General Properties of Detectors

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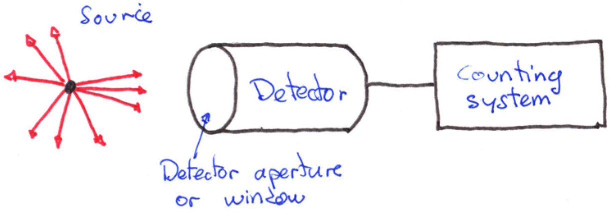
Section 1

Introduction

1. Introduction

Detector model (simplified)

- Before discussing different types of detectors let's study some general properties that apply to all of them.
- Let's consider a simplified view of a detector system:



$S = \#$ part/s emmitted by the source

$r = \#$ part/s recorded by the system

$$r = f_1 f_2 f_3 \cdots f_n S$$

- The f's are represent the effects of the setup on the measurement:
  - Geometry effects
  - Source effects
  - Detector effects
- As important as these effects is the way the detector is operated.

Absolute and relative measurements

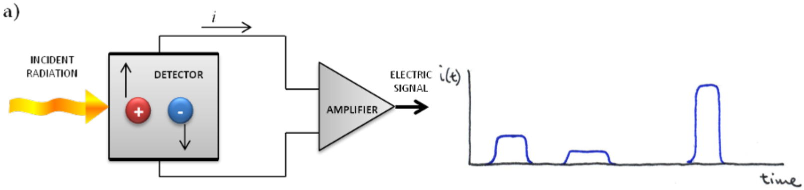
- An absolute measurement is one in which the exact number of particles emitted or the exact numbers of events taking place is determined:
  - Determination of the activity of a source
  - Measurement of a cross section
- In a relative measurement, we are not interested in the exact number of events or particles emitted. A fraction of these is measured.
  - The presence or not of a certain type of radiation
  - Variation of flux of particles
- In case of relative measurement there is a relationship with the absolute measurement, but it's not necessary to know it.
- In general, relative measurements are easier than absolute measurements.
  - In absolute measurements we need to know ALL f factors
  - In relative measurements don't.

Section 2

Modes of detector operation

Modes of detector operation

- Net result of radiation interaction with matter is the appearance of a given amount of charge Q:
  - Charge collection thanks to electric field between electrodes
  - It appears current in the electrodes
  - Collection time depends on each detector



- We are going to study three modes of detector operation
  - Pulse mode: designed to record each individual quantum of radiation
  - Current mode: current average taken over many events
  - Mean Square voltage: average sensitive to Q^2

Signal Generation

- Signals from detectors arise because of the motion of charge carriers after they are formed by incident radiation
  - Induced charge in electrodes due to moving charges
  - Signal induced is created as soon as charge carrier starts to move
- General method to calculate induced charge on electrodes

$$i(t) = q\vec{v}\cdot\vec{E}_w \qquad \rightarrow \qquad \text{Shockley-Ramo theorem}$$

q : is the charge of the charge carrier

$\vec{v}$: velocity of charge carrier. $\vec{v} = \mu\vec{E}$

$$\vec{E} = -\nabla\phi$$

$\vec{E}_w$: Weighting field. How moving particles couples to the electrode

$$\vec{E}_w = -\nabla\phi_w$$

- The induced charge on the electrode by a charge carrier is given by:

$$Q = q\Delta\phi_0$$

$\Delta\phi_0$: Difference in weighting potential from beginning to the end of carrier path

Weighting field

- Knowing ϕ and ϕ_w we can know/simulate induced charge in electrodes
- To compute ϕ we need to solve the Poisson equation

$$\nabla^2\phi = \frac{\rho}{\epsilon}$$

with all contour conditions imposed by the detector.

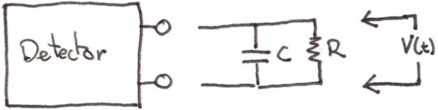
- To compute the weighting field of an electrode we have to:
  - Set to 1 V the voltage of the electrode for which we can calculate the induced current
  - Set to 0 V the rest of the voltages
  - Any trapped charge is ignored in the calculation.

$$\nabla^2\phi_w = 0$$

- Best method to calculate ϕ and ϕ_w → Finite element methods
- Simple cases can be calculated analytically.

