Two Body Scattering

Radiation-Matter Interaction

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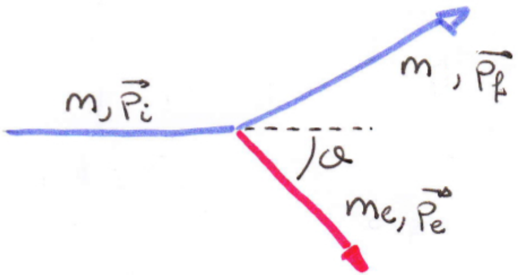
Two Body Scattering

- One of the most important processes in considering the radiation-matter interaction is the energy losses by collision
- It can be considered as a two body scattering:
  - the target (usually an electron) can be considered to be at rest
- Relativistic kinematics should be used to study this process



Two Body Scattering

1) Relation between kinetic energy T and rest mass

$$T + m_e c^2 = \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$p_e^2 = \frac{(T + m_e c^2)^2 - m_e^2 c^4}{c^2} \qquad \text{(A)}$$

2) Momentum conservation

$$\vec{p}_i = \vec{p}_f + \vec{p}_e \rightarrow \vec{p}_f = \vec{p}_i - \vec{p}_e$$

$$p_f^2 = (\vec{p}_i - \vec{p}_e)^2 = p_i^2 + p_e^2 - 2\vec{p}_i \cdot \vec{p}_e \cos\theta \qquad \text{(B)}$$

3) Energy conservation

$$\sqrt{p_i^2 c^2 + m^2 c^4} + m_e c^2 = \sqrt{p_f^2 c^2 + m^2 c^4} + T + m_e c^2$$

$$p_f^2 = \frac{p_i^2 c^2 + T^2 - 2T\sqrt{p_i^2 c^2 + m^2 c^4}}{c^2} \qquad (C)$$

Two Body Scattering

(A) → (B)

$$p_f^2 = p_i^2 + \frac{(T + m_e c^2)^2 - m_e c^4}{c^2} - 2p_i \cos\theta \sqrt{\frac{(T + m_e c^2)^2 - m_e c^4}{c^2}} \quad \text{(D)}$$

(B) = (D)

$$T\sqrt{p_i c^2 + m^2 c^4} = -T m_e c^2 + 2 p_i c \cos\theta \sqrt{(T + m_e c^2)^2 - m_e^2 c^4}$$

$$p_i c \cos\theta \sqrt{\frac{T^2 + 2 T m_e c^2}{T^2}} = m_e c^2 + \sqrt{p_i^2 c^2 + m^2 c^4}$$

$$\boxed{T = \frac{2 m_e c^4 p_i^2 \cos^2\theta}{\left(m_e c^2 + \sqrt{p_i^2 c^2 + m^2 c^4}\right)^2 - p_i^2 c^2 \cos^2\theta}}$$

Two Body Scattering: Maximum Energy Transfer

- Maximum energy transfer occurs in head-on collision: $\cos\theta = 1$

$$T_{max} = \frac{p_i^2 c^2}{\frac{1}{2} m_e c^2 + \frac{1}{2} \frac{m^2}{m_e} c^2 + \sqrt{p_i^2 c^2 + m^2 c^4}}$$

- taking into account that $E_i = m\gamma c^2 = \sqrt{p_i^2 c^2 + m^2 c^4}$

$$T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{m} + \left(\frac{m_e}{m}\right)^2}$$

$$T_{max} \approx \begin{cases} 2 m_e c^2 (\gamma\beta)^2 & \text{for } \gamma m_e << m \\ \gamma m c^2 = E & \text{for } \gamma \to \infty \\ m_e c^2 (\gamma - 1) = E - m_e c^2 & \text{for } m = m_e \end{cases}$$

Two Body Scattering: Maximum Energy Transfer

- Let's consider the case $m_e << m$: collision of a heavy particle with an orbital electron
  - In case of very high energy $p >> \frac{m^2}{m_e} c$

$$T_{max} \simeq p_i c \simeq E_i$$

$\frac{m^2}{m_e} c$=36 GeV/c for pions

=1.7 TeV/c for protons

This means that a massive particle (i.e. proton) can be stopped by an electron.

  - For lower energies: $p << \frac{m^2}{m_e} c$

$$T_{max} \simeq 2 m_e c^2 \beta^2 \gamma^2$$

- In the case that $m = m_e$ we have:

$$T_{max} = m_e c^2 (\gamma - 1) = E - m_e c^2$$