron-Electron Bremstrahlung

- The above formulas represent energy loss in nuclear e.m. field
- In an atom there are also electrons
- Cross sections for electron-electron bremsstrahlung are the same, but does not scales with Z^2 but only on Z
- Summing up both contributions we just need to substitute Z^2 by $Z(Z+1)$

$$-\left(\frac{dE}{dx}\right)_{rad} = \frac{\rho N_A}{A} E_0 \phi_{rad}$$

no screening $\qquad \phi_{rad} = 4Z(Z+1) r_e^2 \alpha \left(\ln \frac{2E_0}{m_e c^2} - \frac{1}{3} - f(Z) \right)$

total screening $\qquad \phi_{rad} = 4Z(Z+1) r_e^2 \alpha \left(\ln(183\, Z^{1/3}) + \frac{1}{18} - f(Z) \right)$

Collision vs Radiative Energy loss

- Main differences between collision and radiative energy losses are:
  - Ionization increases logarithmically with energy
  - Radiation losses increases linearly with energy
  - With ionization particles loose energy "continuously"
  - With radiation, particles can loose all energy with just one or two photons emitted

- There are empirical formulas that relate both energy losses

$$\frac{\left.\frac{dE}{dx}\right|_{rad}}{\left.\frac{dE}{dx}\right|_{col}} \simeq \frac{(Z+1.2)E[MeV]}{800}$$

http://physics.nist.gov/Star

Critical Energy

- Critical energy (E_c) is the energy at which radiation energy loss equals ionization energy loss

$$\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{col} \qquad \text{for } E = E_c$$

- Empirical formulas to estimate E_c are:

$$E_c = \frac{610\,MeV}{Z + 1.24} \qquad \text{solid and liquids}$$
$$E_c = \frac{710\,MeV}{Z + 0.92} \qquad \text{gases}$$

http://pdg.web.cern.ch/pdg/2020/AtomicNuclearProperties

Critical Energy

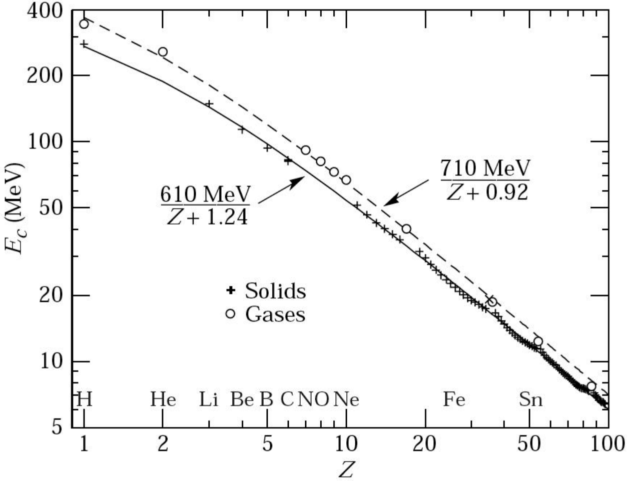


Figure 27.13: Electron critical energy for the chemical elements, using Rossi's definition [4]. The fits shown are for solids and liquids (solid line) and gases (dashed line). The rms deviation is 2.2% for the solids and 4.0% for the gases. (Computed with code supplied by A. Fassó.)

Radiation length

- We define the radiation length (X_0) as the mean distance over which an electron losses all but $\frac{1}{e}$ of its energy by bremsstrahlung.

$$-\frac{dE}{dx} = N\,E\,\phi_{rad}$$

$$\frac{dE}{E} = -N\phi_{rad}\,dx \quad \rightarrow \quad E = E_0 e^{-\frac{x}{X_0}}$$

$$X_0 = \frac{1}{N\phi_{rad}}$$

- X_0 has been calculated and tabulated

$$\frac{1}{X_0} = \underbrace{4\alpha r_e^2 N_A}_{716.4\,g\,cm^2} \frac{1}{A}\{Z^2[L_{rad} - f(Z)] + Z\,L'_{rad}\}$$

$$f(Z) = a^2[(1+a^2)^{-1} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6] \qquad a = \alpha Z$$

Radiation length

Element	Z	L_{rad}	L'_{rad}
H	1	5.31	6.144
He	2	4.79	5.621
Li	3	4.74	5.805
Be	4	4.71	5.924
Others	> 4	$\ln(184.15\, Z^{-1/3})$	$\ln(1194\, Z^{-2/3})$

- An empirical estimation that agrees with upper formula better than 2.5% is:

$$X_0 = 716.4[g\, cm^{-2}] \frac{A}{Z(Z+1)\ln(287/\sqrt{Z})}$$

- The radiation length in a mixture may be approximated by:

$$\frac{1}{X_0} = \sum \frac{w_j}{X_j}$$

where w_j is the fraction by weight for the jth element

Range of electrons

- The range of beta particles is defined in the same way as heavy particles
- Unlike heavy particles its range is a poor indicator
- Actual ranges can be 20-400% different from dE/dx integration
  - Because of multiple scattering betas does not follow rectilinear trajectories
  - "Catastrophic" energy loss due to bremsstrahlung.

Empirical formulas

R: in $g\,cm^{-2}$
T: in MeV

$$R = \begin{cases} 0.412\,T^{1.27-0.0954\ln T} & \text{for } 0.01 \le T \le 2.5\,M \\ 0.530\,T = 0.106 & \text{for } T > 2.5\,MeV \end{cases}$$