Pulse mode

- In this mode we want to keep track of individual radiation events
- We will have access to the amplitude and time information.
- Signal pulse will depend on the input circuit connected to the detector



- Preamplifier (see Electronics chapter) = net of resistors/capacitors
  - R=Input Resistance of the circuit
  - C=Total capacitance (circuit+detector)
  - Basic parameter $\tau = RC$
  - Signal obtained $V(t)$ along R

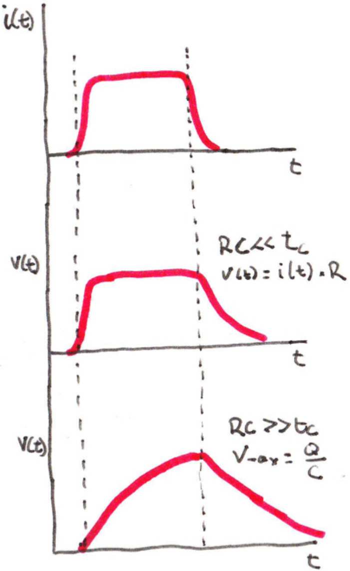
Pulse mode

- $RC << t_c$:

$$i_R(t) \simeq i(t) \rightarrow v_R(t) = i(t)R$$

  - This is the configuration if we want to access to timing info.
  - Time-of-flight
  - High-rate scintillators

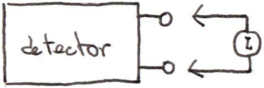
- $RC >> t_c$:
  - $t < t_c$: charge integration in C
  - $t > t_c$: C discharge through C
  - Leading edge depends on detector (t_c)
  - Trailing edge depends on the circuitry
  - $V_{max} = \frac{Q}{C} \propto$ charge deposited
  - Info about energy deposited, particle type,etc
  - Requires use of a CSA

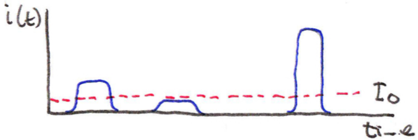


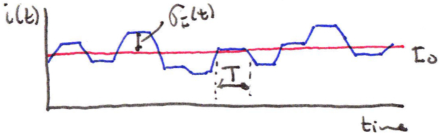
Current mode

- The measuring circuitry has a fixed response time $T >> t_c$
- It averages the current over this time T

$$I(t) = \frac{1}{T}\int_{t-T}^{t} I(t')dt'$$







- Suitable for high fluxes where pulse mode cannot be applied
  - Radioprotection → Immediate control if ionization

Current mode

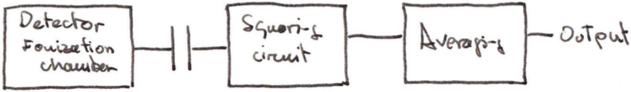
- In case of steady radiation we can calculate I_0 and $\sigma_i(t)$

$$I_0 = rQ = r\frac{E}{w}q \quad \rightarrow \quad \begin{aligned} r &= \text{event rate} \\ Q &= \text{charge produced for each event} \\ E &= \text{average energy deposited per event} \\ w &= \text{average energy to produce an ion pair} \\ q &= 1.6 \times 10^{-19}\, C \end{aligned}$$

- ... and the standard deviation $\sigma_i(t)$
  - From Poisson statistics, the σ over the number of events is
  $$n = rT \quad \Rightarrow \quad \sigma_n = \sqrt{n} = \sqrt{rT}$$
  - If all events leave in average same charge Q
  $$\frac{\overline{\sigma_i(t)}}{I_0} = \frac{\sigma_n}{n} = \frac{1}{\sqrt{rT}} \qquad \rightarrow \qquad \overline{\sigma_i(t)} = \frac{I_0}{\sqrt{rT}}$$

Mean Square Voltage Mode

- Statistical properties of the signal in current mode allow this mode
- Signal is sent through an element that blocks the DC part of I_0, and allow only pass the fluctuation $\sigma_i(t)$
- Additional signal processing can square this signal and average it



$$\overline{\sigma_i^2(t)} \simeq \overline{\sigma_i(t)}^2 = \frac{I_0^2}{rT} = \frac{rQ^2}{T}$$