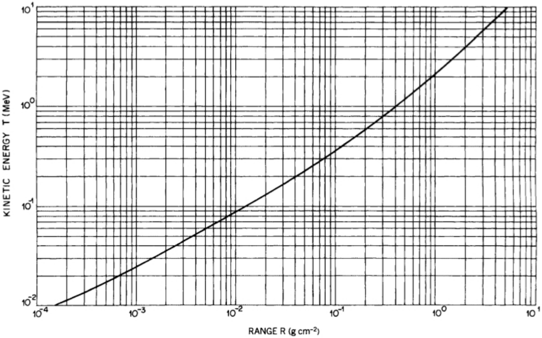
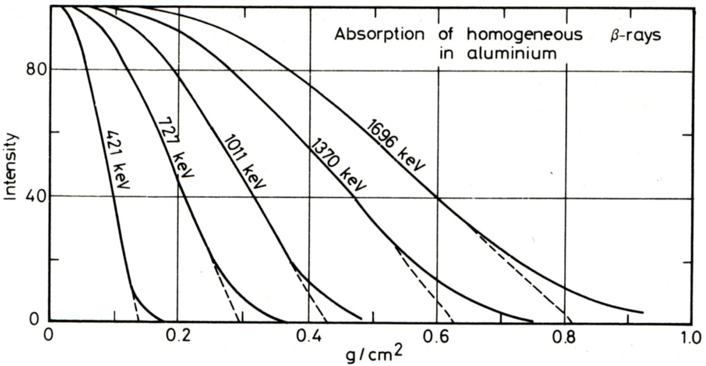


Fig. 6.4 Beta-particle range–energy curve for materials of low atomic number. [From U.S. Public Health Service, *Radiological Health Handbook*, Publ. No. 2016, Bureau of Radiological Health, Rockville, MD (1970).]

Range of electrons



Attenuation of betas

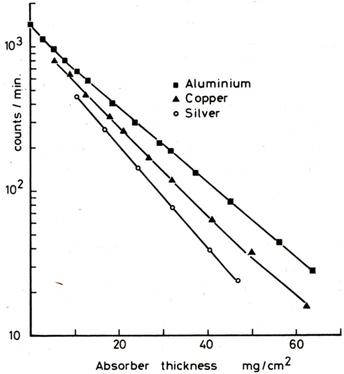
- Absorption of β particles follows an exponential behavior

$$I = I_0 e^{-\mu x} = I_0 e^{-\mu_\beta(\rho x)}$$

$\mu_\beta[\text{cm}^2/\text{g}]$: absorption coefficient

- Relationship between μ_β and $E_{\beta max}$

$$\mu_{\beta,air} = 16(E_{\beta max} - 0.036)^{-1.4}$$
$$\mu_{\beta,tissue} = 18.6(E_{\beta max} - 0.036)^{-1.37}$$
$$\mu_{\beta,i} = 17(E_{\beta max})^{-1.14}$$



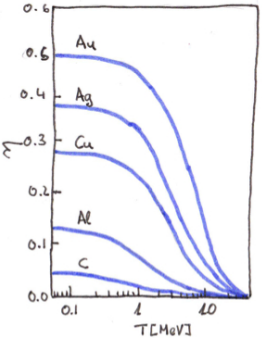
- Absorption is not truly exponential (probabilistic). It's an artifact produced by
  - continuous energy spectra
  - scattering of particles by the absorber

Backscattering

- Because of its small mass, electrons are susceptible of large angle deflections
- The probability of backscattering is not negligible
  - Specially important for low energy electrons
  - It depends on the incidence angle. (harder for perpendicular incidence)
  - Increases with increasing Z
- Backscattering coefficient or albedo η

$$\eta = \frac{\#\ \text{backscattered electrons}}{\#\ \text{incident electrons}}$$

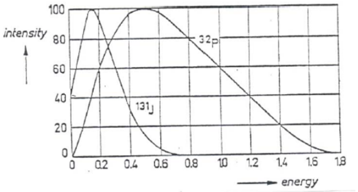
- Important parameter for detectors:
  - Bad geometry can backscatter most electrons
  - Non-collimated electrons on NaI → up to 80% backscattering



Protection against electrons

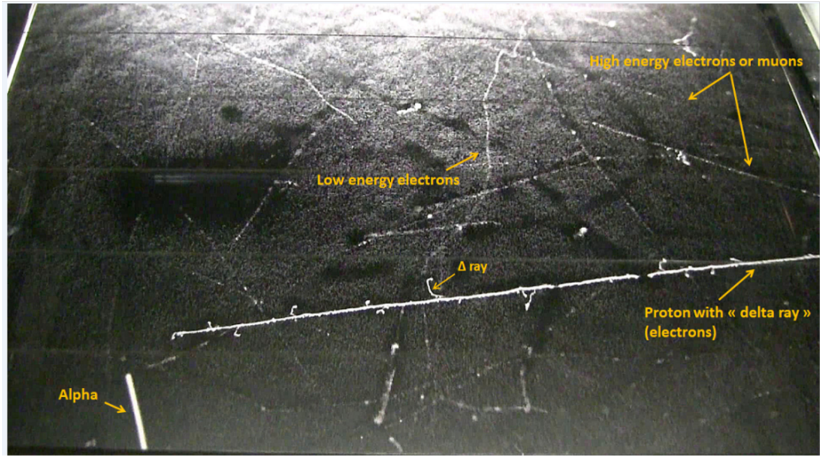
- Ionization is 100 to 1000 less than for alpha particles
  - penetrate deeper into matter → keep distance to source
- Plate of a few mm of aluminium stops the beta particles,
  - Glasses stop most beta particles
  - High-Z materials stop more efficient than low-Z materials but also produce more bremsstrahlung!
  - Choice : few cm plexiglass

- Remember: β sources have a continuous energy distribution:
  - Different ranges → use average energy as a guess of the range



Isotope	E_{max} (MeV)	E_{av} (MeV)		E_{av}/E_{max}
^{3}H	0.018	0.0057		0.31
^{14}C	0.156	0.05		0.32
^{22}Na	0.575	0.225		0.39
^{24}Na	1.39	0.57		0.39
^{32}P	1.71	0.695		0.40
^{35}S	0.168	0.55		0.33
^{45}Ca	0.250	0.1		0.40
^{59}Fe	0.46	0.150	0.120	0.32
	0·255	0.085		0.30
^{64}Cu	0·578	0.175	0.205	0.33
	0.659	0.265		0.40
^{130}I	1.02	0.360	0.270	0.35
	0.60	0.195		0.32
^{131}I	0.61	0.205		0.34
RaE	1.17	0.330		0.28