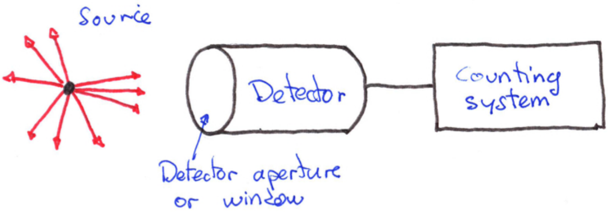
- Signal is proportional to mean charge Q produced in each event
- Enhanced events where Q is large
  - Useful in case of mix radiation
  - In a nuclear reactor, neutron signal will be larger than photon signal

Section 3

Geometry effects

Geometry effects

- Geometry effects take into account factors as the size and shape of both source and detectors as well as the distance between them
- The geometry can affect to the measurement in two ways:
  - The influence of the medium between the source and the detector
  - The exact size and shape of the source and the detector as well as the distance between them. This a purely geometrical effect that it will be taken into account with the solid angle.

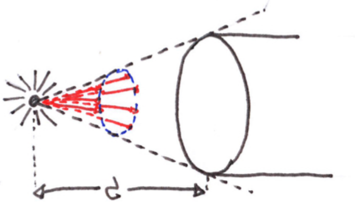


Effect of the Medium

- Let's consider a source and a detector separated a distance d.
- In most of the cases, the medium is either air or other low density medium
- The effect of the medium depend on various factors
  - The medium: both Z and its density.
    Differential cross sections depends on Z
    Macroscopic cross sections depend of ρ
  - The type of particle.
    Different energy loss behaviors for charged and neutral particles.
    Absorption and scattering processes.
    Buildup effects.
- Important effect that should be taken into account specially when dealing with low energy or low intensity phenomena.
- Influence of the medium can always be avoided performing the measurement in vacuum, at the price of increase the complexity of the setup.

Solid Angle: Intuitive definition

- The concept of solid angle will allow us to determine the fraction of particles emitted by the source that will enter into the detector
- For an intuitive definition, let's consider an isotropic point source.
- Since particles are emitted with equal probability in all directions, only a fraction of them will reach the detector.

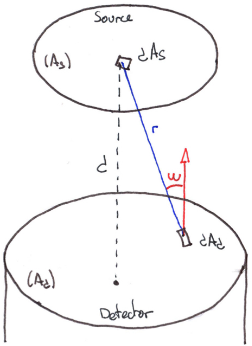


- This fraction of particles is equal to the solid angle (Ω) subtended by the detector.
- In a general case of an extended source we can apply also this definition.

$$\Omega = \frac{\text{\# particles emitted inside the aperture of the detector}}{\text{\# particles emitted by the source}}$$

Solid Angle

- To derive the mathematical expression, let's consider the following:
  - A plane source of area A_s emitting S_0 particles/(m^2 s) isotropically
  - A detector of area A_d at a distance d
  - Let's consider two differential areas dA_s and dA_d and integrating



$$\Omega = \frac{\int_{A_s}\int_{A_d}\left(S_0 \frac{dA_s}{4\pi r^2}\right) dA_d (\hat{n}\cdot\hat{r})}{S_0 A_s}$$

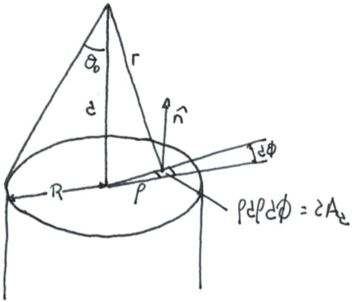
$$\downarrow \quad \hat{n}\cdot\hat{r}=\cos w$$

$$\Omega = \frac{1}{4\pi A_s}\int_{A_s} dA_s \int_{A_d} dA_d \frac{\cos w}{r^2}$$

- This equation is valid for any shape of source and detector
- Ω is the fractional solid angle $(0 \leq \Omega \leq 1)$