Cloud chamber: seeing particles

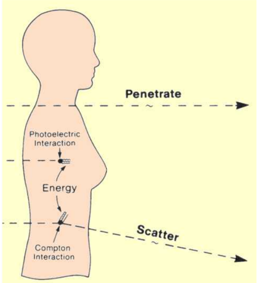


Rare picture shows in a single shot the 4 particles that are detectable in a cloud chamber : proton, electron, muon (probably) and alpha at Pic du Midi (2877 m)
See also 11 min of Cosmic Ray in a Cloud Chamber

Section 4

Energy loss by Photons

Basic concepts of γ-interaction with matter



- Transmission
  - Not interaction at all
- Attenuation
  - Absorption: γ fully loses its energy
  - Scatter: γ loses part of its energy
- Behaviour of γ in matter is quite different from charged particles
  - Photons are much more penetrating (smaller cross sections)
  - Photons does not loose energy continuously → intensity is attenuated

$$I = I_0 e^{-\mu x}$$

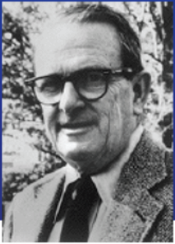
- Remember: Mean free path concept:

$\lambda = \dfrac{1}{\mu}$ average distance traveled into the absorber until the photon interacts

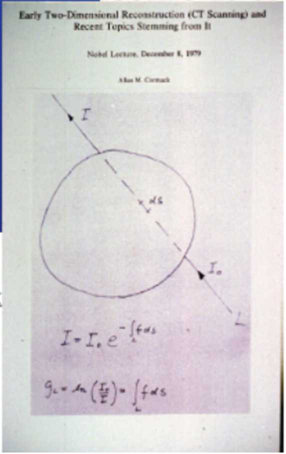
$\lambda \equiv$ mm - 10's cm

CT principles

Nobel Price Physiologiy and Medecine 1979



Allan MacLeod Cormack
Physicien Nucléaire
Cape Town
Harvard University
Tufts University



Early Two-Dimensional Reconstruction (CT Scanning) and Recent Topics Stemming from It

Nobel Lecture, December 8, 1979

Allan M. Cormack

Sir Godfrey N. Hounsfield
Electrical engineer
EMI Research

Interaction of Photons

- Behaviour of γ in matter is quite different from charged particles
  - Photons are much more penetrating (smaller cross sections)
  - Photons does not loose energy continuously $\rightarrow$ intensity is attenuated
    - Main processes of photons in matter, removes them completely
    - Photons that does not interact, just pass through retaining its original energy
    - Attenuation is exponential

$$I = I_0 e^{-\mu x}$$

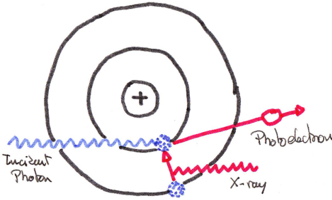
Absorption coefficient is related to cross section and is characteristic for each material

- Main interaction of photons with matter

$$\left.\begin{aligned} \text{Photoelectric effect: } & \sigma_{ph} \\ \text{Compton scattering: } & \sigma_c \\ \text{Pair production: } & \sigma_{pp} \\ \text{Nuclear dissociation: } & \sigma_{nuc} \end{aligned}\right\} \rightarrow \begin{aligned} & \mu \propto \sigma_{ph} + \sigma_c + \sigma_{pp} \\ & \frac{\mu}{\rho} \propto \frac{\sigma_{ph}}{\rho} + \frac{\sigma_c}{\rho} + \frac{\sigma_{pp}}{\rho} \end{aligned}$$

Photoelectric effect

- Absorption of a photon by an atomic electron:
  - Photo fully loses its energy
  - Emission of a photo-electron: $E_e = E_\gamma - BE$



- This process cannot take place with a free electron (nucleus has to absorb recoil momentum)
- 80% collisions with K-shell
- $\sigma_{ph} \propto Z^5$
- E_γ =BE$_K$ $\rightarrow$ new process $\rightarrow$ sharp peak
- $E_\gamma \gtrsim$BE$_K$ $\rightarrow \sigma_{ph} \propto \frac{1}{E^{3.5}}$ quick drop.
- K-shell electrons no longer available
- Similar behaviour with other shells.

$$\sigma_{ph}^K = \left(\frac{32}{\epsilon^7}\right)^{\frac{1}{2}} \alpha^4 Z^5 \sigma_{Th}^e \qquad \epsilon \simeq \text{K-shell}$$

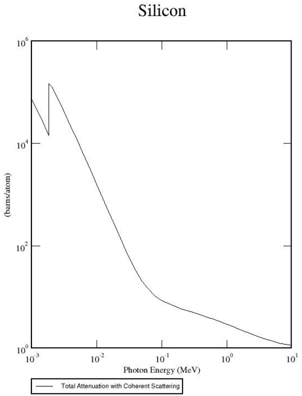
$$\sigma_{ph}^K = 4\pi r_e^2 Z^5 \frac{1}{\epsilon} \qquad \epsilon >> \text{K-shell}$$

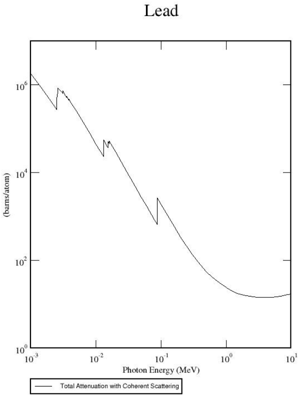
$$\epsilon = \frac{E_\gamma}{m_e c^2} \qquad \sigma_{Th}^e = \frac{8}{3}\pi r_e^2 = 665 \text{ mbarn}$$

Complexity of atomic shells: $Z^5 \rightarrow Z^{4-5}$

Photoelectric effect

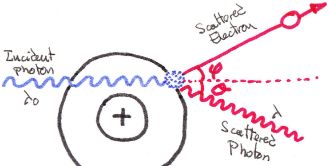
http://physics.nist.gov/PhysRefData/Xcom/Text/XCOM.html