Solid Angle: Example

- Point Isotropic Source and a Detector with a Circular Aperture



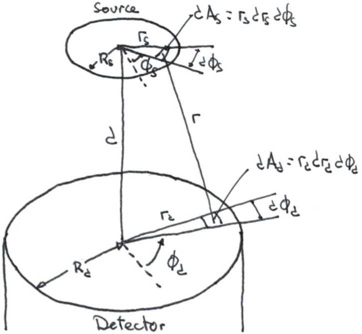
$$\Omega = \frac{1}{2}\left(1 - \frac{d}{\sqrt{d^2 + R^2}}\right)$$

$$\cos\theta_0 = \frac{d}{\sqrt{d^2 + R^2}}$$

$$\Omega = \frac{1}{2}(1 - \cos\theta_0)$$

Solid Angle: Example

- Disk Source Parallel to a Detector with a Circular Aperture



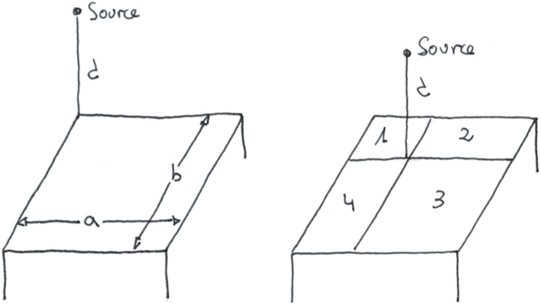
$$\Omega = \frac{w^2}{4}\left\{1 - \frac{3}{4}(\phi^2 + w^2) + \frac{15}{8}\left(\frac{\phi^4 + w^4}{3} + \phi^2 w^2\right) - \frac{35}{16}\left[\frac{\phi^6 + w^6}{4} + \frac{3}{2}\phi^2 w^2(\phi^2 + w^2)\right]\right\}$$

$\phi = R_s / d$

$w = R_d / d$

Solid Angle: Example

- Point Source and a Detector with a Rectangular Aperture



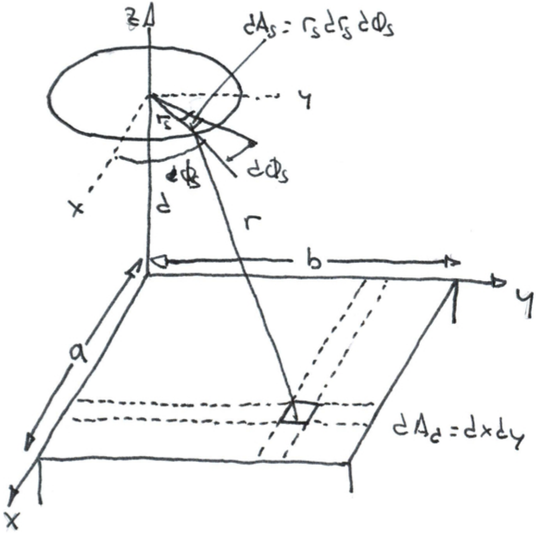
$$\Omega = \frac{1}{4\pi}\arctan\frac{ab}{d\sqrt{a^2+b^2+d^2}}$$

- If the source is located at an arbitrary point above the detector, the solid angle is the sum of four terms.

$$\Omega = \Omega_1 + \Omega_2 + \Omega_3 + \Omega_4$$

Solid Angle: Example

- Disk Source and a Detector with a Rectangular Aperture



Solid Angle: Example

- Disk Source and a Detector with a Rectangular Aperture

$$\Omega = \frac{w_1 w_2}{4\pi}\Big[1 + \frac{3}{4}\phi^2 - \frac{1}{2}(w_1^2 + w_2)^2 + \frac{1}{8}(5\phi^4 + 3w_1^4 + 3w_2^4)$$
$$+ \frac{5}{4}\phi^2(w_1 + w_2) - \frac{35}{64}\phi^6 + \frac{5}{12}w_1^2 w_2^2 - \frac{35}{16}\phi^4(w_1^2 + w_2^2)$$
$$- \frac{7}{32}\phi^2(9w_1^4 + 9w_2^4 + 10w_1^2 w_2^2) - \frac{7}{16}w_1^2 w_2^2(w_1^2 + w_2^2)$$
$$- \frac{5}{16}(w_1^6 + w_2^6)\Big]$$

$$w_1 = a/d \quad w_2 = b/d \quad \phi = R_s/d$$

- If the source is located at an arbitrary point above the detector, the solid angle is the sum of four terms.

Solid Angle calculation by Monte Carlo methods

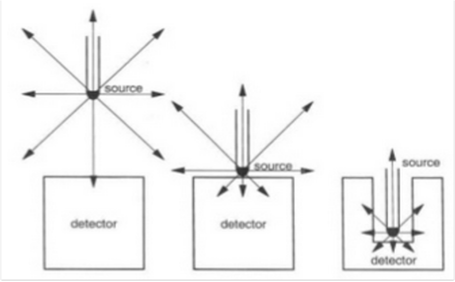
- The basic equation of the solid angle can be solved analytically in very few simple and symmetric cases
- In some other cases, approximate solutions can be obtained by series expansions or numerical integration
- The most general method that can be used with any geometry is based on Monte Carlo calculation
  - Easy source-detector geometry definition
  - Initial particle position and angle determined from random numbers
  - Track particle to see if intersects detector volume
  - Solid angle defined as

$$\Omega = \frac{\#\ \text{particles hitting detector}}{\#\ \text{particles generated}}$$

  - Error decreases with increasing number of generated particle.

Solid angle = Geometric efficiency

- Be careful:
  - The same detector and source may have different solid angles depending on the experimental setup.
  - The geometric efficiency is NOT an intrinsic characteristic of a detector.
  - It takes into account the relative position between the detector and the radiation source



Section 4

Source effects

4. Source effects

Source effects

- Radioactive deposit
  - Self Absorption factor
  - Backscattering factor

- Beams
  - Size
  - Intensity
  - In-beam interaction

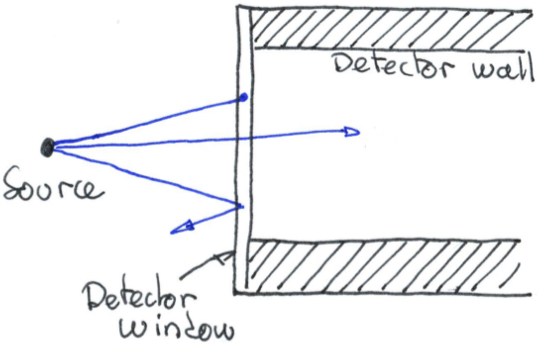
Section 5

Detector effects

5. Detector effects
    - Efficiency
    - Energy resolution. Fano Factor
    - Dead Time

Detector effects

- The detector itself may affect the measurement in various ways.
  - Through its walls and entrance windows
  - Its intrinsic detection efficiency
  - Different detectors can react differently to the same radiation
  - The recovery time (if any) after a particle is detected
  - The readout mode



Efficiency

- One of the main characteristics of a detector system
- It accounts the number of events registered by a detector wrt certain numbers of events.
  - Both terms numerator and denominator should always be well defined
  - It depends on: radiation type, rate, detector, readout, geometry, ...

Intrinsic	$\varepsilon_{int} = \frac{\#\text{ events registered}}{\#\text{ events impinging the detector}}$ $\varepsilon_{int} = 1 - e^{-\mu x}$
Absolute	$\varepsilon_{abs} = \frac{\#\text{ events registered}}{\#\text{ events emitted by source}}$
Relative	$\varepsilon_{rel} = \frac{\#\text{ events registered by det 1}}{\#\text{ events registered by det 2}} = \frac{\varepsilon_{int,1}}{\varepsilon_{int,2}}$
Peak	$\varepsilon_{peak} = \frac{\#\text{ events registered in a peak}}{\#\text{ events impinging the detector}}$

- BE CAREFUL: It's a relative measurement
- Definitions are related:

$$\varepsilon_{abs} = \varepsilon_{int}\Omega$$

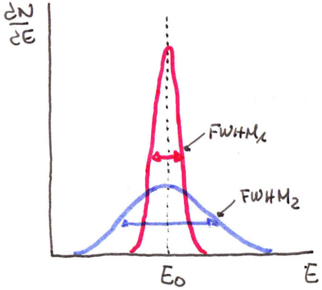
Energy resolution

- This characteristics is applicable only if we measure the energy.
- It measures the ratio of the FWHM of an energy peak with the mean of this peak

$$R = \frac{\Delta E}{E}$$

$$R_1 = \frac{\mathrm{FWHM}_1}{E_0} < \frac{\mathrm{FWHM}_2}{E_0} = R_2$$

$\mathrm{FWHM} = 2.35\sigma$ if gaussian distribution



- Cuased by:
  - drifts of detector properties
  - random noise
  - statistical noise