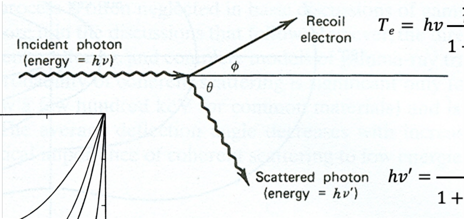


Compton scattering

- Inelastic scattering of γ's on free or loosely bound electrons at rest
  - Photon is scattered atomic electron
  - Photon loses part of its energy
  - If E_γ is large enough it does not matter if electron is bound

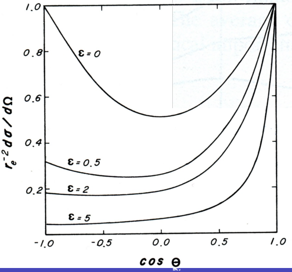


$$E_\gamma = E_e + E_{\gamma'}$$



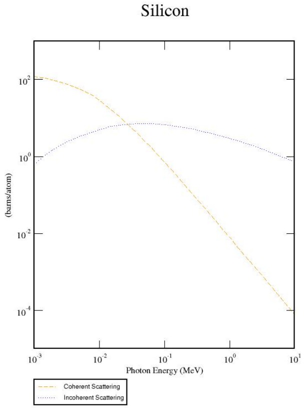
$$T_e = h\nu \frac{\frac{h\nu}{mc^2}(1 - cos\theta)}{1 + \frac{h\nu}{mc^2}(1 - cos\theta)}$$

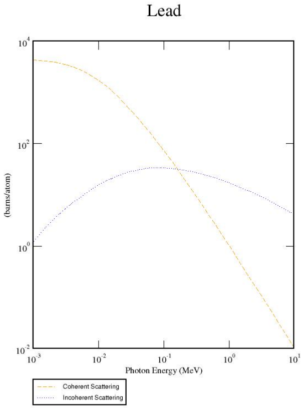
$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1 - cos\theta)}$$



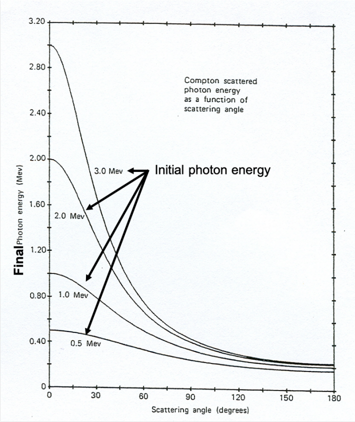
Compton scattering

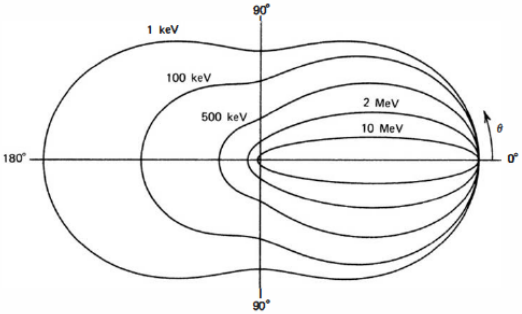
http://physics.nist.gov/PhysRefData/Xcom/Text/XCOM.html





Compton scattering





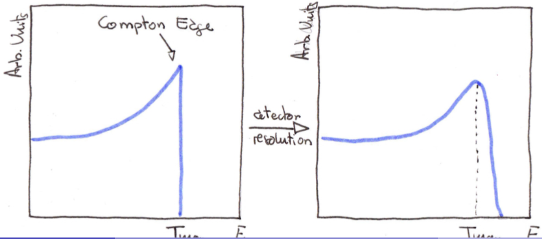
Compton scattering: Recoil electrons spectra

- An important feature in Compton scattering is the energy distribution of the recoil electrons
- It can be obtaining substituting T in Klein-Nishina cross-section

$$T = h\nu \frac{\gamma(1-\cos\theta)}{1+\gamma(1-\cos\theta)} \quad \rightarrow \quad \begin{array}{c} \gamma = \dfrac{h\nu}{m_e} \\ T_{max} \rightarrow \cos\theta = -1 \rightarrow h\nu \dfrac{2\gamma}{1+2\gamma} \end{array}$$

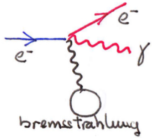
$$\frac{d\sigma}{dT} = \frac{\pi r_e^2}{m_e c^2 \gamma^2}\left[2 + \frac{s^2}{\gamma^2(1-s)^2} + \frac{s}{1-2}\left(s - \frac{2}{\gamma}\right)\right] \qquad s = \frac{T}{h\nu}$$

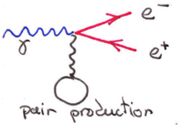
- Important in spectroscopy!!!



Pair Production

- Transformation of a photon in an $e^+ - e^-$ pair
- Only possible in presence of matter.
- $E_\gamma > 2m_e = 1.022$ MeV
- Process similar to bremsstrahlung.





$$\xi = \frac{100\, m_e c^2\, h\nu}{E_+ E_- Z}$$

E_+ =Energy positron

E_- =Energy electron

no screening $\quad \sigma_{pair} = 4Z^2 r_e^2 \alpha \left[\frac{7}{9} \ln\left(\frac{2h\nu}{m_e c^2} - f(Z)\right) - \frac{109}{54}\right] \quad m_e c^2 < h\nu < 137 m_e c^2 Z^{1/3}$

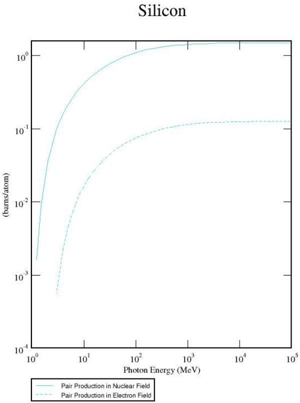
screening $\quad \sigma_{pair} = 4Z^2 r_e^2 \alpha \left[\frac{7}{9} \ln\left(183 Z^{1/3} - f(Z)\right) - \frac{1}{54}\right] \quad h\nu > 137 m_e c^2 Z^{1/3}$

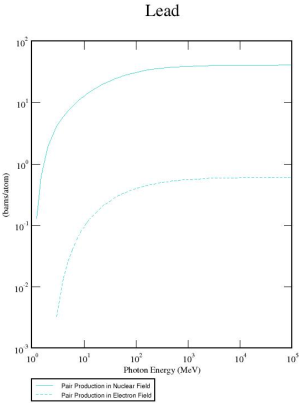
- Pair production occurs also in the atomic field: $Z^2 \to Z(Z+1)$
- Mean free path of a γ-ray due to pair production

$$\frac{1}{\lambda_{pair}} = N\sigma_{pair} \simeq \frac{7}{9} Z(Z+1) r_e^2 \alpha N \left(183 Z^{1/3} - f(Z)\right) \simeq \frac{7}{9} X_0$$

Pair Production

http://physics.nist.gov/PhysRefData/Xcom/Text/XCOM.html



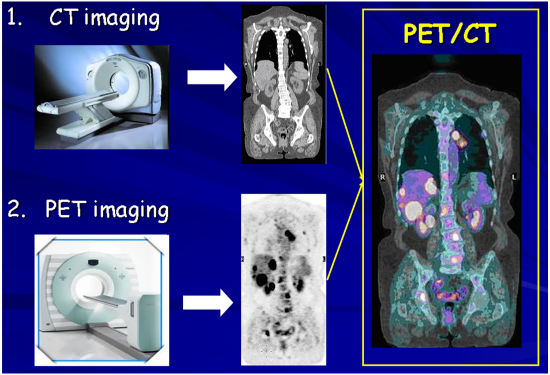


Positron-electron annihilation

- Similar process as pair production
- Positrons annihilates on an electron in the material

$$e^+ + e^- \rightarrow 2\gamma \qquad E_\gamma = 511 \text{ KeV}$$

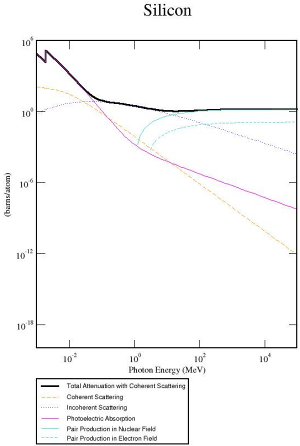
- This is the basis of PET (Positron Emission Tomography)

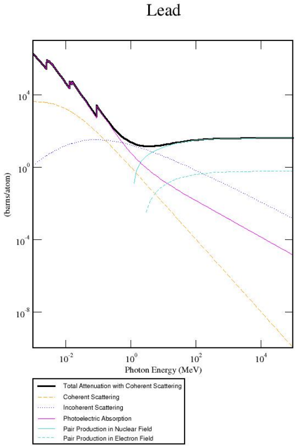


Photon Total Cross-Section

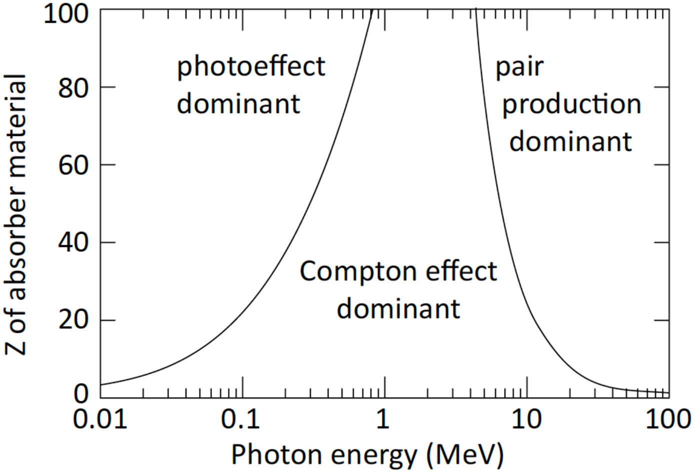
$$\sigma_{tot} = \sigma_{ph} + Z\sigma_c + \sigma_{pair}$$

Low energies < few keV	→	photoelectric effect
Medium energies	→	Compton scattering
High energies > 10-100 MeV	→	Pair creation





Dependence of Photon interaction with Energy



Absorption coefficient

- we can define the absorption coefficient (μ) as the probability per unit length for an interaction:

$$\mu = N\sigma = \frac{N_A \rho}{A}\sigma = \frac{1}{\lambda}$$

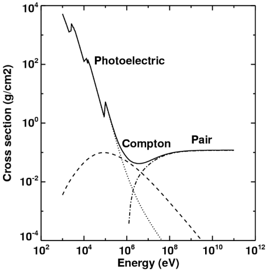
$$\frac{\mu}{\rho} \propto \frac{\sigma_{ph}}{\rho} + \frac{Z\sigma_c}{\rho} + \frac{\sigma_{pair}}{\rho}$$

- For $E \simeq$1-10 MeV:

$$\mu_{ph} \sim Z_m^5 / E_\gamma^{3.5}, \quad \mu_c \sim Z/E, \quad \mu_{pair} \sim EZ^2$$

- The fraction of photons surviving a distance x is:

$$I(x) = I_0 e^{-\mu x}$$



- For mixtures we can use the Bragg's rule:

$$\frac{\mu}{\rho} = w_1 \frac{\mu_1}{\rho_1} + w_2 \frac{\mu_2}{\rho_2} + \cdots \qquad w_i = \frac{a_i A_i}{A_m}$$

Half-value layer (CDA)

❒ Sometimes linear absorption coefficients are given in terms of half-thickness ($X_{1/2}$)

$$I(x) = I_0 e^{-\mu x}$$

$$\frac{I(x)}{I_0} = e^{-\mu X_{1/2}} = \frac{1}{2}$$

$$\mu X_{1/2} = \ln 2$$

$$X_{1/2} = \frac{\ln 2}{\mu} = \frac{0.693}{\mu}$$

$\frac{I(x)}{I_0}$	CDA
1.0	0
0.50	1
0.25	2
0.10	3.3
0.01	6.6
0.001	10

Half-value layer (CDA)

Table 8-3. Half-value layers (in cm) versus photon energy for various materials[a].