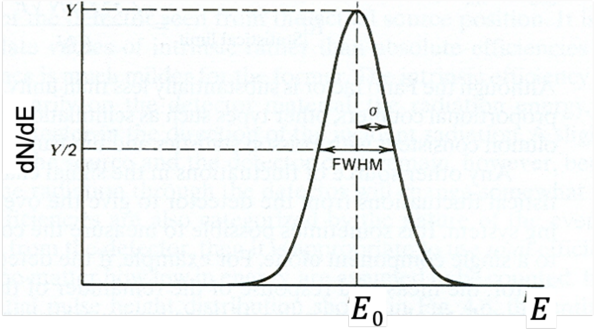
Energy resolution: Statistical approach

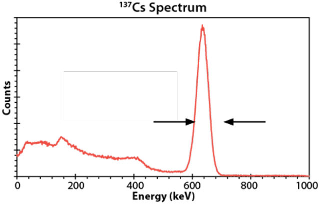
- Calculation of R as a function of N is based on Poisson statistics:

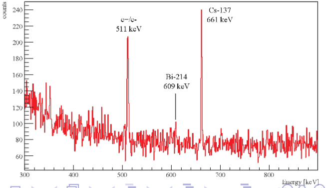
$$R = \frac{\Delta E}{E} = 2.35\frac{\sigma}{N} = \frac{2.35}{\sqrt{N}}$$

$N = E/w$: Number ions pairs generated
$\sigma_N = \sqrt{N}$ (Poisson statistics)
$E = kN \quad \rightarrow \quad \sigma_E = k\sigma_N = k\sqrt{N}$

$$\Delta E = FWHM$$







Fano Factor

- The expression $R = \frac{2.35}{\sqrt{N}}$ is indeed an upper limit of the energy resolution.
  - Experience shows that some detectors can achieve energy resolution smaller than the one predicted
  - The reason of this behavior is that the processes generating the charge carriers are not independent.

- Fano factor has been introduced to take into account this deviation.

$$F = \frac{\text{observed variance in } N}{\text{Poisson variance in } N} = \frac{\sigma^2_{obs}}{\sigma^2_{Poisson}}$$

Scintillators	$F = 1$
Semiconductors	$F \sim$ 0.083-0.143
Prop. Counters	$F \sim$ 0.12

$$\sigma^2_{obs} = F\sigma^{Poisson}$$

Fano Factor

$$F = \frac{\text{observed variance in } N}{\text{Poisson variance in } N} = \frac{\sigma^2_{obs}}{\sigma^2_{Poisson}}$$

Scintillators	$F = 1$
Semiconductors	$F \sim 0.083\text{-}0.143$
Prop. Counters	$F \sim 0.12$

$$\sigma_{obs} = \sqrt{F}\sigma_{Poisson}$$

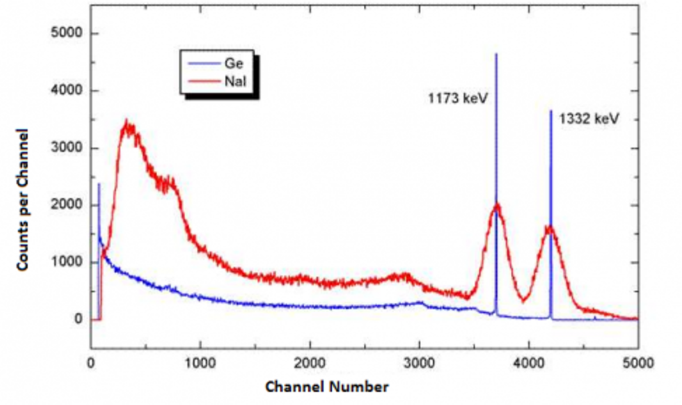
$$R_{Poisson} = \frac{2.35}{\sqrt{N}}$$

$$R_{stat} = \frac{2.35\sigma_{obs}}{N} = \frac{2.35\sqrt{F}\sigma_{Poisson}}{N} = 2.35\sqrt{\frac{F}{N}}$$

- In case other sources of uncertainty they will add quadratically

$$\text{FWHM}^2_{total} = \text{FWHM}^2_{stat} + \text{FWHM}^2_{noise} + \text{FWHM}^2_{drift} + \cdots$$

Why is the energy resolution for a Ge-detector much better than for a scintillator?