Energy (MeV)	Lead (11.35 g/cm^3)	Iron (7.874 g/cm^3)	Aluminum (2.699 g/cm^3)	Water (1.00 g/cm^3)	Air (0.001205 g/cm^3)	Stone concrete (2.30 g/cm^3)
0.1	0.011	0.237	1.507	4.060	3.726×10^3	1.734
0.3	0.151	0.801	2.464	5.843	5.372×10^3	2.747
0.5	0.378	1.046	3.041	7.152	6.600×10^3	3.380
0.662	0.558	1.191	3.424	8.039	7.420×10^3	3.806
1.0	0.860	1.468	4.177	9.802	9.047×10^3	4.639
1.173	0.987	1.601	4.541	10.662	9.830×10^3	5.044
1.332	1.088	1.702	4.829	11.342	1.047×10^4	5.368
1.5	1.169	1.802	5.130	12.052	1.111×10^4	5.698
2.0	1.326	2.064	5.938	14.028	1.293×10^4	6.612
2.5	1.381	2.271	6.644	15.822	1.459×10^4	7.380
3.0	1.442	2.431	7.249	17.456	1.604×10^4	8.141
3.5	1.447	2.567	7.813	19.038	1.747×10^4	8.828
4.0	1.455	2.657	8.270	20.382	1.868×10^4	9.366
5.0	1.429	2.798	9.059	22.871	2.094×10^4	10.361
7.0	1.348	2.924	10.146	26.860	2.449×10^4	11.846
10.0	1.228	2.940	11.070	31.216	2.817×10^4	13.227

a Calculated from attenuation coefficients listed in Table 8-2.
Source: Data from Hubbell and Seltzer (1995).

Energy transfer coefficient

- Energy transfer coefficient μ_{tr} is the fraction of the incident photon energy that is transmitted to kinetic energy of secondary charged particles

$$\frac{\mu_{tr}}{\rho} = \frac{\sigma_{ph,tr}}{\rho} + \frac{Z\sigma_{c,tr}}{\rho} + \frac{\sigma_{pair,tr}}{\rho}$$

- Energy absorption coefficient (μ_{en}) is the fraction of energy removed from the photons by the medium.
- μ_{en} does not take into account:
  - The KE of secondary particles quitting the medium
  - The energy lost in radiative interactions
- Both coefficients are related

$$\frac{\mu_{en}}{\rho} = \frac{\mu_{tr}}{\rho}(1-g)$$

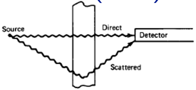
where g is the energy lost in radiative interactions

Build-up

- In general, the transfer of energy from the photon beam to the electrons at a particular location does not lead to the absorption of energy by the medium at the same location
- Reason: Compton photons and the range of electrons
- In the interaction of a photon beam with a medium:
  - Only a part of the energy carried by the secondary electrons is absorbed in the volume element surrounding this point.
  - The rest of the transferred energy is absorbed somewhere else
  - In a deeper layer, the electron fluence and the absorbed energy will be higher:
    - The electrons released in the layer itself by direct photons
    - The electrons released in the layer by Compton photons.
    - The electrons coming from the previous layers.
- Process known as "build-up": accounted by an extra factor $B(d,E)$

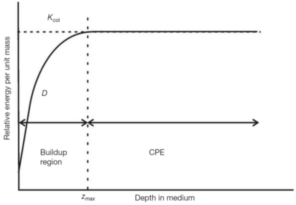
$$I_d = I_0 B(d,E) e^{-\mu d}$$

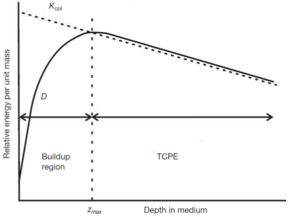
$B(d,E)$ depends on geometry, material and energy.



Electronic equilibrium

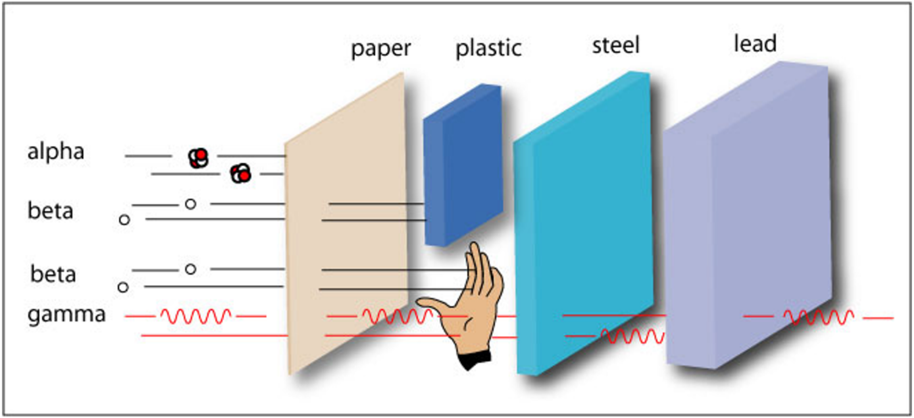
- The build-up continues until the number of secondary electrons entering the considered volume element is identical to the number of electrons leaving this element.
  - This process depends on the photon energy.
  - Electronic equilibrium → CPE=Charged Particle Equilibrium
- In addition there is a competitive process: attenuation
  - Equilibrium only at z_{max}: maximal dose
  - $z > z_{max}$: constant relationship between absorbed and transferred energy
  - Transient equilibrium → TCPE=Transient Charged Particle Equilibrium





Protection against radiation

The range of a radiation depends on the type of particle, on its initial energy and on the material which it passes



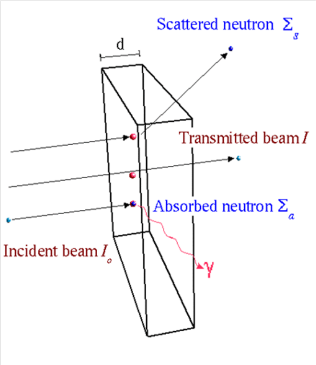
Section 5

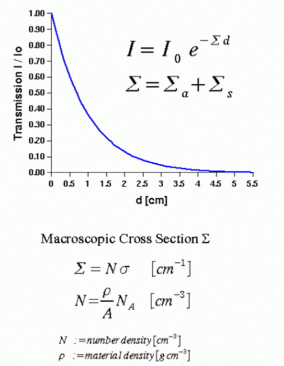
Interaction of neutrons with matter

5. Interaction of neutrons with matter

Interaction of neutrons with matter

- Neutrons have not Coulombian interction.
- Interaction with matter is via collisions with atomic nuclei
  - Neutrons disappear completely or their energy/direction changes
  - Cross section changes dramatically with neutron energy: $\sigma \propto \frac{1}{E^n}$
- As photons, a neutron beam is attenuated: $I = I_0 e^{-\Sigma d}$



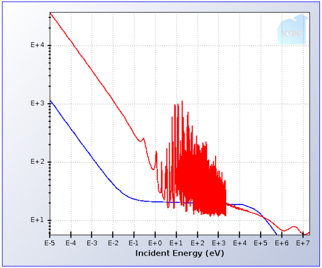


Interaction of neutrons with matter

- Neutrons are classified according with their energy as:

Relativistic	$E_n > 50\,MeV$
Fast	$500\,keV < E_n < 50\,MeV$
Intermediate	$1\,keV < E_n < 500\,keV$
Slow	$0.025\,eV < E_n < 1\,keV$
thermal	$E_n < 0.025 eV$

- Explore cross-sections at: https://www.nndc.bnl.gov/



Slow neutrons (E<0.5 eV)

- thermalisation via elastic collisions in absorber → E≃0.025 eV
- move then "freely" and behave like molecules in a gas
- can make nuclear reaction nuclei: (n,γ) but also (n,p),(nα),(n,fission)
- In tissue mainly n + p → d + γ(2.2 MeV; this also causes ionization!)

Fast neutrons (E>0.5 eV)

- elastic collisions energy loss in collisions with atomic nuclei which now clearly recoil (due to the higher energy of the neutrons!)
  - "moderation" - most efficient with hydrogen (water)
  - the recoiling nuclei cause a lot of ionization along their path!!
- inelastic scattering (high energy)→ atomic nucleus in excited state → emits γ-rays

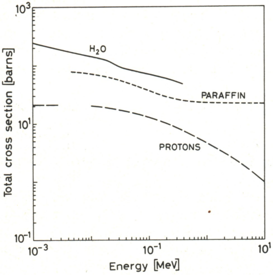
Fast neutrons can create big damage deep inside living tissue!!

A 2 MeV neutron travels about 6 cm in tissue before being thermalized.

- In tissue, containing mainly H, C, N and O atoms.
- Elastic collision with nucleus or an (n,γ)-reaction causes a large ionisation-density

Protection agains neutrons

- Stay as short as possible in a room where neutrons are present
  - Neutrons are distributed over the full space by the thermalisation process, then behave as a gas
  - They can then in principle interact via (n,γ) reactions with any object in the room.
- Protection against neutrons is done
  - slowing them down (moderation, thermalisation)
  - remove them via a neutron capture reaction (mainly (n,γ))



Material	Number of collisions
hydrogen, ^1H	18
deuterium, ^2H	25
helium, ^4He	43
carbon, ^{12}C	110
oxygen, ^{16}O	150
uranium, ^{238}U	2200

Number of collisions required to slow neutrons of 2 MeV down to thermal

Section 6

Radiation Measurement Units

6. Radiation Measurement Units

Radiation Measurement Units

- Capital importance in Radiation Protection and Medical Applications, but also important when talking about radiation detection.
- Radiation Units measure:
  - The quantity of ionization produced
  - The amount of energy deposited in the material
- ATTENTION: Please distinguish between:
  - quantity: velocity
  - units: m/s, cm/μs, etc...
- Definitions of these units/quantities are continuously evolving
  - Adapted to new situations
  - Adapted to new discoveries/research
- Definitions regulated by international organizations
  - ICRP: International Commission on Radiological Protection
  - NCRP: National Council on Radiation Protection and Measurement (only USA)
  - ICRU: International Commission on Radiation Units and Measurement

LET and Kerma

- LET = Linear Energy Transfer = Truncated energy loss

$$L_\Delta = \left(\frac{dE}{dx}\right)_{T<\Delta}$$

- It takes into account the average energy locally imparted into a medium
- It does not take into account processes that release energy non-locally
  - δ-rays with energy $> \Delta$
  - Radiative photons

- Kerma (K) is the sum of the kinetic energies of all those charged particles released by neutral particles.
  - In case of photons related with mass energy transfer and photon energy fluence

$$K = \mu_{tr}\Psi \qquad \Psi = \Phi E$$

Exposure

- Defined for X-rays on terms of the amount of ionization they produce in air
- Probably de oldest quantity:
  - Easy to measure → with ionization chambers
  - ... but not possible to apply to other media (i.e. tissues, solids)
- Unit: ROETGEN(R) → charge/volume
  - Quantity of X-rays producing an ionization of 1 esu/cm^3 in air at STP
    1 esu = 3.34 10^{-10}C = 2.08 10^9 ion pairs
    STP: T=0C and P=1 atm
  - 1R = 2.58 10^{-4} C/kg
- The exposure rate of a source of activity A at a distance d is given by

$$\text{Exposure Rate} = \frac{\Gamma A}{d^2}$$

- where Γ is the exposure rate constant
- It depends on the source
- Units= $(R \cdot cm^2)/(hr \cdot mCi)$