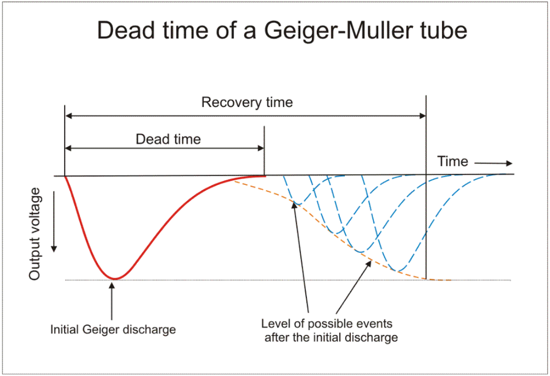


$$R = 2.35\sqrt{\frac{Fw}{E}}$$

Scintillators:	$R \sim 5 - 10\%$	$w = 30 - 100$ eV	$F \sim 1$
Germanium:	$R < 1\%$	$w = 2 - 3$ eV	$F \sim 0.1$

Dead Time

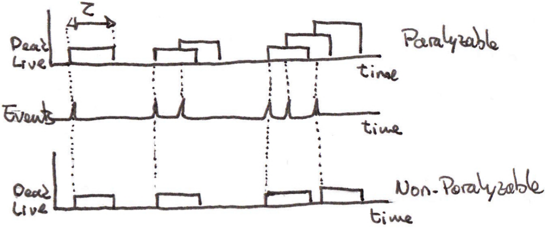
- It's the finite time required by a detector system to process an event
- During this time the detector is insensitive.
- Any event arriving to the detector during this time will not be detected.



Dead Time: Causes and Types

- Major causes of dead time:
  - Detector itself: i.e. electrical discharge in a GM ~400 μs to recover
  - Readout electronics: Gate generation
  - Logic: Vetos from external systems

- We can distinguish two types of dead time behaviour:
  - Paralyzable or extendable
  - Non-Paralyzable



Dead Time: Non-paralyzable case

- Let's consider n = true interaction rate
  m = recorded count rate
  τ = system dead time

- Non-paralyzable case:
  - Fraction of time the detector is "dead" = $m\tau$
  - Rate at which event are lost = $nm\tau$

$$n = m + nm\tau \quad \rightarrow \quad n = \frac{m}{1 - m\tau}$$

Dead Time: Paralyzable case

- Paralyzable case:
  - Dead time periods are not of fixed length
  - m = rate of occurrences of time intervals between true events larger than τ
  - Distribution on intervals between random events is:

$$P_1(t)dt = ne^{-nt}dt$$

$P_1(t)dt$ = distribution of intervals between random events occurring at an average rate n

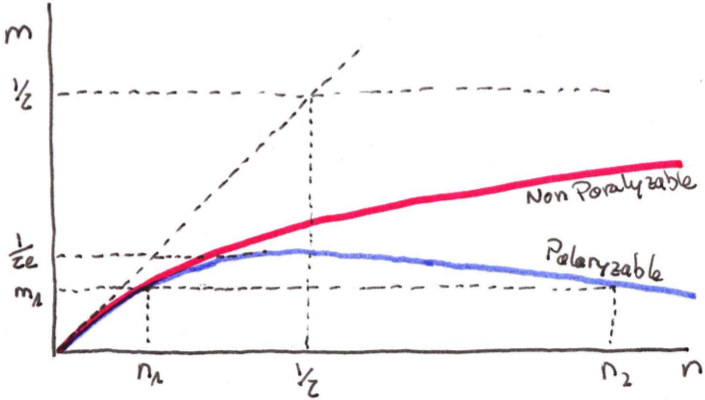
  - Probability of intervals larger than τ