Nuclide	Γ
^{124}Sn	9.8
^{137}Cs	3.3
^{60}Co	13.2
^{22}Na	12.0
^{222}Ra	8.2

Absorbed Dose

- Energy absorbed per unit mass from any kind of radiation $D = \dfrac{\Delta E}{\Delta m}$
- Units:

	SI	1 Gray (Gy)	=	1 Joule/kg
	CGS	1 rad	=	100 erg/gr

$$1 \text{ Gy} = 100 \text{ rad}$$

- More general quantity to measure radiation effects in all materials with all kind of radiations

Categorization of dose:

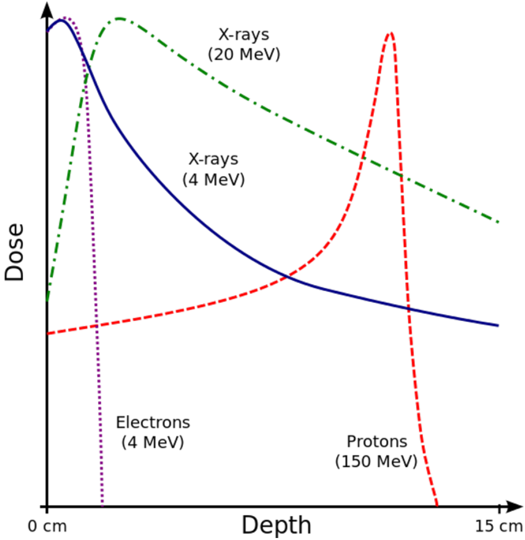
<1 Gy:	LOW DOSE
1-10 Gy:	MODERATE DOSE
>10 Gy:	HIGH DOSE

- The absorbed dose is the primary quantity used in dosimetry
- Typical doses:

Radiotherapy total:	40 Gy to tumor (over several weeks)
Radiotherapy single dose:	4 Gy
Mammography:	9 mGy

- ATTENTION: The absorbed dose gives
  - NO indication about the rate at which radiation occurs
  - NO indication about the radiation type

Doses in radiotherapy



Example

Calculate the absorbed dose in air for 1 R of γ-rays.

$$1\,R = \frac{1\,esu}{cm^3} = 2.08 \times 10^9 \text{ ion pairs/cm}^3$$

Average energy to create an ion-pair in air ~32 eV

$$\begin{aligned}\text{Energy absorbed} &= 32 \text{ eV} \cdot 2.08 \times 10^9 \text{ ion pairs/cm}^3 \\ &= 6.66 \times 10^4\,\text{MeV/cm}^3\end{aligned}$$

$$\rho_{air} = \frac{1.2\,mg}{cm^3}$$

$$1\,MeV = 1.6 \times 10^{-6}\,\text{erg}$$

$$\begin{aligned}1\,R &= 89\,\text{erg/gr} \\ &= 0.89\,\text{rad} \\ &= 8.9 \times 10^{-3}\,\text{Gy}\end{aligned}$$

Equivalent dose (H_T)

- The equivalent dose $H_{T,R}$ in a tissue T due to radiation R is equal to the absorbed dose by the tissue T caused by radiation type R multiplied by the quality factor w_R of the radiation.

$$H_{T,R} = w_R D_{T,R} \quad \rightarrow \quad H_T = \sum_R w_R D_{T,R}$$

- Units: SI Gray (Gy) → Sievert (Sv) 1 Joule/kg
  CGS rad → rem 100 erg/gr

$$1\ \text{Sv} = 100\ \text{rem}$$

Radiation		w_R
γ,X-rays,e$^{\pm}$		1
Neutrons, energy <10 keV		5
	>10 keV to 100 keV	10
	>100 keV to 2 MeV	20
	>2 MeV to 20 MeV	10
	>20 MeV 1	5
Protons, other than recoil and energy >2 MeV		2 (NCRP) 5 (ICRP)
α, fission fragments and non-relativistic nuclei		20

Effective Dose (H_E)

- Different tissues respond differently to radiation and the
- The probability of stochastic effects is then different for the different tissues.
- ICRP,NCRP have assigned dimensionless weighting factors (w_T) for the different tissues
- w_T defined to take into account the overall detriment of an individual

Tissue or Organ	w_T
Gonads	0.20
Bone marrow	0.12
Colon	0.12
Lung	0.12
Stomach	0.12
Bladder	0.05
Breast	0.05
Liver	0.05
Esophagus	0.05
Thyroid	0.05
Skin	0.01
Bone surface	0.01
Remainder	0.05

$$H_E = \sum_T w_T H_T \qquad \sum w_T = 1$$

In case of uniform irradiation

$$H_T = \text{cte} \quad \rightarrow \quad H_E = \sum_T w_T H_T = H_T \sum_T w_T = H_T$$

Typical doses from common sources

Natural sources	
Cosmic rays	28 mrem/yr
Natural background (U,Th, Ra)	26
Internal radioactive sources (^{40}K, ^{14}C)	26
Environmental sources	
Technologically enhanced	4 mrem/yr
Global fallout	4
Nuclear power	0.3
Medical	
Diagnostic	78 mren/yr
1 x-ray	100-200 mrem
Pharmaceuticals	14
Occupational	1
Consumer products (TV, etc..)	5

Exposure limits

	NCRP	ICRP
Occupational Exposure		
Effective Dose		
Annual	50 mSv	50 mSv
Cumulative	10 mSv × age (y)	100 mSv in 5 y
Equivalent Dose		
Annual	150 mSv lens of eye; 500 mSv skin, hands, feet	150 mSv lens of eye; 500 mSv skin, hands, feet
Exposure of Public		
Effective Dose		
Annual	1 mSv if continuous 5 mSv if infrequent	1 mSv; higher if needed, provided 5-y annual average ≤ 1 mSv
Equivalent Dose		
Annual	15 mSv lens of eye 50 mSv skin, hands, feet	15 mSv lens of eye; 50 mSv skin, hands, feet

Radiation Protection: ALARA principle

We live with 1-3 mSv

Where to stop?
Where is the safe point?
What are the effects of radiation?

4000 mSv can kill