$$P_2(\tau) = \int_{\tau}^{\infty} P_1(t)dt = e^{-nt}$$

  - Rate of occurrence of such intervals:

$$m = ne^{-n\tau}$$

Dead Time: Paralyzable vs Non-Paralyable



Dead Time: Two sources method

- Based on counting rates of two sources individually and in combination.
  - n_1, n_2, n_{12}, n_b true counting rates
  - m_1, m_2, m_{12}, m_b observed rates

$$n_{12} - n_b = (n_1 - n_b) + (n_2 - n_b)$$

$$n_{12} + n_b = n_1 + n_2$$

- Assumed non-paralyzable model $n = \frac{m}{1-m\tau}$

$$\frac{m_{12}}{1 - m_{12}\tau} + \frac{m_b}{1 - m_b\tau} = \frac{m_1}{1 - m_1\tau} + \frac{m_2}{1 - m_2\tau}$$

- Solving this equation gives

$$\tau = \frac{X(1 - \sqrt{1 - Z})}{Y}$$

$$X = m_1 m_2 - m_b m_{12}$$

$$Y = m_1 m_2 (m_{12} + m_b) - m_b m_{12}(m_1 + m_2)$$

$$Z = \frac{Y(m_1 + m_2 - m_{12} - m_b)}{X^2}$$

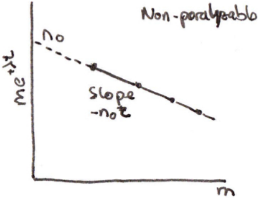
Dead Time: Decaying source method

- Measure the departure of observed counting rate from a the known exponential decay of a short lived radioisotope: $n = n_0 e^{-\lambda t} + n_b$
- In the limit of negligible background $n \simeq n_0 e^{-\lambda t}$

- Non-paralyzable case $n = \frac{m}{1-m\tau}$

$$m e^{\lambda t} = -n_0 \tau m + n_0$$

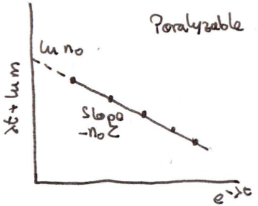
  - $y = m$ and $x = m e^{\lambda t}$
  - Fit to a straight line



- Paralyzable case $m = n e^{-n\tau}$

$$\lambda t + \ln m = -n_0 \tau e^{-\lambda t} + \ln n_0$$

  - $x = \lambda t + \ln m$ and $y = e^{-\lambda t}$
  - Fit to a straight line



Section 6

Detector Response Function

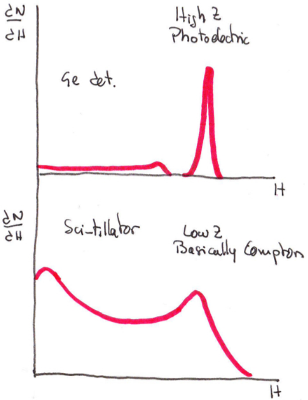
6. Detector Response Function

Pulse Height Spectra

- When operating the detector in pulse mode we have access to energy distribution
- We define the response function for an energy E as the spectrum of pulse heights observed from the detector when it's reached by a mono-energetic radiation

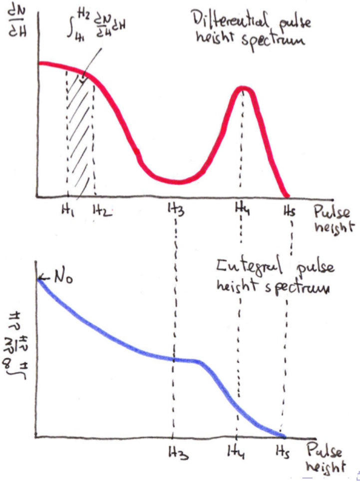
$$PH(E) = \int S(E')R(E,E')dE'$$

- $S(E')$ = Spectrum of incident particles
- $R(E,E')$ = Response function



Counting Curves and Plateau

- In pulse mode, events fed a counting system if pulse heights $> H_t$
- Operating mode (Gain + threshold) should be determined to assure stability of the measurements



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